Ratios and Proportional Relationships

In seventh grade, students extend their understanding of ratios and develop understanding of proportionality to solve a variety of single- and multi-step ratio and percent problems. A **ratio** is a comparison of two numbers or quantities through division and is typically expressed in simplest fraction form (e.g., the ratio of boys to girls in a class is 10 to 15, which would be expressed as 2 to 3). A **proportion** is an equation setting two ratios equal. Blueprint drawings and the actual structures they represent look alike because they are proportionate to one another (e.g., a scale of $\frac{1}{4}$ inch on the drawing to 1 foot of actual length is expressed as $\frac{1}{4}$ = 1′0″). A **proportionality** is a relationship in which the ratio between two quantities does not vary—the ratio remains constant.

At this grade level, students also compute **unit rates**. A *rate* is a ratio that compares two different kinds of quantities—for example, miles per hour or dollars per pound. The rate "miles per hour" gives distance traveled per unit of time, and problems using this type of unit rate can be solved using proportions. Students will graph proportional relationships and understand the unit rate informally as a measure of the steepness of a related line—called *slope*. Students distinguish proportional relationships from other relationships.

Additionally, students use their understanding of ratios and proportionality to solve a variety of percent problems. They will solve problems that include simple interest, taxes, tips, commissions, mark-ups, discounts, and percent increases or decreases.

The Grade 7 Common Core State Standards for Ratios and Proportional Relationships specify that students should—

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The following hands-on activities will help students explore the concepts of ratios and proportional relationships in a meaningful way. The concrete experiences that the activities provide will strengthen students' ability to recognize and work flexibly with these concepts.

Ratios and Proportional Relationships

Contents

LESSON 1

Ratios and Proportional Relationships

Proportional Relationships I

Objective

Determine whether a relationship is proportional by checking for a straight-line graph.

Common Core State Standards

- **7.RP.2a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Proportional relationships can be shown arithmetically, graphically, or algebraically. Students at this level will determine if two quantities are in a proportional relationship by graphing their values to see if they form a straight line through the origin. In the future, they will use the equation of a line, $y = mx + b$, to express a proportional relationship.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What does the* x*-axis represent?* (time in minutes) *What does the y-axis represent?* (distance in kilometers) *What is the scale on each axis?* (x-axis: one space equals 10 minutes; y-axis: one space equals one kilometer)
- **Say:** *Two points always define a line. Some lines represent proportional relationships and some do not. A line that represents a proportional relationship goes through the origin.*
- **Ask:** *Does the line go through the origin?*

Solve It

Reread the problem with the students. Have students set up a graph and plot the points on the Centimeter Grid (BLM 1). Ask them to use a ruler to draw a line through the points and show that the line goes through the origin.

More Ideas

For other ways to teach about proportional relationships and straight-line graphs—

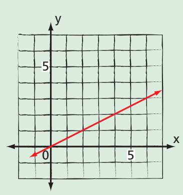
- Have students work a similar problem that starts with just one point: that a mackerel swims 22 km in 2 hours. Ask them to make a line showing other points that are proportional using the XY Coordinate Pegboard or the Centimeter Grid (BLM 1). Then have them list the points that form a proportional relationship.
- Have students start by making a straight line on their XY Coordinate Pegboard that will pass through the origin. Have them describe all of the points that make the proportional relationship of the line.

Formative Assessment

Have students try the following problem.

Which graph represents a proportional relationship?

A.



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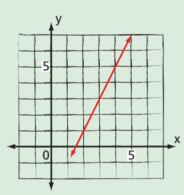

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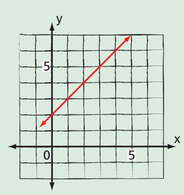

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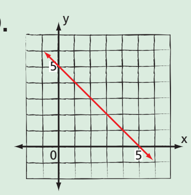
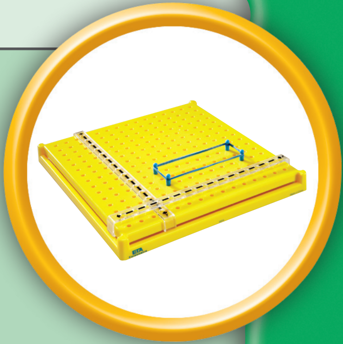




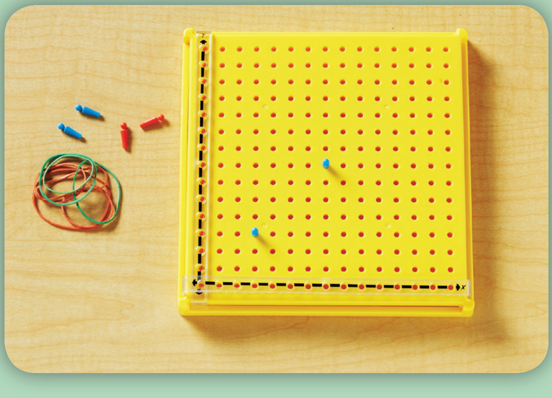
Try It! 20 minutes | Pairs

Here is a problem about proportional relationships.

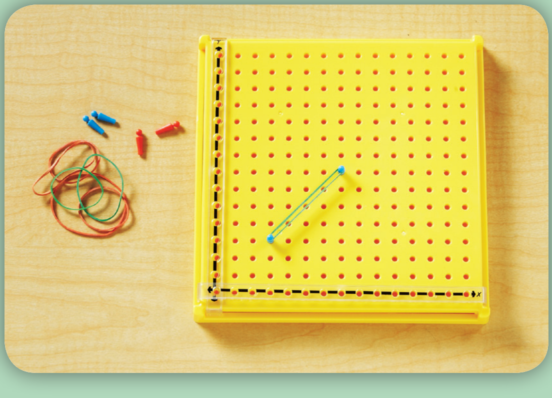
A herring swims 3 kilometers in 30 minutes. Another day, the herring swims 7 kilometers in 70 minutes. Is this a proportional relationship?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

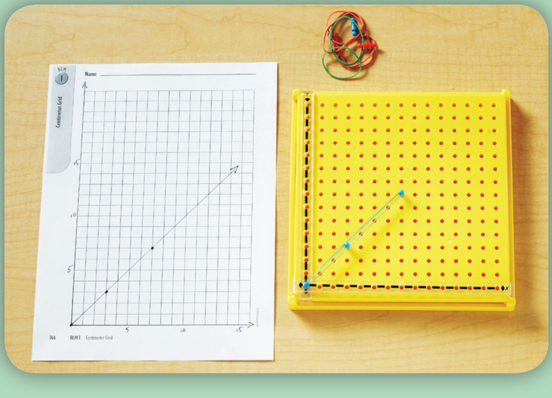
Materials
- XY Coordinate Pegboard
- BLM 1
- ruler



1. Have students set the axes to show the first quadrant of a coordinate plane. **Say:** *The x-axis will represent time in minutes, and we'll have 1 space equal 10 minutes. The y-axis will represent distance, and we'll have 1 space equal 1 kilometer.* Then have students plot the points (30, 3) and (70, 7) on their XY Coordinate Pegboards.



2. Ask students to connect the points with a rubber band to make a line segment on the pegboard. Remind students that any two points define a line.



3. **Ask:** *Where is the origin?* Elicit that the origin is (0, 0). Encourage students to extend their lines to the origin. **Ask:** *Are all three points on a straight line?* Explain that since the points (30, 3) and (70, 7) lie on a straight line that goes through the origin, they express a proportional relationship.

Look Out!

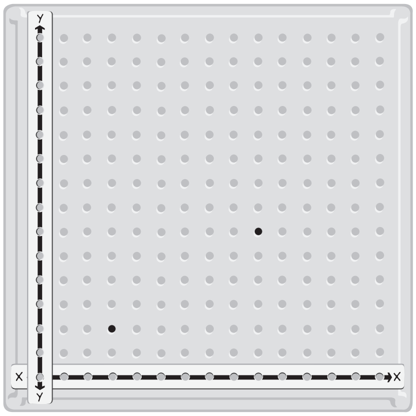
Students might think that any straight line indicates a proportional relationship. Emphasize that the line must go through the origin for the data to express a proportional relationship.

Answer Key

Use an XY Coordinate Pegboard. Complete the model to answer the question.

(Check students' work.)

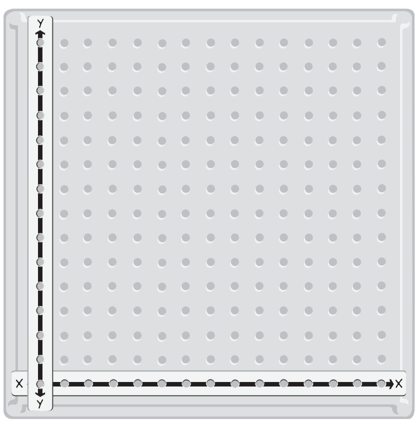
1. You can buy 3 pounds of bananas for $2 or 9 pounds for $6.
Is the relationship proportional?



Yes

Using an XY Coordinate Pegboard, model the problem. Draw the model and use it to answer the question.

2. In a bag, there are 4 red balls and 6 blue balls. In a second bag, there are 12 red balls and 8 blue balls. Is the relationship proportional?



No

Use Centimeter Grid Paper to determine if the relationship is proportional.

3. A baseball player got 14 hits in 35 turns at bat and 32 hits in 80 turns.
Is the relationship proportional?

Yes

4. If 20 people are ahead of you in the lunch line, it takes 12 minutes to get your lunch.
If 30 people are ahead of you, it takes 18 minutes. Is the relationship proportional?

Yes

5. It rained 15 times in 40 days, and it rained 45 times in 100 days.
Is the relationship proportional?

No

Answer Key

Challenge! Explain how you make a graph to determine if some data are in a proportional relationship. Make up an example of a proportional relationship.

Challenge: Graph each data point on a coordinate grid. Draw a line through the points. If the straight line goes through the origin, then the relationship is proportional.

LESSON

2

Objective

Determine whether a relationship is proportional by checking for equivalent ratios.

Common Core State Standards

- **7.RP.2a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Proportional Relationships II

A proportion is an equation that sets two ratios equal to each other. If that equation is true, then the relationship is proportional. Students have checked to see if relationships are proportional by graphing, but now they will check by using their skills with equivalent fractions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What was the first ratio, or fraction, you built?* (2:8, or $\frac{2}{8}$) *What does the ratio represent?* (the number of green apples to the total number of apples) *What was the second ratio, or fraction, you built?* (1:4, or $\frac{1}{4}$) *What does the ratio represent?* (the number of green apples to the total number of apples)
- **Ask:** *How can you tell if the two ratios are equivalent?* (you can build the fractions and compare or simplify)
- **Ask:** *Is this a proportional relationship? Why or why not?* (yes; the ratios are equivalent)

Solve It

Reread the problem with the students. Have students build the two ratios and draw them on the Fraction Squares BLM. Ask students to explain whether the ratios are equivalent and write an equation to represent the equivalent ratios. (2:8 = 1:4, or $\frac{2}{8} = \frac{1}{4}$)

More Ideas

For other ways to teach about proportional relationships and equivalent ratios—

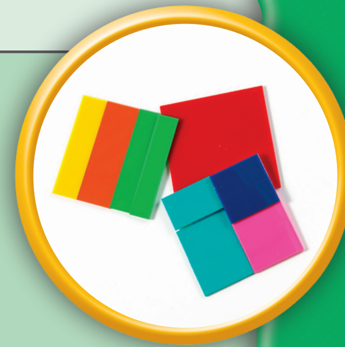
- Have students use Fraction Tower® Equivalency Cubes to build each ratio. Then, they can compare the heights of the towers to see if they are equivalent and therefore represent a proportional relationship.
- Have students make the fractions using Deluxe Rainbow Fraction® Circles. They can measure the fractions with Rainbow Fraction Circle Rings or compare by stacking to determine whether the fractions are equivalent and therefore represent a proportional relationship.

Formative Assessment

Have students try the following problem.

In Rob's group, there are 2 boys and 3 girls. In Caren's group, there are 4 boys and 6 girls. Which equation shows that the groups are proportional?

A. $\frac{2}{6} = \frac{1}{3}$ **B.** $\frac{2}{5} = \frac{4}{10}$ **C.** $\frac{2}{4} = \frac{3}{6}$ **D.** $\frac{2}{3} = \frac{4}{6}$



Try It! 15 minutes | Groups of 4

Here is a problem about proportional relationships.

In a bag of 8 apples, 2 of the apples are green. In a bag of 4 apples, 1 is green. Is this a proportional relationship?

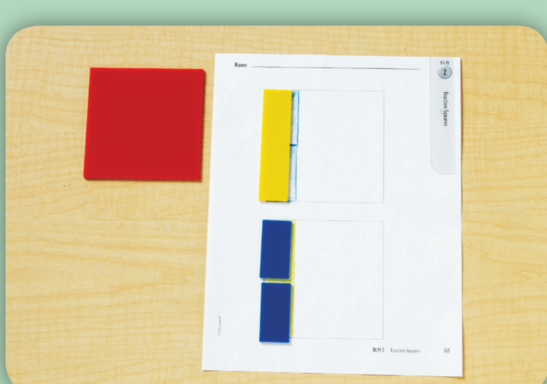
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

- Deluxe Rainbow Fraction® Squares
- BLM 2

1. Ask: *In the first bag, what is the ratio of green apples to all apples?* Have students use Fraction Squares to represent the ratio 2:8, or $\frac{2}{8}$, on their Fraction Squares BLM.

2. Ask: *In the second bag, what is the ratio of green apples to all apples?* Have students use Fraction Squares to represent the ratio 1:4, or $\frac{1}{4}$, on their Fraction Squares BLM.



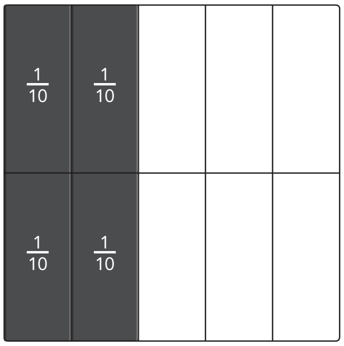
3. Ask: *How can you tell if these ratios are equivalent?* Encourage students to stack the fractions to show they are equivalent. Explain that since the ratios are equivalent, the relationship is proportional.

Look Out!

Students might orient the Fraction Squares differently and therefore think they are not equivalent. Encourage students to align the Fraction Squares pieces vertically and start in the top left corner each time.

Use Fraction Squares. Complete the model to answer the question. (Check students' work.)

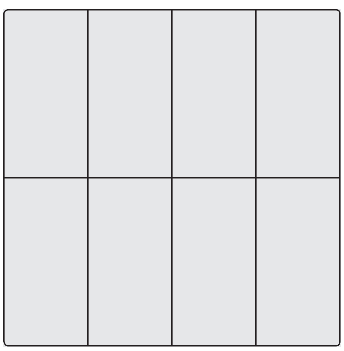
1. In the teacher's pencil jar, there are 10 pencils, 4 of which do not have an eraser. In Julio's pencil bag, there are 5 pencils, 3 of which do not have an eraser. Is the relationship proportional?



No

Using Fraction Squares, model the problem. Draw the model and use it to answer the question.

2. In a bag of 4 instruments, 2 instruments are shakers. In a box of 8 instruments, 4 are shakers. Is the relationship proportional?



Yes

Use Fraction Squares to determine if the relationship is proportional.

3. In PE, 5 of every 6 girls finished a run in less than 10 minutes. Two of every 3 boys finished in less than 10 minutes. Is the relationship proportional?

No

4. Sal paid \$2 for 4 pounds of grapes. Bo paid \$1 for 2 pounds of grapes. Is the relationship proportional?

Yes

Use equivalent ratios to determine if the relationship is proportional.

5. Roberto can ride his bike 4 miles in 20 minutes. Patricia can ride her bike 10 miles in 50 minutes. Is the relationship proportional?

Yes

6. Pearla answered 4 of the 5 questions right on the quiz. Then, on the test, she answered 15 of the 20 questions right. Is the relationship proportional?

No

Answer Key

Challenge! Ms. Turny's class ratio of boys to girls is 2:3, and Mr. Straight's class ratio of boys to girls is 8:12. Explain how you know if the data portrays a proportional relationship.

Challenge: If the data forms equivalent ratios, then the relationship is proportional. Since 2:3 = 8:12, the data does portray a proportional relationship.

LESSON 3

Objective

Identify the constant of proportionality for a proportional relationship.

Common Core State Standards

- **7.RP.2b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **7.RP.2d** Explain what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate.

Constant of Proportionality

A relationship is proportional if its graph is a straight line through the origin. The slope of this line is the constant of proportionality, or unit rate, for the relationship. The unit rate is also revealed on the line by the point (1, *r*), where *r* is the unit rate. Graphing a proportional relationship is the most powerful way to help students visualize what the constant of proportionality represents.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *The problem says the number of blocks is proportional to the number of minutes, so the number of blocks goes on the* y*-axis and time goes on the* x*-axis.*
- Explain that the calculation of the constant of proportionality will produce the same result regardless of the two points chosen from the line. **Ask:** *Which point on the line will give you the constant of proportionality directly, without having to calculate?* Elicit that the answer is the point whose *x*-coordinate is 1. That point is (1, *r*), where *r* is the unit rate, or constant of proportionality.

Solve It

Reread the problem with the students. Have students show the graph on grid paper. Ask them to find the vertical distance and the horizontal distance between the two given points and divide to find the constant of proportionality. Help them recognize that the constant is the *y*-value that the line crosses at $x = 1$.

More Ideas

For another way to teach about the constant of proportionality—

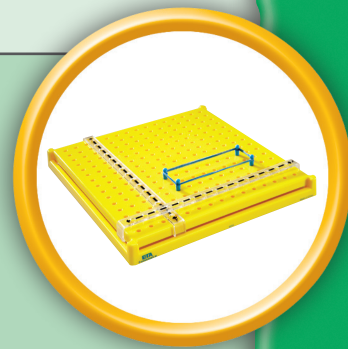
- Have students use Fraction Tower® Equivalency Cubes to solve the problem. Start by asking students to build a whole tenths tower and a whole fifths tower. Have students lay down the fifths tower horizontally and then lay down the tenths tower horizontally immediately below it. Tell students the towers represent the ratio 5:10 (5 blocks in 10 minutes) because there are 5 fifths and 10 tenths. Next have students break the towers to show the ratio 2:4, then 1:2. Elicit from students that to find the unit rate, they need to find the number of green cubes for each purple cube. Help students see that they need $\frac{1}{2}$ of a green cube for each purple cube and that this $\frac{1}{2}$ is the unit rate, or constant of proportionality in the problem.

Formative Assessment

Have students try the following problem.

Teo has 3 trophies and 6 medals. Brandi has 5 trophies and 10 medals. If the number of medals is proportional to the number of trophies, which point will be on the graph of this relationship?

A. (0, 1) **B.** (1, 2) **C.** (4, 7) **D.** (12, 5)



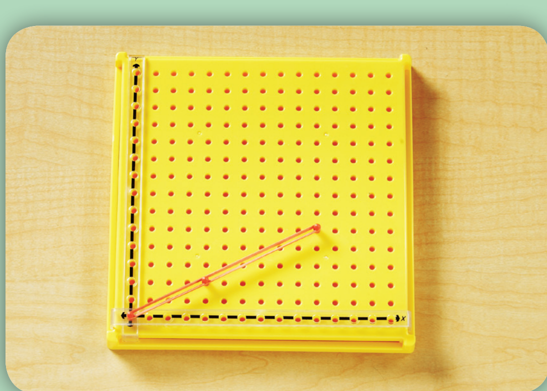
Try It! 15 minutes | Pairs

Here is a problem about proportional relationships and the constant of proportionality.

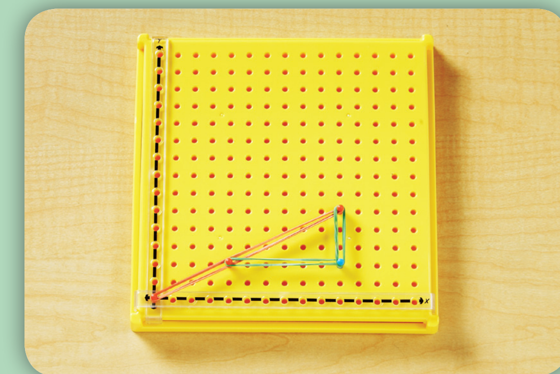
Liam took his dog for a walk. In 4 minutes, he had walked 2 blocks. In 10 minutes, he had walked 5 blocks. If the number of blocks is proportional to the number of minutes, what is the constant of proportionality for the relationship?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

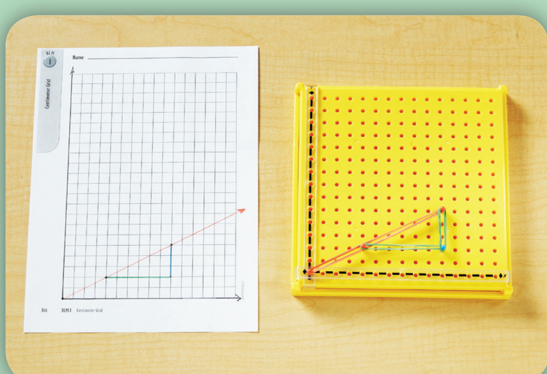
Materials
- XY Coordinate Pegboard
- BLM 1



1. Say: *Let's graph the relationship on the pegboard. Put distance on the y-axis and time on the x-axis.* **Ask:** *What two points will you graph?* Have students place pegs at (4, 2) and (10, 5) and connect the points with a rubber band. **Ask:** *Can you confirm that the relationship is proportional?* Ask students to extend their bands to the origin. Elicit that the relationship is proportional.



2. Ask: *What is the horizontal distance between the two points?* Have students use a rubber band and peg to mark the horizontal distance between (4, 2) and (10, 5). **Ask:** *What is the vertical distance between the two points?* Have students use another band to mark the vertical distance between (4, 2) and (10, 5).



3. Say: *Divide the vertical distance by the horizontal distance. The quotient is called the* constant of proportionality. *It is also called* the unit rate, *because it tells how much the y-coordinate changes for each unit of change in the x-coordinate.* Have students confirm the unit rate by inspecting the graph.

⚠ Look Out!

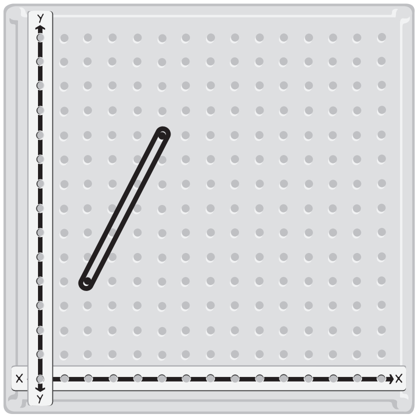
Students might have difficulty finding the vertical and horizontal distances between points. Point out that the rubber bands that represent the distances form a right triangle with the line.

Answer Key

Use an XY Coordinate Pegboard. Build the model and use it to answer the question.

(Check students' work.)

1. For every 2 apples in Kali's orchard, there are 4 pears. In Sam's orchard, there are 10 pears for every 5 apples. If the number of pears is proportional to the number of apples, what is the constant of proportionality?



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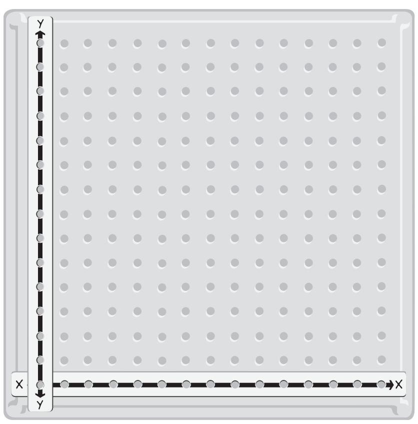
Using an XY Coordinate Pegboard, model the problem. Draw the model and use it to answer the question.

2. Yesterday, Maria had 4 nickels and 3 dimes in her wallet. Today, she has 12 nickels and 9 dimes. If the number of dimes is proportional to the number of nickels, what is the constant of proportionality?



$\frac{3}{4}$

Use Centimeter Grid Paper to find the constant of proportionality for the relationship.

3. Carson can jump 30 times in 18 seconds. He can jump 20 times in 12 seconds. Assume that the number of times Carson can jump is proportional to the number of seconds he is given.

$\frac{5}{3}$

4. Cleo bought 36 stamps and paid $12. Ned paid $6 for 18 stamps. Assume the amount paid for stamps is proportional to the number of stamps purchased.

$\frac{1}{3}$

5. Sheila made 14 three-point shots in 35 attempts. She made 32 three-point shots in 80 attempts. Assume the number of three-point shots Sheila makes is proportional to her number of attempts.

$\frac{2}{5}$

Answer Key

Challenge! Caitlyn read 2 books in 8 days. She read 4 books in 16 days. Assume that the number of books Caitlyn reads is proportional to the number of days she spends reading. Find the constant of proportionality for the relationship. Identify the point on a graph of the relationship that directly names the value of the constant. Explain why that point works.

Challenge: The two points represented in the problem are (8, 2) and (16, 4). The vertical distance between them is 4 – 2 = 2. The horizontal distance is 16 – 8 = 8. The constant of proportionality is $\frac{1}{4}$. The point that directly names the constant is $(1, \frac{1}{4})$. This is so because the constant of proportionality is the change in *y* for each unit change in *x*. The point $(1, \frac{1}{4})$ represents a 1-unit change in *x* relative to the point (0, 0). The vertical change is $\frac{1}{4} - 0 = \frac{1}{4}$ and the horizontal change is 1 – 0 = 1, so the *y*-coordinate of the point $(1, \frac{1}{4})$ literally names the constant of proportionality.

LESSON 4

Equations of Proportional Relationships

Objective

Develop a set of coordinate pairs for a proportional relationship; write and graph an equation for the relationship.

Common Core State Standards

- **7.RP.2b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **7.RP.2c** Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

In previous lessons, students have learned about proportional relationships and have learned to find the constant of proportionality for a relationship using coordinate pairs. These coordinate pairs are related linearly, meaning that the graph of their relationship forms a straight line. In this activity, students will use a set of related coordinate pairs to graph and write the equation for a proportional relationship.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Why aren't we pegging the number of people that Kelly meets each day?*
- **Ask:** *How do you know the graph shows a proportional relationship? Could we have known this without graphing the coordinates? Explain.*
- **Ask:** *What is the constant of proportionality?*

Solve It

Ask students to explain why Kelly's plan to meet three new people every two days is a proportional relationship. Encourage them to use their graph to explain their answer. Have students write the equation for the relationship.

More Ideas

For another way to teach about proportional relationships and equations—

- Give students a related set of ordered pairs. Have students use their XY Coordinate Pegboard to determine whether the set of ordered pairs defines a proportional relationship. Have them use the graph to explain their answer.

Formative Assessment

Have students try the following problem.

Which of the following lists of values shows a proportional relationship?

A.

x	*y*
1	1
2	4
3	9
4	16
5	25

B.

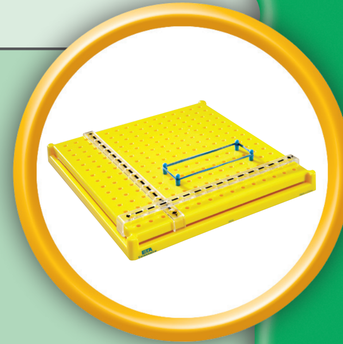
x	*y*
4	2
8	4
12	6
16	4
24	2

C.

x	*y*
0	–5
1	–4
2	–3
3	–4
4	–1

D.

x	*y*
0	0
3	6
6	12
9	18
12	24



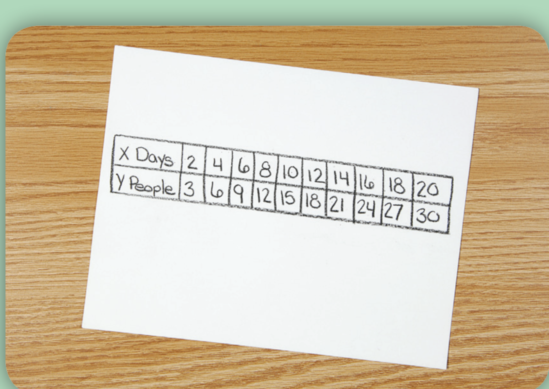
Try It! 20 minutes | Pairs

Here is a problem about proportional relationships and equations.

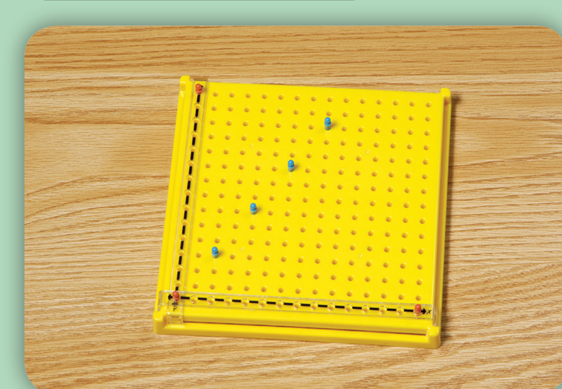
Kelly loves to meet new people. When she moved to a new school, she decided to meet three new people every two days. How many people will she have met after 10 days? After 16 days? Write an equation for the number of people Kelly will have met after x days.

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

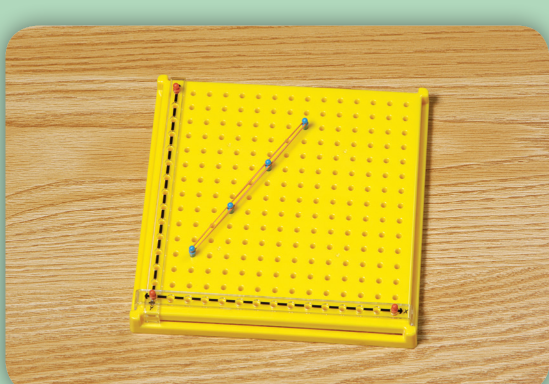
Materials
- XY Coordinate Pegboard
- paper



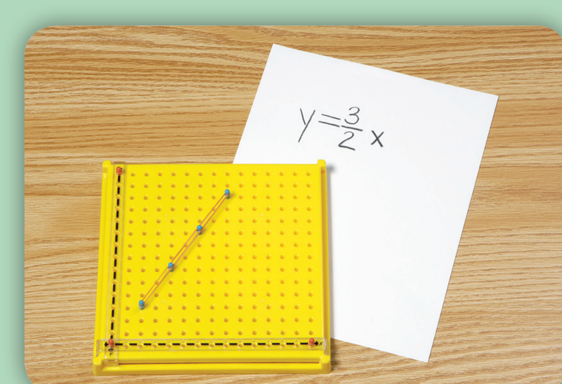
1. Have students set up a table of values showing each day and the number of new people Kelly has met by the end of that day. Have them start at $x = 2$ and continue through $x = 20$.



2. Have students peg the first few sets of coordinate pairs from their table of values.



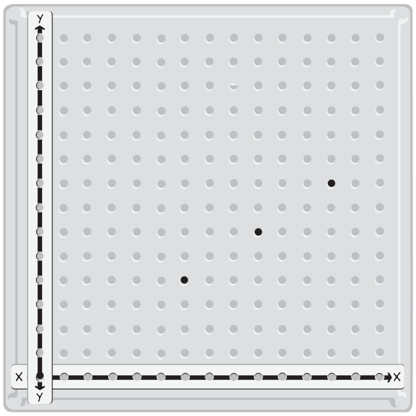
3. Have students find the constant of proportionality. Tell them to look for a pattern. **Ask:** *What is the rule for moving from one point on the graph to the next?* Elicit from them that the pattern is “up 3, over 2.”



4. Have students write their solutions on a sheet of paper. Their formulas should be in the form $y = mx$.

Use an XY Coordinate Pegboard to plot the points shown. Make a table of ordered pairs. Graph the line. Write an equation. (Check students' work.)

1.

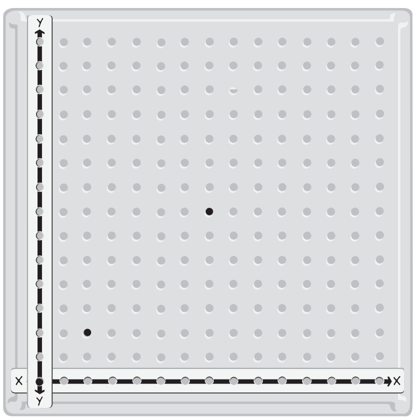


x	y
0	0
6	4
9	6
12	8
15	10
18	12

$y = \frac{2}{3}x$

Using an XY Coordinate Pegboard, graph the line that passes through the points given on the grid. Sketch the line. Make a table of ordered pairs. Write an equation.

2.

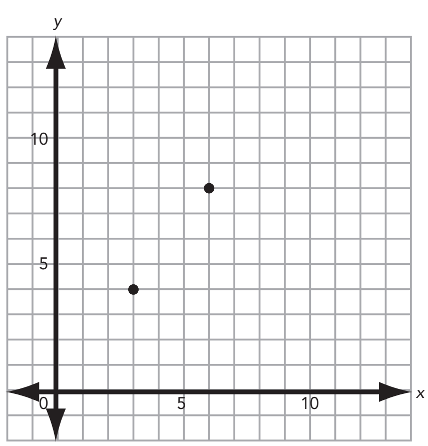


x	y
0	0
2	2
4	4
7	7
9	9
11	11

$y = x$

Graph a line that passes through the given points. Make a table of ordered pairs. Write an equation.

3.



x	y
0	0
3	4
6	8
9	12
12	16
15	20

$y = \frac{4}{3}x$

Answer Key

Challenge! How many points must you have to make a line? Why is it good to have three points to make a line?

Challenge: (Sample) Two points make a line. When you graph three points on a line, it is a good way to check that the first two points are graphed accurately.