Statistics and Probability

We use statistics and probability often in our daily lives. When we hear about the "average" price per gallon of gasoline or the "chance" of snowfall, we are dealing with statistics and probability. **Statistics** is the study of how data is collected, summarized, and presented. **Probability** is the likelihood that something will happen. Together, statistics and probability are used with data to draw conclusions, and/or make predictions.

Probability is expressed mathematically as a number between 0 and 1 (e.g., a chance of rainfall of $\frac{7}{10}$, or 0.7, or 70%). A probability near 0 means an event is unlikely, a probability of $\frac{1}{2}$ means the event is neither likely nor unlikely, and a probability near 1 means the event is likely to occur. Probabilities are useful for predicting what may happen in the long term, such as realizing trends in sales, human populations, and weather.

At this level, students build on their previous work with single data distributions and address differences between populations–that is, between two data sets. Students recognize that it is difficult to gather statistics on an entire population but that a random sample can be representative of the total population and will generate valid results.

Additionally, students develop probability models and use them to find probabilities of future events (such as finding the probability that a spinner will land on a certain spot or that a penny will be tossed as a "heads" or as a "tails"). Students also find probabilities of compound events. A **compound event** consists of two or more **simple events**. Tossing a die is a simple event. Tossing two dice is a compound event.

The Grade 7 Common Core State Standards for Statistics and Probability specify that students should–

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

The following hands-on activities will enable teachers to help students develop statistics and probability skills and concepts in a meaningful way. The activities will help students gain an appreciation for how mathematics can be used to model the real world.

Statistics and Probability

Contents

LESSON

1

Objective

Estimate the size of a subgroup by sampling the larger population.

Common Core State Standards

- **7.SP.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- **7.SP.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Population Sampling

The media often present data from polls and sample populations—data intended to help sway or justify a variety of actions or positions. It is important for students to understand the processes associated with population sampling and the mathematics involved in analyzing the data. In this activity, students will simulate a survey of a wildlife population by collecting and analyzing a series of representative samples.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What does the container of colored cubes represent? What does it mean to "take a sample"? When you take a sample, what data do you need to record?*
- Have the various groups of students report their findings. Then reveal that there are 165 frogs in the pond and that 15 (9.09%) of them have mutations. Have students discuss how different groups of researchers might get different results, even though everyone followed the same directions.

Solve It

Reread the problem with students. Ask them to write to Chen explaining their procedure for answering his question. Have them include their results.

More Ideas

For other ways to teach about population sampling—

- Encourage students to look for examples of the use of population sampling (wildlife studies, election polls, and so forth) in the media. Whenever possible, discuss the sample size and number of samples taken in relation to the size of the total population.
- Have students repeat the exercise. This time, however, they should take 10 samples instead of four. Are the results the same? **Ask:** *Do you think your results are likely to be more accurate this time, less accurate, or about the same? Explain.*

Formative Assessment

Have students try the following problem.

In a sampling of 63 squirrels observed in a forest preserve, 9 were red squirrels. If the total squirrel population is estimated to be 550, approximately how many of them are red squirrels?

A. 14 **B.** 80 **C.** 470 **D.** 540

Try It!

40 minutes | Groups of 4

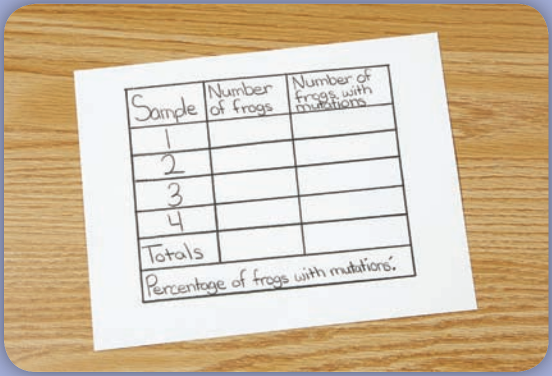
Here is a problem about population sampling.

Chen has noticed that some of the frogs in a nearby pond have malformed legs. He wants to know what percentage of the frog population has these mutations, but he realizes that he can't catch and examine every frog. How can Chen determine the percentage of mutant frogs in the pond without counting every frog?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

- Centimeter Cubes (165 cubes, 15 of which are red)
- $\frac{1}{4}$-cup measuring cup



Sample	Number of frogs	Number of frogs with mutations
1		
2		
3		
4		
Totals		

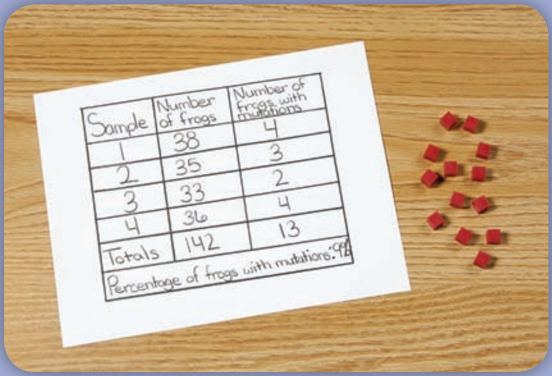
Percentage of frogs with mutations:

1. Explain to students that they are going to simulate a sampling process. Tell them that the red cubes will represent the mutant frogs. Have students set up a table for recording their data. The table should accommodate four samples.

Sample	Number of frogs	Number of frogs with mutations
1	38	4
2		
3		
4		
Totals		

Percentage of frogs with mutations:

2. Have students scoop out a sample of Centimeter Cubes with the measuring cup, count them, and record the total number of cubes and the number of red cubes in the sample. Then have them return the cubes to the container and mix them together. (You may explain that this is the equivalent of "catch and release.") Have students repeat the process three more times.



Sample	Number of frogs	Number of frogs with mutations
1	38	4
2	35	3
3	33	2
4	36	4
Totals	142	13

Percentage of frogs with mutations: 9%

3. Have students calculate the total number of frogs captured and the total number of mutant frogs in all four samples. **Ask:** *According to your research, what percentage of frogs in the pond has mutations?*

⚠ Look Out!

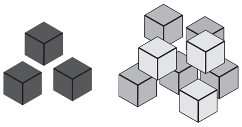
Remind students as necessary to return the sample to the container and mix the cubes thoroughly before taking another sample.

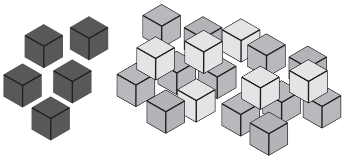
Some students will report mutation rates of more or less than 9 percent. Have them confirm that they followed the procedure and that their calculations are correct. Reassure them that a sampling is not a literal count of every individual in the population and that samples will vary.

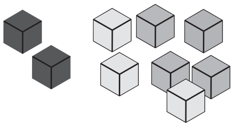
Use Centimeter Cubes to represent votes from a subgroup of a larger population. In the sample shown, the red cubes are modeled by the dark cubes and represent a *yes* vote. Record your results.

(Check students' work.)

1. Three samples are shown. Complete the table.







Sample	Number of Votes	Number of *Yes* Votes	Number of *No* Votes
1	11	3	8
2	23	5	18
3	9	2	7

How many votes were cast based on the samples? 43

How many votes were *yes*? 10

What percent of the votes cast were *yes* votes? 23.3%

Using Centimeter Cubes, represent votes from a subgroup of a larger population. Take three samples from a large pile of cubes. Choose a color to represent *yes* votes. Record your results.

2.

Sample	Number of Votes	Number of *Yes* Votes	Number of *No* Votes
1		Answers will vary.	
2			
3			

How many votes were cast based on the samples? ________

How many votes were *yes*? ______

What percent of the votes cast were *yes* votes? ______________

3.

Sample	Number of Votes	Number of *Yes* Votes	Number of *No* Votes
1		Answers will vary.	
2			
3			

How many votes were cast based on the samples? ________

How many votes were *yes*? ______

What percent of the votes cast were *yes* votes? ______________

Answer Key

Challenge! Why it is necessary to use a sample when seeking results from a large poplulation?

Challenge: (Sample) A population is often too large to test or survey. A sample can be tested or surveyed and the results can be used to generalize the results of the larger population.

LESSON

2

Objective

Build a spinner that models a set of probabilities.

Common Core State Standards

- **7.SP.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Modeling Probability: Building Spinners

A spinner is a convenient way to model probability—the likelihood that a specified event will occur. Probability can be expressed as a fraction, decimal, or percent. In this lesson, students will build spinners to model probabilities expressed as fractions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Discuss with students the fractions they were given in the problem and have students tell you what the fractions represent.
- Have students write an equation for each spinner showing the sum of the probabilities. Elicit that the sum is always 1. **Ask:** *Can a probability ever be greater than 1?* Elicit that probability is always between 0 and 1. Explain that a value near 0 means an event is unlikely, that a value near 1 means the event is likely, and that a value near $\frac{1}{2}$ means the event is neither unlikely nor likely.

Solve It

Reread the problem with students. Have them use unit fractions or a combination of unit fractions to create the spinners as described. Have students use the rings to check the accuracy of the spinners and to help them draw the spinners on a piece of paper.

More Ideas

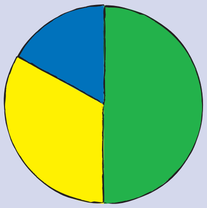
For other ways to teach about modeling probability—

- Have students extend this activity by creating a spinner that represents the following probabilities: $P(\text{color } 1) = \frac{1}{10}$; $P(\text{color } 2) = \frac{2}{5}$; $P(\text{color } 3) = \frac{1}{5}$; $P(\text{color } 4) = \frac{3}{10}$.
- Have students design a number cube with the following probabilities: $P(1) = \frac{1}{6}$; $P(2) = \frac{1}{2}$; $P(3) = \frac{1}{3}$.

Formative Assessment

Have students try the following problem.

Which of the following probabilities is correct for the spinner shown here?



A. $P(\text{green}) = \frac{1}{3}$ **B.** $P(\text{yellow}) = \frac{1}{2}$

C. $P(\text{yellow}) = \frac{1}{6}$ **D.** $P(\text{green}) = \frac{1}{2}$

Try It!

30 minutes | Groups of 4

Here is a problem about building spinners to model probabilities.

Jake found an old board game in the attic that he'd like to play. The problem is that all of the spinners for the game are missing. Luckily, he found a description of them in the directions. He needs to replace the three spinners described here:

Spinner 1: P(color 1) = $\frac{1}{3}$*; P(color 2) =* $\frac{1}{2}$*; P(color 3) =* $\frac{1}{6}$

Spinner 2: P(color 1) = $\frac{5}{12}$*; P(color 2) =* $\frac{1}{4}$*; P(color 3) =* $\frac{1}{3}$

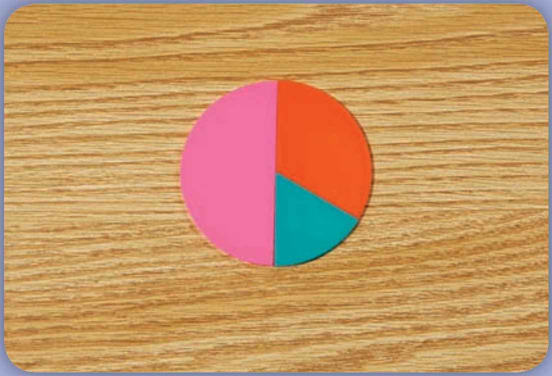
Spinner 3: P(color 1) = $\frac{3}{10}$*; P(color 2) =* $\frac{1}{5}$*; P(color 3) =* $\frac{1}{2}$

Help Jake accurately re-create the spinners he will need.

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

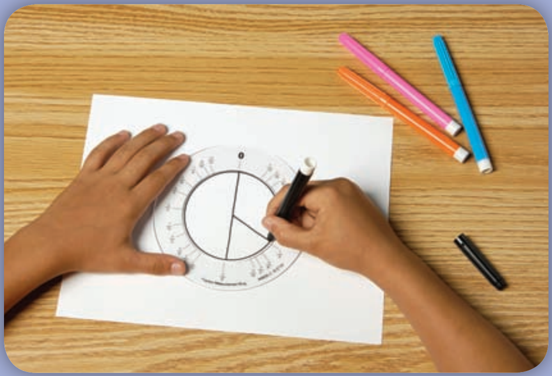
Materials

- Deluxe Rainbow Fraction® Circles
- Rainbow Fraction Circle Rings
- drawing compass
- colored markers



1. Have students build Spinner 1, using the appropriate Fraction Circles to represent each probability. **Ask:** *What color fraction piece will you use to represent* $\frac{1}{3}$*?*

2. Next have students build Spinners 2 and 3, using the appropriate Fraction Circles. **Ask:** *How will you represent* $\frac{5}{12}$*?*



3. Have students use the Measurement Ring to confirm that the spinner sections represent the correct probabilities. Have them use the ring as an aid to draw the spinners. Students may use markers to color in the sections. Students should label each section with the appropriate fraction.

Look Out!

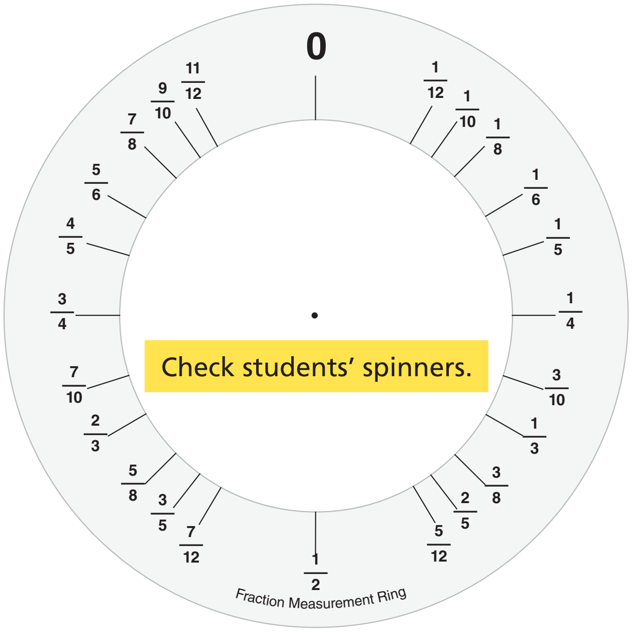
Some students might struggle to associate the value of a probability with the idea of an event being likely, unlikely, or neither. Use Fraction Towers with a 0–1 number line to demonstrate the concept.

Use Fraction Circles and the Measurement Ring to make a spinner for the probabilities given.

(Check students' work.)

1.

Color	Probability
Black	$\frac{1}{12}$
Gray	$\frac{2}{3}$
White	$\frac{1}{4}$

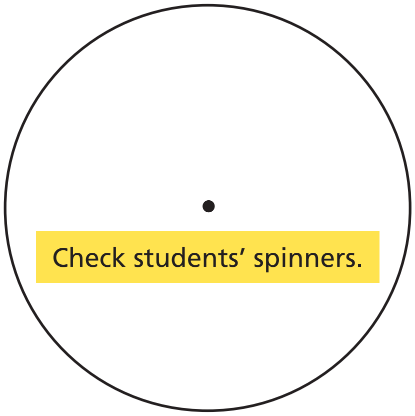


Check students' spinners.

Using Fraction Circles and the Measurement Ring, make a spinner for the probabilities given.

2.

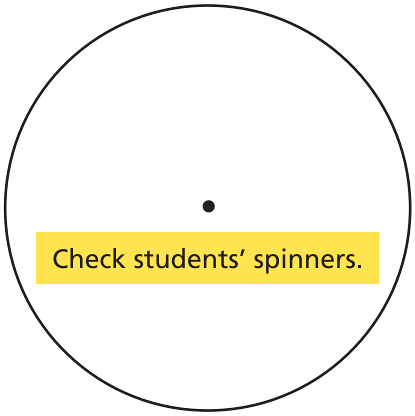
Pattern	Probability
dotted	$\frac{1}{4}$
striped	$\frac{1}{8}$
clear	$\frac{3}{8}$
solid	$\frac{1}{4}$



Check students' spinners.

3.

Letter	Probability
A	$\frac{3}{10}$
B	$\frac{2}{5}$
C	$\frac{1}{10}$
D	$\frac{1}{5}$



Check students' spinners.

Answer Key

Challenge! When making a spinner showing certain probabilities, what must the sum of the probabilities equal? Explain. Draw a picture to help.

Challenge: (Sample) The probabilities must have a sum of 1 because together they represent the whole circle.

LESSON

3

Objective

Find the theoretical and experimental probabilities of an event involving a spinner.

Common Core State Standards

- **7.SP.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- **7.SP.7a** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

Theoretical and Experimental Probability with Spinners

Students learn that they can use theoretical probability to predict the results of an experiment and that it may or may not be the same as the experimental probability. Here they use spinners to learn how to distinguish and compare theoretical and experimental probability and how to express the probability as a fraction, decimal, or percent.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Discuss that the likelihood that an event will occur is indicated by a number from 0 to 1. Zero means the event is impossible, and 1 means the event is certain.
- **Ask:** *Given the theoretical probability of the spinner landing on a number less than 4, how many times would you expect the spinner to land on one of these numbers in 10 spins? 30 spins? 50 spins?*
- **Ask:** *As you run more trials, what do you notice about the theoretical and experimental probabilities?*
- **Ask:** *Who do you think will earn more points, Thom or Maya? Why?*

Solve It

Reread the problem with students. Ask them to find the theoretical and experimental probabilities of Thom getting a point, and to use their experiment to compare and contrast theoretical and experimental probability.

More Ideas

For other ways to teach about theoretical and experimental probabilities—

- Give polyhedral dice to pairs of students. Have one student find the experimental probability of rolling a prime number on 10, 20, and 30 rolls while the other finds and uses the theoretical probability to determine the expected results.

Formative Assessment

Have students try the following problem.

A spinner is numbered 1 through 12. Lauren spins it 15 times and it lands on a number greater than 3 ten times. What is the theoretical probability that the spinner lands on a number greater than 3?

A. $\frac{1}{5}$ **B.** $\frac{1}{4}$ **C.** $\frac{3}{4}$ **D.** $\frac{4}{5}$

Try It!

30 minutes | Pairs

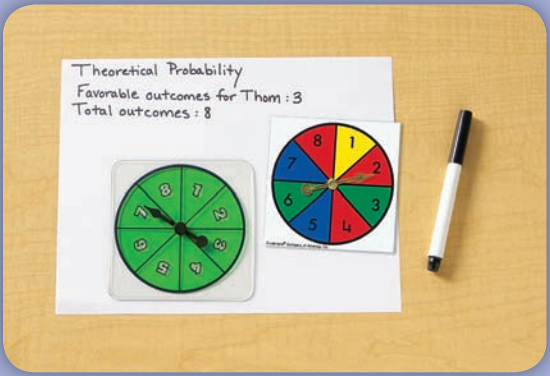
Here is a problem about theoretical and experimental probabilities.

Thom and Maya are playing a game with a spinner numbered 1–8. Thom gets a point if the spinner lands on a number less than 4. Maya gets a point if the spinner lands on 4 or greater. Compare the theoretical probability that Thom will get a point with the experimental probability using 10, 30, and 50 spins.

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials. Explain the difference between *theoretical probability*, which is a calculated number, and *experimental probability*, which describes what actually occurs in an experiment.

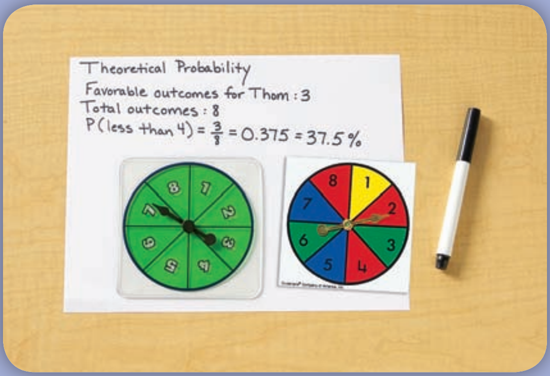
Materials

- Spinners
- paper (1 sheet per pair)



Theoretical Probability
Favorable outcomes for Thom : 3
Total outcomes : 8

1. Say: *The theoretical probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes.* **Ask:** *How many favorable outcomes are there for Thom? How many possible outcomes are there?*



Theoretical Probability
Favorable outcomes for Thom : 3
Total outcomes : 8
P(less than 4) = $\frac{3}{8}$ = 0.375 = 37.5%

2. Say: *You can express probability as a fraction, decimal, or percent.* Help students express as a fraction the theoretical probability of the spinner landing on a number less than 4. Then have them convert the fraction to a decimal and a percent.

3. Say: *Experimental probability is the ratio of favorable trials to the total number of trials.* Have students tally favorable trials (numbers less than 4) for 10, 30, and 50 spins, and express the experimental probabilities in three ways. Then have them compare the theoretical and experimental probabilities.

⚠ Look Out!

Some students may confuse theoretical and experimental probabilities. Emphasize that theoretical probability tells what would happen if each possible outcome appears the same number of times. For example, if a spinner has five equal sections and you spin it five times, each number would appear once, or 1 out of 5 times. In an actual experiment, a number might appear more than or less than once in five spins. What actually happens is experimental probability.

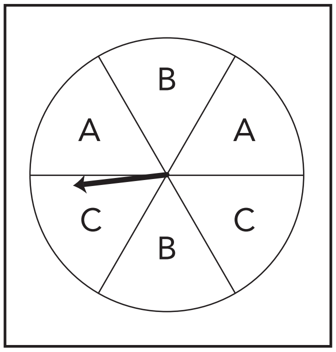
Use a spinner to model probability. Find each probability.

(Check students' work.)

1. *P*(B) $\frac{1}{3}$

P(A) $\frac{1}{3}$

P(C) $\frac{1}{3}$



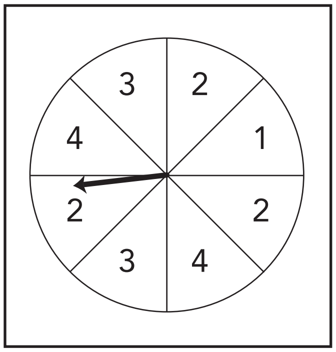
Using a spinner, model each probability. Find each probability.

2. *P*(1) $\frac{1}{8}$

P(4) $\frac{1}{4}$

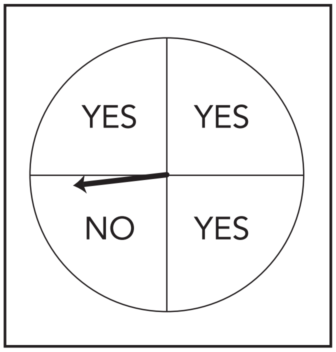
P(number < 5) 1

P(0) 0



Find each probability

3.

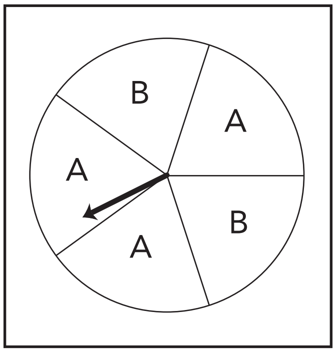


P(YES) $\frac{3}{4}$

P(NO) $\frac{1}{4}$

P(MAYBE) 0

4.



P(A) $\frac{3}{5}$

P(B) $\frac{2}{5}$

P(A or B) 1

Answer Key

Challenge! How do you use the number of sections in a spinner when finding the probability of an event?

Challenge: (Sample) The number of sections in the spinner is the denominator for the fraction when finding probability.

LESSON

4

Objective

Model relationships between events using random drawings from a bag.

Common Core State Standards

- **7.SP.7a** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
- **7.SP.8a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

Statistics and Probability

Modeling Probability: Relationships Between Events

Once students have a basic grasp of probability concepts, they can extend their understanding. They can reason, for example, about relationships between events, such as the probability that an event will occur and the probability it will not occur. Random drawings from a bag are a good way to model real relationships, such as between randomly selected people.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is the sum of the probability that an event will occur and the probability the event will not occur?*
- Reiterate the idea that picking a blue duck and picking a green duck cannot happen at the same time. Explain that this point is not as trivial as it seems. Ask students whether picking a blue tile and picking a square tile can happen at the same time. Mention to students that sometimes they will need to deal with questions like this.

Solve It

Reread the problem with students. Have them do the activities and explain how they determined the probabilities. Have them identify other probabilities that add up to 1.

More Ideas

For other ways to teach about relationships between events—

- Give spinners to pairs of students. Have students find probabilities such as the probability of spinning a 5 and the probability of not spinning a 5, or the probability of spinning a 4 or a 5.
- Have pairs of students use polyhedral dice to determine the probabilities, for example, of rolling and not rolling a multiple of 3 and the probability of rolling a prime number or a 4.

Formative Assessment

Have students try the following problem.

A bag contains 3 blue tiles, 4 green tiles, 1 yellow tile, and 2 red tiles. What is the probability of drawing a green or red tile from the bag?

A. $\frac{1}{5}$ **B.** $\frac{2}{5}$ **C.** $\frac{1}{2}$ **D.** $\frac{3}{5}$

Try It!

25 minutes | Groups of 4

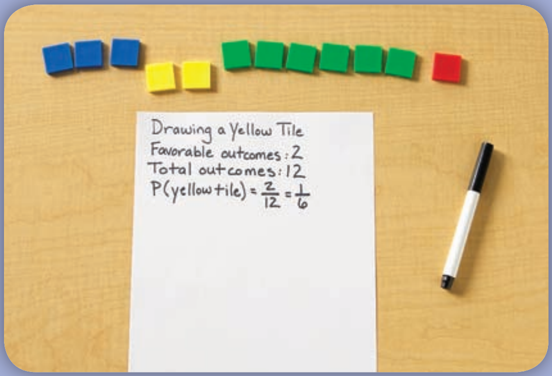
Here is a problem about relationships between events.

Eve picks a prize from a bag filled with 3 blue ducks, 2 yellow ducks, 6 green ducks, and 1 red duck. What is the probability that Eve picks a yellow duck and what is the probability that she does not pick a yellow duck? What is the probability that Eve picks either a blue duck or a green duck?

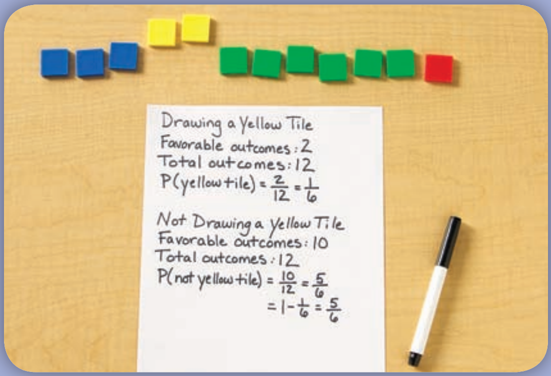
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

- Color Tiles (6 blue, 4 yellow, 2 red, and 12 green per group)
- paper (1 sheet per group)

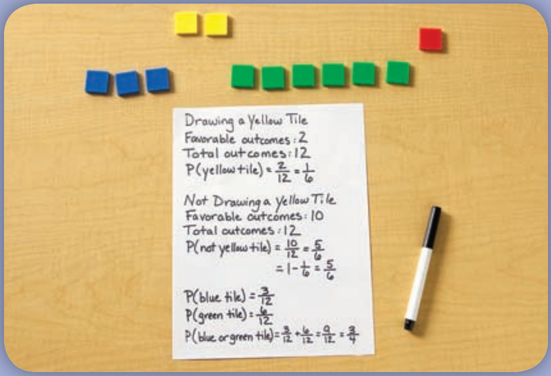


1. Say: *Use Color Tiles to model the ducks in the bag.* **Ask:** *What is the probability of drawing a yellow tile?* Have students model the possibilities and then count and record the favorable outcomes and the total outcomes. Have them express the probability as a fraction in simplest form.



2. Ask: *What is the probability of drawing a tile that is not yellow?* Have students show the non-yellow tiles. **Say:** *Find and record the probability by listing and counting outcomes and then by subtracting from 1 the probability of drawing a yellow tile.*

3. Ask: *Can drawing a yellow tile happen at the same time as drawing a non-yellow tile? Can drawing a blue tile happen at the same time as drawing a green tile?* Have students model the events to show that both cannot occur at the same time.



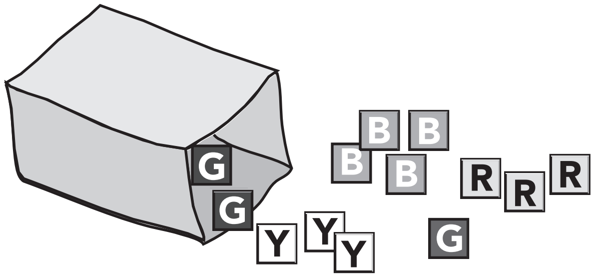
4. Ask: *What is the probability of drawing either a blue or a green tile?* Have students model the number of favorable selections—3 blue plus 6 green. Help them conclude that P(blue or green) is $\frac{9}{12}$, or $\frac{3}{4}$. Elicit, further, that this is the same as P(blue) + P(green)—that is, $\frac{3}{12} + \frac{6}{12} = \frac{9}{12} = \frac{3}{4}$.

Lesson 4 Statistics and Probability

Answer Key

Use Color Tiles to model a set with 3 yellow, 3 red, 4 blue, and 3 green. Find the probability of each event. (Check students' work.)

1.



P(Y) $\frac{3}{13}$

P(not Y) $\frac{10}{13}$

P(Y or G) $\frac{6}{13}$

P(G or R) $\frac{6}{13}$

P(not G and not B) $\frac{6}{13}$

Using Color Tiles, model the set described. Then find the probability of each event.

2. Bag with 4 red tiles, 5 blue tiles, 6 green tiles, and 2 yellow tiles.

P(yellow or blue) $\frac{7}{17}$

P(red) $\frac{4}{17}$

P(green, red, or blue) $\frac{15}{17}$

3. Bag with 5 red tiles, 3 blue tiles, and 3 yellow tiles.

P(not blue and not red) $\frac{3}{11}$

P(yellow) $\frac{3}{11}$

P(red or yellow) $\frac{8}{11}$

Find each probability given the set described.

4. Bag with 10 red marbles, 12 blue marbles, 8 white marbles, 6 green marbles, and 4 yellow marbles.

P(not yellow) $\frac{9}{10}$

P(not red and not white) $\frac{11}{20}$

P(green or blue) $\frac{9}{20}$

P(not green) $\frac{17}{20}$

P(not green and not blue) $\frac{11}{20}$

P(green) $\frac{3}{20}$

5. Bag with 1 red marble, 1 blue marble, 1 white marble, 8 green marbles, and 10 yellow marbles.

P(blue) $\frac{1}{21}$

P(red) $\frac{1}{21}$

P(green) $\frac{8}{21}$

P(white) $\frac{1}{21}$

P(red, blue, white, green, or yellow) 1

P(not yellow) $\frac{11}{21}$

Answer Key

Challenge! If you have 20 items in a set and 4 of the items are red, what do you know about the probability of red and the probability of not red? Show your work.

Challenge: (Sample) The sum will be 1. Red is 4 out of 20 or $\frac{1}{5}$ and not red is 16 out of 20, or $\frac{4}{5}$; $\frac{1}{5} + \frac{4}{5} = 1$.

LESSON

5

Statistics and Probability

Probability and Fairness

Objective

Determine whether a spinner is fair by comparing its fractional parts.

Common Core State Standards

- **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- **7.SP.7b** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

Using an area model to determine theoretical probability involves the understanding that the sum of the fractional parts of a whole must be 1. Using an area model also facilitates an understanding of "fairness." This activity uses dartboards as a starting point for the investigation of these concepts.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What point values could you assign to each of the colored areas in order to make the second dartboard fair?*
- **Ask:** *In 20 tries, about how many times should you have hit red, blue, green, and yellow on the first dartboard? The second dartboard?*
- **Ask:** *What is the result if you add together the fractional parts of the first board? If you add together the fractional parts of the second board? In general, what should be the sum of the fractional parts of any board?*

Solve It

Reread the problem with students. Have students explain in writing how they determined whether the boards were fair or unfair.

More Ideas

For another way to teach about fair and unfair outcomes—

- Set up an experiment that will produce unfair outcomes. Place 2 yellow, 4 green, and 6 red Centimeter Cubes in a paper bag. Tell students not to look in the bag until the end of the activity. Have students pull out a cube, record its color, return the cube to the bag, and mix the cubes. Students should repeat the process 50 times. **Say:** *Examine the data from your experiment.* **Ask:** *What colors of cubes are there in the bag? Are the colors represented equally? If not, estimate the proportions that are represented.* When students have made and defended their predictions, have them open the bag and check their work.

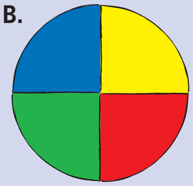
Formative Assessment

Have students try the following problem.

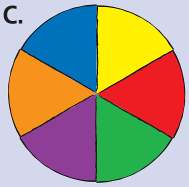
Which of the following dartboards is unfair?

A.

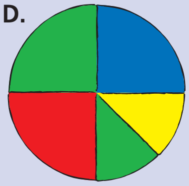
B.



C.



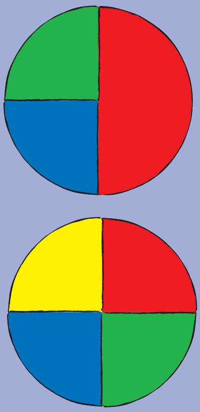
D.



Try It!

30 minutes | Pairs

Here is a problem about determining whether a spinner is fair or unfair.

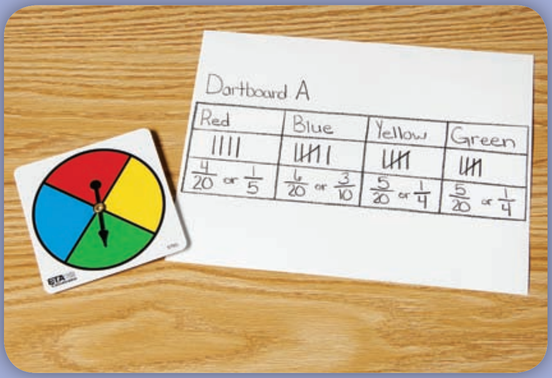


James and his friends play a magnetic dart game at their school's afternoon recreational program. Each player picks a different color as his or her target. The dartboard is mounted on a stand and spun. The players are blindfolded and take turns throwing darts at the spinning board, hoping to hit their target section. The players have two boards they can use. What is the probability of hitting each of the colors on the dartboards? Are the boards fair?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

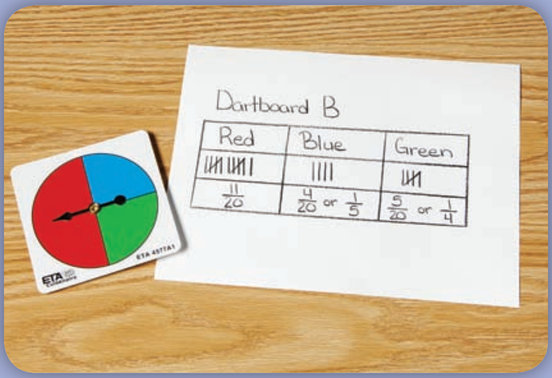
- Spinners



Dartboard A

Red	Blue	Yellow	Green
𝍷𝍷𝍷𝍷	𝍸𝍷	𝍸	𝍸
$\frac{4}{20}$ or $\frac{1}{5}$	$\frac{6}{20}$ or $\frac{3}{10}$	$\frac{5}{20}$ or $\frac{1}{4}$	$\frac{5}{20}$ or $\frac{1}{4}$

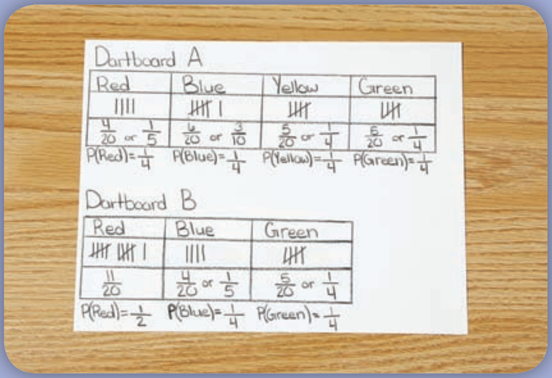
1. Have students spin the spinner, which is divided into four equal parts, 20 times and record the results on a tally chart. **Ask:** *Based on your results, what is the probability of hitting each of the colors on this spinner?*



Dartboard B

Red	Blue	Green
𝍸𝍸𝍷	𝍷𝍷𝍷𝍷	𝍸
$\frac{11}{20}$	$\frac{4}{20}$ or $\frac{1}{5}$	$\frac{5}{20}$ or $\frac{1}{4}$

2. Now have students repeat the experiment with the spinner that is divided into one half and two quarters. **Ask:** *Based on your results, what is the probability of hitting each of the colors on this spinner?*



Dartboard A

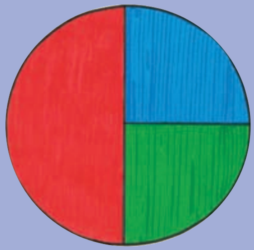
Red	Blue	Yellow	Green
𝍷𝍷𝍷𝍷	𝍸𝍷	𝍸	𝍸
$\frac{4}{20}$ or $\frac{1}{5}$	$\frac{6}{20}$ or $\frac{3}{10}$	$\frac{5}{20}$ or $\frac{1}{4}$	$\frac{6}{20}$ or $\frac{1}{4}$
$P(Red)=\frac{1}{4}$	$P(Blue)=\frac{1}{4}$	$P(Yellow)=\frac{1}{4}$	$P(Green)=\frac{1}{4}$

Dartboard B

Red	Blue	Green
𝍸𝍸𝍷	𝍷𝍷𝍷𝍷	𝍸
$\frac{11}{20}$	$\frac{4}{20}$ or $\frac{1}{5}$	$\frac{5}{20}$ or $\frac{1}{4}$
$P(Red)=\frac{1}{2}$	$P(Blue)=\frac{1}{4}$	$P(Green)=\frac{1}{4}$

3. Have students compare the results of their experiments to the actual probabilities of hitting each color. Ask them to summarize their findings. **Ask:** *Are both of the dartboards fair? Explain.*

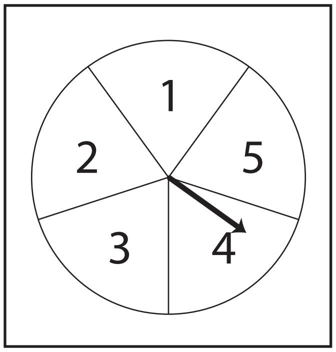
⚠ Look Out!



Some students may think that if an outcome is possible, that it is as likely to occur as any other possible outcome. Help students realize that the area a section covers influences the probability that the spinner will land there.

Use the spinner collection to model fair and unfair spinners. Find a spinner whose sections match each spinner below. Answer the questions.

1.



Find *P*(1). $\frac{1}{5}$

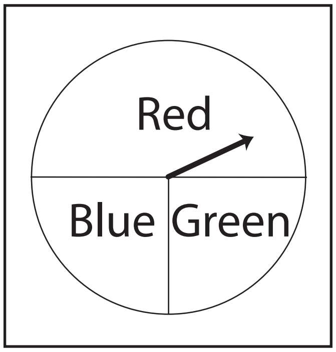
Find *P*(3). $\frac{1}{5}$

Find *P*(4). $\frac{1}{5}$

Is the spinner fair? yes

Why or why not? all sections have the same probability

2.



Find *P*(red). $\frac{1}{2}$

Find *P*(blue). $\frac{1}{4}$

Find *P*(green). $\frac{1}{4}$

Is the spinner fair? no

Why or why not? sections have different probabilities

Using the spinner collection, model a fair and an unfair spinner. Sketch the models. Answer the questions.

3. Sketch a fair spinner below.

Check students' models and explanations.

Why is the spinner fair? ______

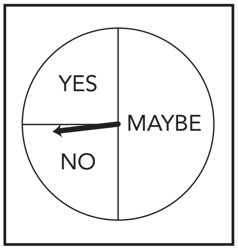
4. Sketch an unfair spinner below.

Check students' models and explanations.

Why is the spinner unfair? ______

Determine if each spinner is fair. Explain your answer.

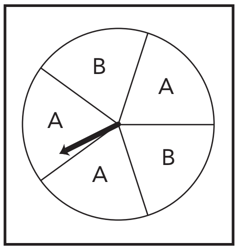
5.



no

the outcomes have different probabilities

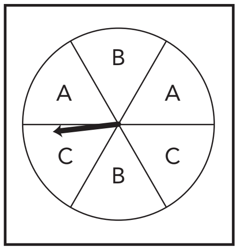
6.



no

the probability is greater for spinning A

7.



yes

the probability for each outcome is the same

Answer Key

Challenge! When a spinner has an odd number of equal-sized sections and the sections are not uniquely labeled, how can you be certain that the spinner is not fair? Are there any odd numbers for which the spinner could be fair? Explain or draw an example.

Challenge: (Sample) The number and size of the sections with different labels have to be equal. If a spinner has 5 sections, you can meet that requirement if none of the sections has the same label. A spinner with 9 equal sections could be fair if the sections were marked with 3 different labels equally. This would be true for a spinner with 15 sections and 5 different labels. This type of pattern is true for spinners that have a number of labels that is a factor of the number of sections.

LESSON

6

Finding Probability Without Replacement

Objective

Determine the probability for a random drawing without replacement.

Common Core State Standards

- **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- **7.SP.8a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.8b** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Students have learned how to find the probability of a single event. In this lesson, students will determine the probability of having the same outcome occur more than once in a row.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *The probability of picking a blue sock can be found by taking the number of blue socks and dividing that number by the total number of socks in the drawer.* **Ask:** *What is the probability of picking a blue sock?* Elicit that the probability is $\frac{2}{6}$ or $\frac{1}{3}$.
- **Say:** *Suppose that Bill picks a blue sock and doesn't return it to the drawer.* **Ask:** *If he picks again, what are his chances of picking another blue sock?* Elicit that because there are now just five socks in the drawer and only one of them is blue, you divide one by five; the answer is $\frac{1}{5}$.
- **Ask:** *How could you use your data from the first step to estimate the probability of drawing two blue socks?* Have students obtain an estimate and compare it with the theoretical probability.

Solve It

Reread the problem with students. Ask them to write to Bill explaining how to determine the probability of picking two blue socks.

More Ideas

For another way to teach about probability for a random drawing—

- Extend the activity by having students determine the probability of picking two blue socks with replacement. Have students re-enact the scenario, but this time they should replace the first sock drawn. Compare the probabilities (both theoretical and experimental) with and without replacement.

Formative Assessment

Have students try the following problem.

Each of the numbers 1–6 is written on a card and put into a hat. Two cards will be drawn, one at a time, without replacement. What is the probability of drawing two odd numbers?

A. $\frac{7}{10}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{5}$ **D.** $\frac{1}{10}$

Try It! 40 minutes | Pairs

Here is a problem about a random drawing without replacement.

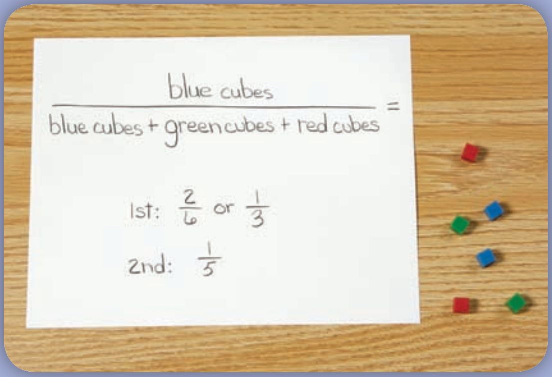
Bill's sock drawer contains 2 blue socks, 2 green socks, and 2 red socks. If he picks one sock at random, what is the probability it will be blue? If he then picks another sock (without returning the first sock), what is the probability the second sock will be blue? What is the probability that both socks will be blue?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

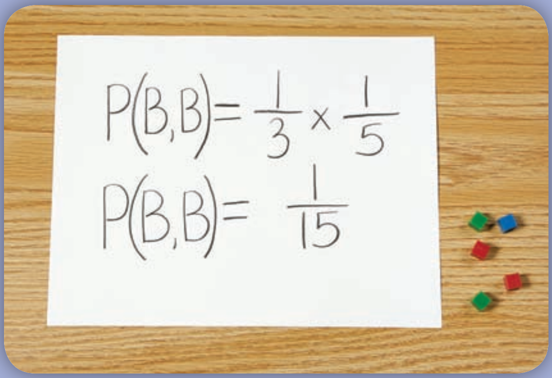
Materials

- Centimeter Cubes (2 blue, 2 green, and 2 red)
- BLM 12
- paper bag

1. Have students place 2 blue, 2 green, and 2 red cubes in a paper bag. One student should pick a cube at random. The other student should record its color on the recording chart. Without returning the cube to the bag, the first student should select another cube at random. Its color also should be recorded on the chart. Students should return the cubes to the bag and repeat the procedure 30 times.



2. Have students determine the probability of randomly choosing a blue sock from the sock drawer. Then have them determine the probability that the second sock drawn will be blue if the first sock drawn is blue and is not replaced.



3. Have students use the formula to determine the probability that both socks Bill draws will be blue. Remind students of the Conditional Probability formula, $P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$.

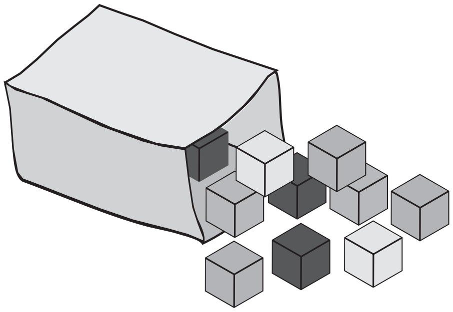
⚠ Look Out!

Watch for students who do not decrease the number of socks available when Bill draws the second sock. Have students re-enact the scenario with cubes to confirm that the number of socks in the drawer is now five and not six.

Answer Key

Use Centimeter Cubes to model the probability of each event, without replacement. Make a bag like the one shown. Answer the questions. (Check students' work.)

1. The bag has 2 yellow cubes, 5 green cubes, and 3 red cubes.



What is the probability of selecting a yellow cube at random? $\frac{1}{5}$

Without replacing the yellow cube, what is the probability of selecting a red cube at random? $\frac{1}{3}$

What is P(yellow, red)? $\frac{1}{15}$

What is P(yellow, yellow)? $\frac{1}{45}$

Using Centimeter Cubes, model each bag described. Find each probability without replacement.

2. A bag with 5 black cubes, 3 pink cubes, and 2 blue cubes

What is P(blue, black)? $\frac{1}{9}$

What is P(pink, blue)? $\frac{1}{15}$

What is P(black, black)? $\frac{2}{9}$

3. A bag with 6 orange cubes, 6 red cubes, and 6 brown cubes

What is P(orange, red)? $\frac{2}{17}$

What is P(red, red)? $\frac{5}{51}$

What is P(brown, red)? $\frac{2}{17}$

Find each probability without replacement.

4. A bag with 5 black marbles, 2 white marbles, and 8 yellow marbles

What is P(yellow, white)? $\frac{8}{105}$

What is P(white, black)? $\frac{1}{21}$

What is P(black, black)? $\frac{2}{21}$

What is P(black, yellow)? $\frac{4}{21}$

5. A bag with 3 solid ribbons, 4 striped ribbons, and 10 checkered ribbons

What is P(solid, solid)? $\frac{3}{136}$

What is P(checkered, striped)? $\frac{5}{34}$

What is P(striped, solid)? $\frac{3}{68}$

What is P(solid, checkered)? $\frac{15}{136}$

6. A bag with 12 red tiles, 10 black tiles, and 20 white tiles

What is P(red, white)? $\frac{40}{287}$

What is P(white, black)? $\frac{100}{861}$

What is P(red, black)? $\frac{20}{287}$

What is P(black, black)? $\frac{15}{287}$

7. A bag with 10 green marbles, 2 clear marbles, and 8 blue marbles

What is P(clear, clear)? $\frac{1}{190}$

What is P(green, clear)? $\frac{1}{19}$

What is P(blue, green)? $\frac{4}{19}$

What is P(green, green)? $\frac{9}{38}$

Answer Key

Challenge! Describe the numbers you multiply in the denominator when you find the probability of two events without replacement. When does the probability in simplest form have a denominator that differs from the product of the numbers you just described?

Challenge: (Sample) The denominator is the product of the number of items and one less than the number of items. When a numerator and a denominator have a common factor and you divide each by the common factor, the denominator of the probability in simplest form is not the product of the number of items times one less than the number of items.

LESSON

7

Objective

Find the theoretical and experimental probabilities of an event involving dice.

Common Core State Standards

- **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- **7.SP.8a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.8b** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Theoretical and Experimental Probability with Dice

Students' experiences with determining theoretical probability continue with this activity in which they play a game involving dice. The students then gather experimental data by playing the game and comparing the results with the mathematically determined theoretical probabilities.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Based on your experiment, what is* P*(multiple of 3) and* P*(multiple of 4)? How do these values compare with the theoretical probabilities?* Have students share their results.

Solve It

Reread the problem with students. Discuss the differences between the experimental and theoretical probabilities. Ask students what they would do to obtain experimental probabilities closer to the theoretical probabilities. Elicit that they could increase the number of rolls.

More Ideas

For another way to teach about theoretical and experimental probabilities—

- Have students complete a similar activity using the 4-sided die and the 8-sided die. This time have them determine *P*(sum of 4) and *P*(sum of 5). Tell students to find the experimental probability by rolling the dice 32 times and the theoretical probability using a table of values.

Formative Assessment

Have students try the following problem.

A green 6-sided number cube and a blue 6-sided number cube are rolled, and the results are added together. All of the possible sums are listed in the table.

Which of the following probabilities is correct?

A. *P*(multiple of 10) = $\frac{1}{4}$

B. *P*(odd number) = $\frac{1}{2}$

C. *P*(even number) = $\frac{1}{36}$

D. *P*(greater than 5) = $\frac{1}{4}$

Green

Blue	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Try It!

25 minutes | Pairs

Here is a problem about theoretical probability vs. experimental probability.

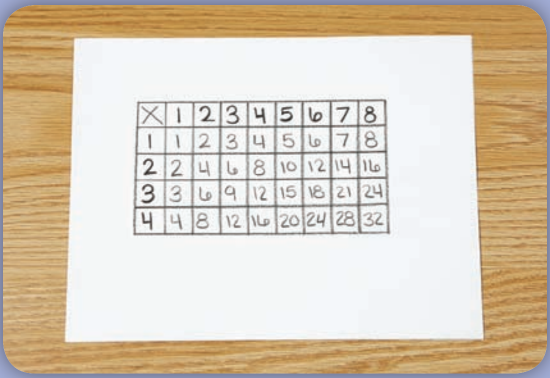
Matt and Jana are playing a game. Matt rolls a 4-sided die, and Jana rolls an 8-sided die. If the product of the rolls is a multiple of 3, Matt gets a point. If the product of the rolls is a multiple of 4, Jana gets a point. A game consists of 32 rolls of the dice. Who has a better chance of winning?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

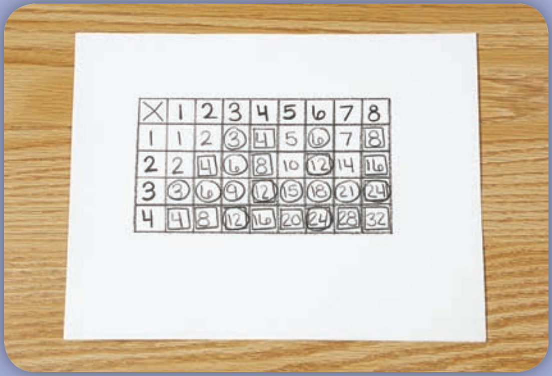
- Polyhedral Dice Set (4-sided die and 8-sided die)

1. Have students take the roles of Matt and Jana and play a round (32 rolls of the dice) of the game. Students should record the product for each of the rolls. **Ask:** *Who won the game—Matt or Jana?*



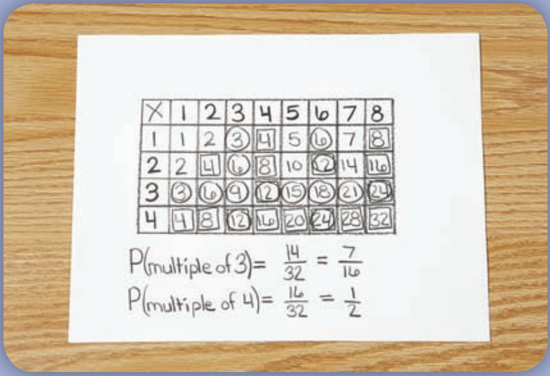
×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32

2. Say: *Now determine the theoretical probability.* Have students set up a table of values and fill in the products.



×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32

3. Have students mark the multiples of 3 and the multiples of 4. **Ask:** *How many values in the table are multiples of 3? How many are multiples of 4?* Elicit from students that there are fourteen multiples of 3 and sixteen multiples of 4.



×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32

$P(\text{multiple of } 3) = \frac{14}{32} = \frac{7}{16}$

$P(\text{multiple of } 4) = \frac{16}{32} = \frac{1}{2}$

4. Have students then determine *P*(multiple of 4) and *P*(multiple of 3) using their table of values. **Ask:** *Who has a better chance of winning a round of this game?*

Use a 4-sided die and a 6-sided die to make a table of products when the dice are rolled. Use the table to find each probability.

(Check students' work.)

1.

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24

P(multiple of 6) $\frac{1}{3}$

P(multiple of 4) $\frac{11}{24}$

P(even product) $\frac{3}{4}$

P(multiple of 10) $\frac{1}{12}$

Using Polyhedral Dice, make a table to find each probability.

2. two 4-sided dice

Check students' tables.

P(multiple of 3) $\frac{7}{16}$

P(product < 15) $\frac{15}{16}$

P(product that is a prime number) $\frac{1}{4}$

P(multiple of 8) $\frac{3}{16}$

3. 6-sided die and 10-sided die

Check students' tables.

P(odd product) $\frac{1}{4}$

P(product < 10) $\frac{1}{3}$

P(product > 40) $\frac{1}{10}$

P(multiple of 5) $\frac{1}{3}$

Answer Key

Challenge! An experiment has you roll an 8-sided die and a 12-sided die and multiply the face values of the dice. What is the number of outcomes for this experiment? What is the smallest product in the table? What is the largest product in the table? How many products are less than 10?

Challenge: 96; 1; 96; 25

LESSON

8

Objective

Find the probability of a compound event; make an organized list.

Common Core State Standards

- **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- **7.SP.8a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.8b** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Statistics and Probability

Compound Events: Making an Organized List

Experience with experiments helps students build on their intuitive sense about probability. In this lesson, students make an organized list to identify outcomes in a sample space and make predictions about their occurrences. Comparing theoretical predictions and observational data enables students to draw new insights and adjust their thinking accordingly.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What fraction do you use to compute the probability of an event?* Point out that the fraction is the number of favorable outcomes over the number of possible outcomes.
- **Ask:** *Besides using an organized list, how else might you find the number of possible outcomes?* If appropriate, elicit that the possibilities include drawing a picture and making a tree diagram.

Solve It

Reread the problem with students. Have them find and write the probability for each single event in the form of a fraction. Guide students to multiply these fractions to find the probability of the compound event. Have students compare the experimental and theoretical probabilities. Ask students to tell how the favorable outcomes would differ if Deana wanted to roll a number greater than 5 OR toss yellow.

More Ideas

For other ways to teach about probabilities of compound events—

- Have students conduct the experiment using a 1–8 spinner instead of the die.
- Use Color Tiles. Place six yellow and four red tiles in a bag. Have students draw one tile, record the color, replace the tile, and repeat. Each draw is a single event. The two draws are a combined event. Have students find the experimental and theoretical probabilities of drawing a yellow tile first and a red tile second.

Formative Assessment

Have students try the following problem.

Johann rolls a number cube with the numbers 1–6. He also tosses a coin. What is the probability that he will roll an even number and toss tails?

A. $\frac{1}{2}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{4}$ **D.** $\frac{1}{12}$

Try It!

30 minutes | Groups of 4

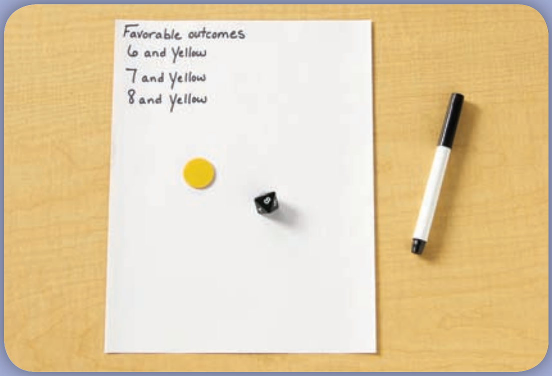
Here is a problem about the probability of a compound event.

Deana has a polyhedral die with faces labeled 1–8 and a counter with one yellow and one red face. What is the probability that she will roll a number greater than 5 and toss a counter yellow-face up?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

- Octahedral Dice (1 per group)
- Two-Color Counters (1 per group)
- cup (optional for rolling die and counters; 1 per group)
- paper (2 sheets per group)



Favorable outcomes
6 and Yellow
7 and Yellow
8 and Yellow

1. Ask: *What are the favorable outcomes for each single event—that is, for just the number and for just the color? What are the favorable outcomes for the compound event?* Have students record the favorable outcomes for the compound event.

2. Guide students to list all the possible outcomes for the compound event. Then have them perform at least 50 trials for this event and record their results. **Ask:** *What are the experimental and theoretical probabilities for the compound event?*

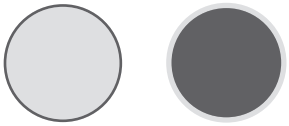
3. Guide students to see that they can determine the number of possible and favorable outcomes by using the Counting Principle. Show how this leads to the rule *P*(*A* and *B*) = *P*(*A*) × *P*(*B*).

⚠ Look Out!

Emphasize that both events must have favorable outcomes to satisfy the conditions: rolling a 2 and yellow is not a favorable outcome because the outcome is not favorable for one of the two single events. Students may think that the theoretical probability and the experimental probability should be the same. Stress that the two results may not be the same.

Use the decahedral die and a Two-Color Counter to model each probability. Find the probability of each compound event. (Check students' work.)

1. 10-sided die numbered 0 to 9 and 1 Two-Color Counter



P(1 and red) $\frac{1}{20}$

P(8 and red) $\frac{1}{20}$

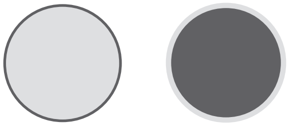
P(4 and not yellow) $\frac{1}{20}$

P(6 and yellow) $\frac{1}{20}$

P(7 or 8 and red) $\frac{1}{10}$

Using a die and a Two-Color Counter, model each probability. Find each probability.

2. 20-sided die numbered 1 to 20 and 1 counter



P(1 and yellow) $\frac{1}{40}$

P(12 and red) $\frac{1}{40}$

P(4, not red) $\frac{1}{40}$

3. 6-sided die numbered 1 to 6 and 1 counter

P(2 and red or yellow) $\frac{1}{6}$

P(not 3, red) $\frac{5}{12}$

P(2 and yellow) $\frac{1}{12}$

P(not 4 or 5, yellow) $\frac{1}{3}$

Find each probability.

4. 8-sided die numbered 1 to 8 and 1 counter

P(1 and yellow) $\frac{1}{16}$

P(7, not red) $\frac{1}{16}$

P(not 9, not yellow) $\frac{1}{2}$

P(5 or 6, red) $\frac{1}{8}$

5. 12-sided die numbered 1 to 12 and 1 counter

P(12 and yellow) $\frac{1}{24}$

P(13 and red) $\frac{1}{2}$

P(not 1, not yellow) $\frac{11}{24}$

P(4 and red or yellow) $\frac{1}{12}$

Answer Key

Challenge! What does the word *compound* mean when finding the probability of an event?

Challenge: (Sample) A compound event is an event with two parts that are independent of each other.

LESSON 9

Objective

Find the probability of a compound event; make a tree diagram.

Common Core State Standards

- **7.SP.8a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.8b** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Statistics and Probability

Compound Events: Making a Tree Diagram

Students have probably noticed a discrepancy between the theoretical probability of a simple event occurring and their actual experimental results. They should understand by now that generally, the more trials one performs, the closer the experimental results will be to the theoretical outcome. This activity lays the foundation for ensuring that the theoretical probability for compound events is correctly calculated.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How many different combinations of colors and numbers do you think there are for the spinner and the die?*
- **Ask:** *What are the possible outcomes for a spin of the spinner? What are the possible outcomes for a roll of the die? Does the outcome of Jackie's spin affect the outcome of Jeff's roll of the die? Does the outcome of Jeff's roll of the die affect the outcome of Jackie's spin?*

Solve It

Have students explain in writing how likely Jeff and Jackie are to spin and roll a red 4. Are their chances of spinning a red 4 different from their chances of spinning and rolling any other combination?

More Ideas

For other ways to teach about probabilities of compound events—

- Have students extend this activity by finding the probabilities of other outcomes such as *P*(red, 3), *P*(blue, 1), *P*(green, 3), and so forth. Gradually increase the complexity to outcomes such as *P*((red, 4) *or* (red, 2)), *P*((green, 3) *or* (blue, 2)), and *P*((yellow, 3) *and* (blue, 4)).
- Have students conduct a similar experiment using two number cubes. Students can find the probability of rolling an odd number, then an even number.

Formative Assessment

Have students try the following problem.

A family has three children. What is the probability that two of the children are boys?

A. $\frac{1}{4}$ **B.** $\frac{1}{3}$ **C.** $\frac{3}{8}$ **D.** $\frac{1}{2}$

Try It! 30 minutes | Pairs

Here is a problem about the probability of a compound event.

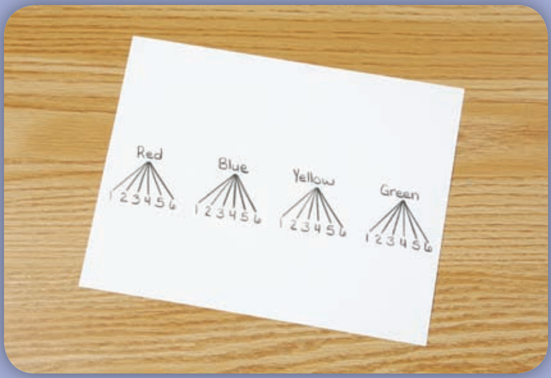
Jeff has a spinner that is divided into four equal sections: red, yellow, green, and blue. Jackie has a 6-sided die. They decide to make up a game of their own in which Jeff must call out a number and Jackie must call out a color. If they call out "red" and "4," what is the probability that they will spin and roll that particular outcome?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

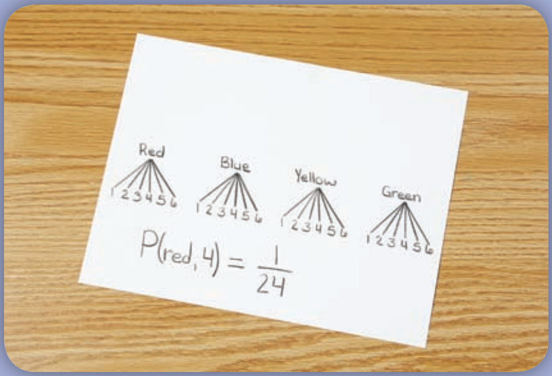
Materials

- Spinners
- Number Cube

1. Have students simulate the game by spinning and rolling until they get "red" and "4." Have them record how many trials it took them to get that result. **Ask:** *How many trials did it take?*



2. Next, have students create a tree diagram, starting with the spinner colors. Then have them complete the tree diagram with the numbers on the die. **Ask:** *What is the probability that you will get "red" and "4" on your first try?*



3. Discuss the tree diagram with students. **Ask:** *How many spins and rolls should it take, on average, to get "red" and "4"?*

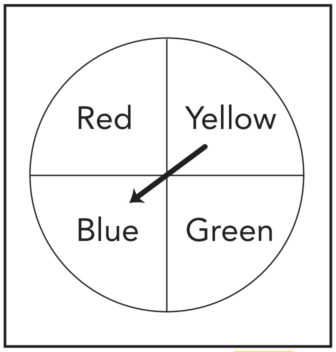
⚠ Look Out!

Students may want to stop drawing the tree diagram once they have reached the outcome (red, 4). They need to understand that in order to find the total number of possible outcomes, they will need to complete the entire tree diagram. Explain to them that the total provides them with the correct denominator for the fraction that represents *P*(red, 4).

Use the 4-section color spinner and a number cube to simulate a game. Make a tree diagram for all possible outcomes. Find each probability.

(Check students' work.)

1. Four-section spinner with red, blue, green, and yellow sections and a number cube labeled 1 to 6



Check students' tree diagram. The diagram should have four branches, one for each color and each branch should have six numbers, 1–6.

P(red and 1) $\frac{1}{24}$

P(green and an even number) $\frac{1}{8}$

Using the 6-section color spinner and a coin, make a tree diagram of all possible outcomes. Find each probability.

2.

Check students' tree diagram. The diagram should have six branches, one for each color and each branch should have 2 options, heads and tails.

P(yellow and heads) $\frac{1}{12}$

P(blue or green and heads) $\frac{1}{6}$

Find each probability given the two elements of chance.

3. 2 coins

P(two heads) $\frac{1}{4}$

P(heads and tails) $\frac{1}{2}$

4. 8-sided die labeled 1–8 and a coin

P(8 and heads) $\frac{1}{16}$

P(tails and odd) $\frac{1}{4}$

5. two-section spinner labeled red and blue, and a 4-sided die labeled 1–4

P(blue and 3) $\frac{1}{8}$

P(red and even) $\frac{1}{4}$

Answer Key

Challenge! Describe a tree diagram for three items of chance: coin, 4-section spinner, and a number cube. Does the number of possible outcomes vary depending on the order in which you make your diagram? Explain.

Challenge: (Sample) The diagram could be in any order of the items of chance. If the diagram is made in the order the items are named, there will be two branches, one heads and one tails. Each of these branches will have four branches, one for each color. Each of the color branches will have six number branches. The order of the branches does not affect the number of possible outcomes. In this case, any order of branches will produce 48 outcomes.