Operations and Algebraic Thinking

In second grade, children continue their work with addition and subtraction. They work on word problems within 100 by representing and solving three types of one- and two-step word problems–*result* unknown, *change* unknown, and *start* unknown.

Children internalize addition and subtraction facts within 20 and develop fluency by repeatedly using strategies that make sense to them. They move beyond counting and counting-on to methods such as make a ten, "doubles," and "near doubles." They are called to know from memory all sums of two one-digit numbers. The goal is to give children many experiences using manipulatives and visual representations, not to simply present a list of facts for them to memorize.

Second graders use their knowledge of "doubles" to understand the concepts of *odd* and *even*. They learn that if a number can be broken into two equal addends, then the number is even. They determine whether a group of objects has an odd or even number of members by using strategies such as pairing objects, counting by 2's, and writing an equation. Children also use arrays to work with repeated addition, a foundational concept necessary for learning multiplication.

The Grade 2 Common Core State Standards for Operations and Algebraic Thinking specify that children should–

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

The following hands-on activities help children internalize addition and subtraction facts and solve one- and two-step problems. Mathematically proficient second graders develop a foundation for applying problem-solving strategies and become independently proficient at using those strategies to solve new tasks. They are expected to persevere while solving tasks when "stuck" by re-examining the task in different ways, figuring what they know and don't know, and continuing to solve it.

Operations and Algebraic Thinking

Contents

LESSON 1

Objective

Solve addition and subtraction number sentences.

Common Core State Standards

- **2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Operations and Algebraic Thinking

Addition and Subtraction

As children become fluent in computation, they begin to apply their understanding of operations to problem-solving situations. Children address equations that have an unknown number and learn to work backward by using the inverse operation to solve for the missing number. In this lesson, children model addition and subtraction number sentences and experience firsthand how the two operations are related.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How do you use subtraction to find a missing addend, or adding number? How do you use addition to solve a subtraction problem?*
- **Ask:** *How can I use subtraction to find out what number plus 13 equals 29?* Write _____ + *13 = 29* on the board. Children should explain that they would subtract 13 from 29. Write *29 – 13 =* _____ on the board. Invite children to solve.
- **Ask:** *How can I use addition to show what number minus 23 equals 15?* Write _____ – *23 = 15* on the board. Children should explain that they would add 23 to 15 to find the answer. Write *15 + 23 =* _____ on the board. Invite children to solve.

Solve It

With children, reread the problem. Have children explain in writing how they solved the problem. Then have children show the solution using both an addition and a subtraction sentence.

More Ideas

For other ways to teach about addition and subtraction number sentences—

- Invite children to solve additional problems using Base Ten Blocks and the Missing Numbers Worksheet (BLM 1). Have children record their completed addition and subtraction sentences for each problem.
- Have children write their own addition and subtraction problems involving families of bears. Have children exchange problems with a partner. Then partners use the inverse operation and Three Bear Family® Counters to model and solve the problems.

Formative Assessment

Have children try the following problem.

Circle the addition sentence that solves this problem: _____ – *11 = 16*

A. 11 + 5 = 16 **B.** 16 + 11 = 27 **C.** 5 + 6 = 11 **D.** 11 + 11 = 22

Try It! 25 minutes | Groups of 4

Here is a problem about addition and subtraction number sentences.

Of the 18 students in our class, 7 students have perfect attendance so far this year. How many students do not have perfect attendance?

Introduce the problem. Then have children do the activity to solve the problem. Distribute Cuisenaire Rods, worksheets, and pencils to children. Remind children that white rods have a value of 1, orange rods have a value of 10, and other colors have values in between.

Materials

- Cuisenaire® Rods (half a set per group)
- Missing Numbers Worksheet (BLM 1; 1 per group)
- pencils (1 per group)

1. Say: *Look at the Missing Numbers Worksheet. We will use it to find what number you add to 7 to get 18.* Have children use rods to model the addition problem on the worksheet.

2. Ask: *How can we solve this addition sentence?* Guide children to see that they should subtract to find the missing number. Have children complete the train on the second line to match the first train.

3. Have children use rods to solve a subtraction problem. Write *20 – _____ = 14* on the board. **Ask:** *How can we solve this subtraction sentence?* Have children model the problem on their worksheet and use addition to solve it.

⚠ Look Out!

Watch out for children who seem to start over from scratch when forming a subtraction problem to solve for a missing addend in an addition problem. Point out that the numbers in both problems should be identical; only their positions and the operation change. Demonstrate by showing how rods placed on the Missing Numbers Worksheet change places.

Answer Key

Use Cuisenaire Rods to build each model. Write each missing number. (Check students' work.)

1.

$9 + \underline{7} = 16$ $16 - 9 = \underline{7}$

2.

$5 + \underline{13} = 18$ $18 - 5 = \underline{13}$

Using Cuisenaire Rods, build a model to find the missing number. Sketch the model. Write the missing number. (Check students' models.)

3. $5 + \underline{12} = 17$ **4.** $17 - \underline{12} = 5$

Find the missing number in each addition or subtraction sentence.

5. $7 + \underline{8} = 15$ **6.** $12 + \underline{11} = 23$

7. $22 - \underline{17} = 5$ **8.** $3 + \underline{15} = 18$

Answer Key

Challenge! Use the model in Question 2 to describe two different ways to find the missing number.

Challenge: (Sample) You can count the number of white rods it takes to make the second train the same length as the first. You can use the lengths of any other combination of rods (e.g., 10 + 3) that you use to make the second train the same length as the first.

LESSON 2

Objective

Write and solve number sentences from problem situations that express relationships involving addition and subtraction.

Common Core State Standards

- **2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **2.NBT.7** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Operations and Algebraic Thinking

Writing Number Sentences

A number sentence is an expression that contains numbers; symbols of operations; and a greater than, less than, or equal sign. Number sentences are used as a way to record the computation process of solving a mathematical problem. In order to write a number sentence from a problem situation, the numbers involved, as well as the relationship between them, must be identified.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Have children look at the Base Ten Blocks they used in the activity.
- **Ask:** *How did you show the number of cookies Danielle baked? What blocks did you use? Why? How did you show the cookies she sold? How can you take away the cookies Danielle sold?*
- **Ask:** *What number sentence did you write to show your solution to the problem? How do you know that is correct? How can you take away the cookies Danielle sold?*

Solve It

With children, reread the problem. Ask children to draw pictures of Danielle's cookies or symbols to represent them. Have children mark the cookies sold in some way. Then, have them label the picture with the number sentence and write a sentence explaining how they know the number sentence is correct.

More Ideas

For other ways to teach about writing number sentences—

- Have children use Snap Cubes® to act out a problem situation and write the corresponding number sentence as the situation is acted out.
- Have children use a Hundred Chart (BLM 2) to represent a problem situation and label the chart with the corresponding number sentence.
- Have children use Ten Frames (BLM 3) to represent the problem. Have them complete one ten frame and partially fill another to represent the cookies. Ask children how they might use the make-a-ten strategy to subtract.

Formative Assessment

Have children try the following problem.

Draw pictures to help solve this problem, and write a number sentence to show your work. Matt has cucumbers in his garden. Yesterday he picked 7, and today he picked 8. How many cucumbers did Matt pick altogether?

Try It! 30 minutes | Independent

Here is a problem about writing number sentences.

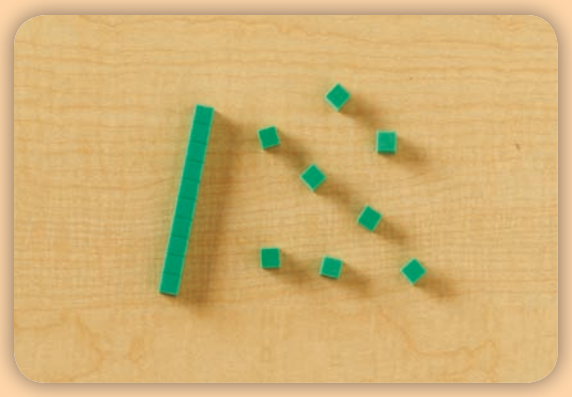
Danielle baked 18 chocolate chip cookies for a bake sale. She sold 9 cookies. How many cookies did she have left?

Introduce the problem. Then have children do the activity to solve the problem.

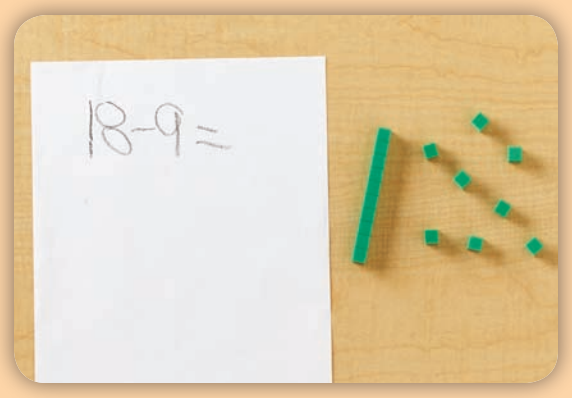
Distribute Base Ten Blocks, paper, and pencils to children.

Materials

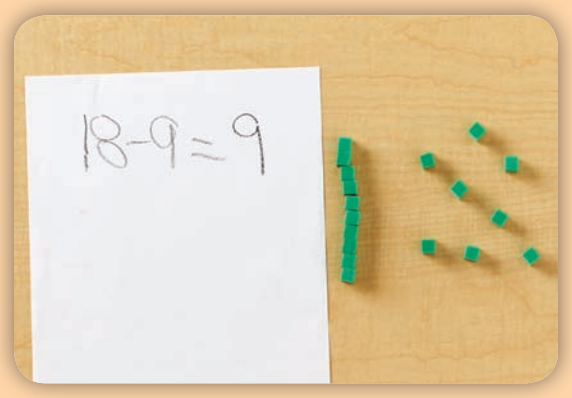
- Base Ten Blocks (rods and units)
- paper (1 sheet per child)
- pencils (1 per child)



1. To begin, have children choose Base Ten Blocks to show the number of cookies Danielle baked. Show how many cookies were sold.



2. Have children write a number sentence on a sheet of paper that can be used to find the number of cookies that were left.



3. Ask: *Do you have to combine two numbers or take a number away from another to find the answer? How can you take away 9 cookies? Do you have to exchange any of the blocks? How many cookies were left?*

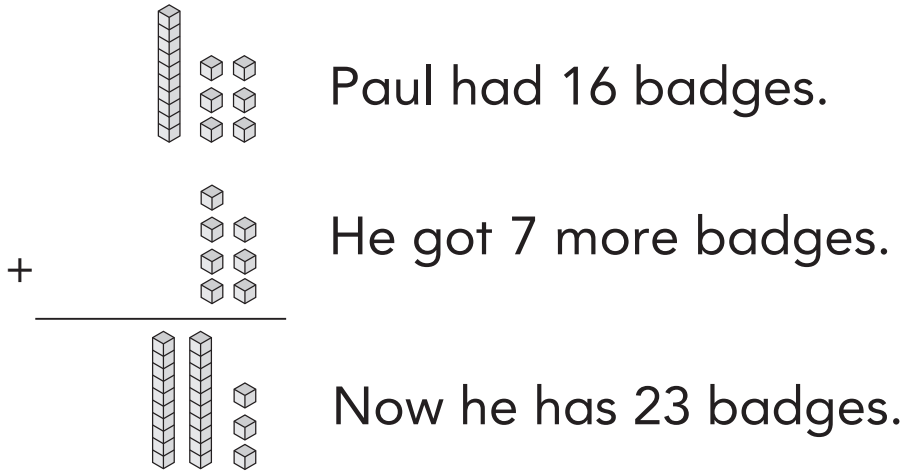
⚠ Look Out!

Watch for children who confuse the operations, either in calculation or in representation with a symbol. Remind children to first decide if they need to combine or take away to find the answer. Prompt them to write the corresponding sequence of numbers and symbols. You also can have children make 9 jumps backward on a naked number line to show the taking away.

Answer Key

Use Base Ten Blocks. Write a number sentence for the model. (Check students' work.)

1.



Number sentence: 16 + 7 = 23

Use Base Ten Blocks. Model the problem. Draw the model. Write a number sentence to solve.

2. Sally had 16 blocks. She gave away 9 of them. How many does she have now?

Number sentence: 16 – 9 = 7

Write a number sentence to solve.

3. I had 7 coins. I got 8 more coins. How many coins do I have now?

Number sentence: 7 + 8 = 15

Answer Key

Challenge! What symbols do you use to write a number sentence for addition? What symbols do you use to write a number sentence for subtraction?

Challenge: + and =; – and =

LESSON 3

Objective

Identify even and odd number patterns.

Common Core State Standards

- **2.OA.3** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Operations and Algebraic Thinking

Even and Odd Number Patterns

Children at this stage have learned to identify some attributes of numbers—such as whether they are greater or less than another number—as well as attributes of geometric shapes. Here, children learn to identify a new attribute of a number: whether it is *odd* or *even.* Learning to recognize odd and even number patterns prepares children for later development of more complex algebraic and geometric concepts.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Have children look at their completed Hundred Charts (BLM 2). **Ask:** *Which numbers from 1 to 10 are odd numbers? Which are even numbers?*
- **Say:** *Look at the even numbers on the Hundred Chart.* **Ask:** *Which digits are in the ones place?* **Say:** *Now look at the odd numbers.* **Ask:** *Which digits are in the ones place?*
- **Ask:** *If a two- or three-digit number has a 0, 2, 4, 6, or an 8 in the ones place, is the number even or odd? If a two- or three-digit number has a 1, 3, 5, 7, or 9 in the ones place, is the number even or odd? What pattern can you see?*

Solve It

With children, reread the problem. **Ask:** *How can Jody find out if everyone in her class will have a buddy?* Ask children to write letters to Jody telling her about even and odd numbers and how she can use what she learns about them to find her answer.

More Ideas

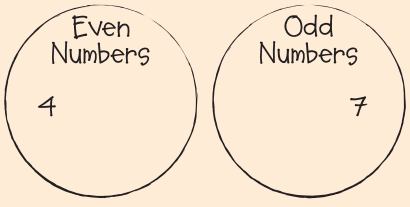
For other ways to teach about even and odd number patterns—

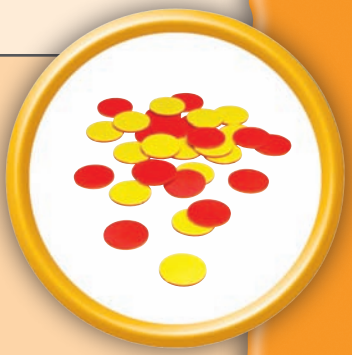
- Have one child pull a handful of Snap Cubes® from a bag. Another child puts the cubes in pairs. Together they determine if the number of cubes is odd or even.
- Distribute copies of Ten Frames (BLM 3) to children. Have children use Two-Color Counters to model numbers in the ten frames. Explain that if a number is even, every counter will have a partner in its row. If a number is odd, there will be one counter without a partner in its row.

Formative Assessment

Have children try the following problem.

Place the following numbers in the sorting circles: 6, 9, 23, 38, 72, 97.





Try It! 30 minutes | Pairs

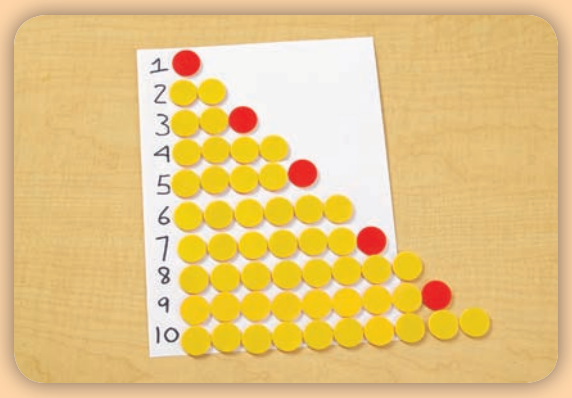
Here is a problem about even and odd number patterns.

Jody is going to the zoo with her second-grade class. Her teacher wants to make sure that everyone has a buddy. There are 27 children in her class. How can Jody find out if every child will have a buddy?

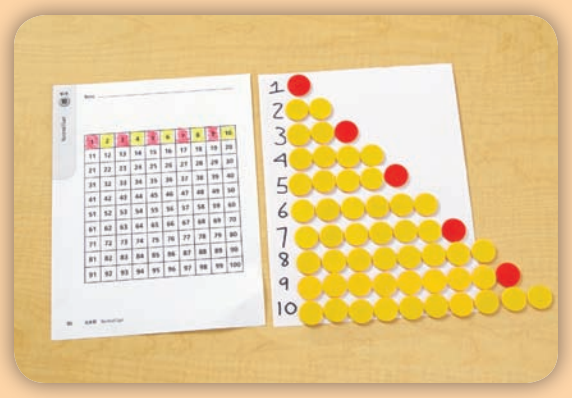
Introduce the problem. Then have children do the activity to solve the problem. Discuss the terms *even* and *odd.* **Say:** *Hold up three fingers.* **Ask:** *Does every finger have a partner?* **Say:** *If every finger has a partner, the number is even. If a finger doesn't have a partner, the number is odd.* Distribute Two-Color Counters, crayons, paper, and a Hundred Chart (BLM 2) to each pair.

Materials

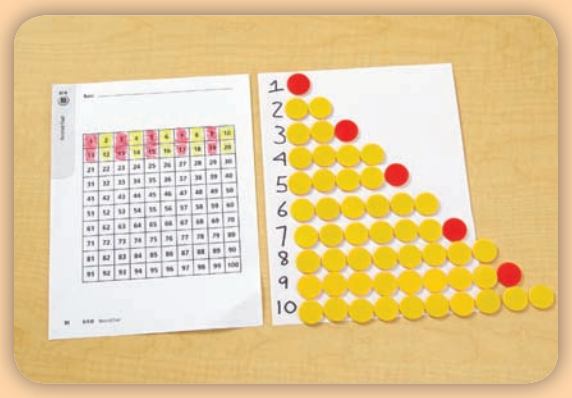
- Two-Color Counters (55 per pair)
- Hundred Chart (BLM 2; 1 per pair)
- paper (1 sheet per pair)
- crayons (1 yellow and 1 red per pair)



1. Have children write the numbers 1 through 10 on the paper, leaving space between numbers, and then model each number with counters. **Say:** *Start with all the counters red-side up. Arrange the counters in pairs when you can.* Tell children to flip the counters yellow-side up whenever they make a pair.



2. Ask: *Which numbers are made up of all pairs?* **Say:** *Use yellow crayon to shade in these number boxes on the Hundred Chart.* **Ask:** *Which numbers in your model have leftover counters that are not in pairs?* **Say:** *Shade those numbers on your Hundred Chart with red crayon.*



3. Ask: *What pattern do you see on the Hundred Chart?* **Say:** *Use the pattern to complete the chart.*

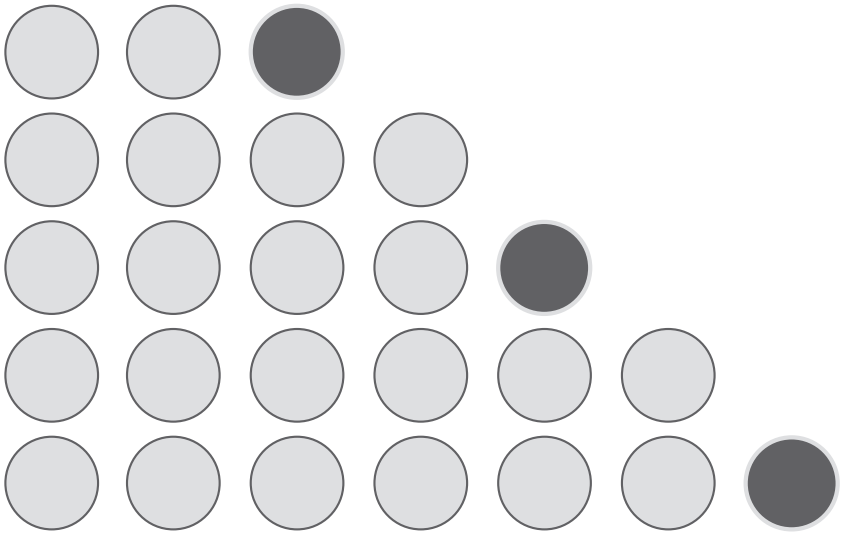
⚠ Look Out!

Watch for children who think numbers ending in zero are neither even nor odd. Have them skip-count by 2s from 2 to 30 and note the numbers that end in zero.

Answer Key

Use Two-Color Counters. Build each number in the rows. Write the number. Circle all odd numbers. (Check students' work.)

1.



3; 4; 5; 6; 7;
3, 5, and 7
should be circled.

Use Two-Color Counters. Build each number. Circle the number if it is odd.

2. 11

Check students' models; number should be circled.

3. 14

Check students' models; number should not be circled

For each number, write odd or even.

4. 2 even

5. 5 odd

6. 19 odd

7. 20 even

8. 1 odd

9. 13 odd

Answer Key

Challenge! What digits can be in the ones place for a number to be even?

Challenge: 0, 2, 4, 6, and 8

LESSON 4

Objective

Relate multiplication to repeated addition.

Common Core State Standards

- **2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Operations and Algebraic Thinking

Repeated Addition

When multiplication is introduced to children as repeated addition, it is not a "new" process, but an expansion of a familiar one. Repeated addition combines identical number groups, for example, 3 + 3 + 3. Multiplication also combines identical number groups, but is more efficient. Children should learn the differences between these two operations while appreciating the equality of the answers.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Have children look at their completed Color Tile Arrays 1 and 2 worksheets (BLM 4, BLM 5). Direct them to Exercise 1. **Ask:** *How many rows are there? How many tiles are in each row? Does each row have exactly the same number of tiles?*
- **Ask:** *How did you use addition to find the number in all?* Invite children to explain how to use repeated addition to find the total number of tiles in the array. Repeat the questions for other exercises on the worksheets.

Solve It

With children, reread the problem. Play an array game like the one described in the problem. For example, **say:** *3 rows with 6 tiles in each row.* Have children build the array and find the number in all. Then invite volunteers to tell how they used repeated addition to find the answer.

More Ideas

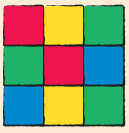
For other ways to teach relating multiplication to repeated addition—

- Have pairs of children use Two-Color Counters to model and solve repeated addition problems. For example, **say:** *Tara saw 2 birds on a tree, 2 birds in a birdbath, and 2 birds on a bird feeder.* **Ask:** *How many birds did Tara see in all?* Have children write an addition sentence (2 + 2 + 2 = 6), and complete this sentence: [3] groups of [2] is [6].
- Have children set out 5 trapezoid Pattern Blocks, cover them with triangle blocks, and find the total number of triangles. Then have them write the addition sentence (3 + 3 + 3 + 3 + 3 = 15) and complete this sentence: [5] groups of [3] triangle blocks is [15].

Formative Assessment

Have children try the following problem.

Circle the number sentence that matches the tiles.



A. 4 + 4 + 4 = 12 **B.** 2 + 2 = 4 **C.** 3 + 3 + 3 = 9

Try It! 30 minutes | Pairs

Here is a problem about relating multiplication to repeated addition.

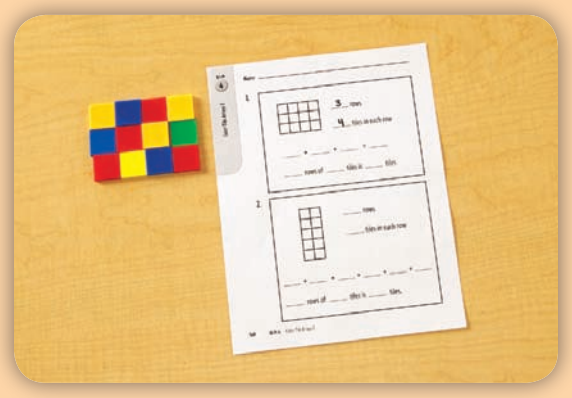
José is playing a game during math club. José's teacher describes a Color Tile array by calling out the number of rows and the number of tiles in each row. Then one child finds the total number of tiles in the array. How can José use the number of rows and the number of tiles in each row to find the number of tiles in all when it's his turn?

Introduce the problem. Then have children do the activity to solve the problem.

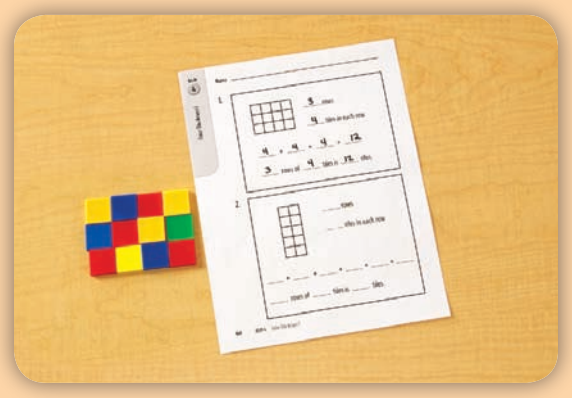
Distribute tiles and copies of the Color Tile Arrays 1 and 2 (BLM 4, BLM 5) to children.

Materials

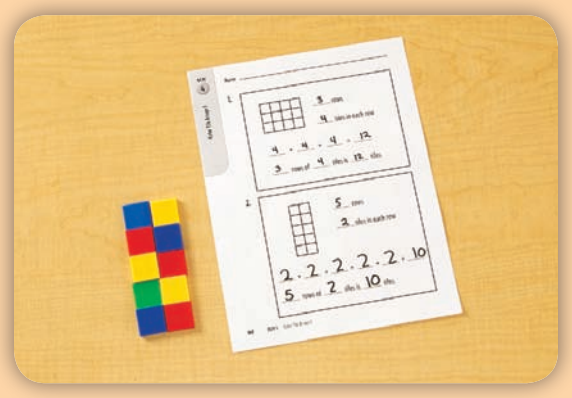
- Color Tiles (50 assorted tiles per pair)
- Color Tile Arrays 1 (BLM 4; 1 per child)
- Color Tile Arrays 2 (BLM 5; 1 per child)



1. Ask: *What do we do when we see the words "in all"? How can we find the number of tiles in all?* Guide children to conclude that they should add. Then instruct children to look at the first array on the worksheet. Have children use their tiles to model the same array. **Ask:** *How many rows are in this array? How many tiles are in each row?* Have children write the correct numbers next to the first array.



2. Explain to children that to find the answer (the number of tiles in all), they need to add the number of tiles in each row (4) 3 times. Have children complete the addition sentence on their Color Tile Arrays 1 worksheet. Then have them complete the final sentence for the exercise by filling in the number of rows and tiles and the number in all.



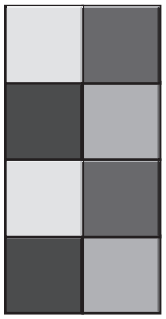
3. Have children repeat steps 1 and 2 to complete Exercises 2 through 4 on the Color Tile Arrays 1 and 2 worksheets.

⚠ Look Out!

Watch out for children who try to add the number of rows instead of adding the number of tiles in each row. Encourage these children to count the number of tiles in row 1 and write that number in the addition sentence, then the number of tiles in row 2, and so on until they have filled in the addition sentence. It might also help these children to build their arrays with a different color for each row. That way, they will more easily see the arrangement of a group of tiles in one row.

Answer Key

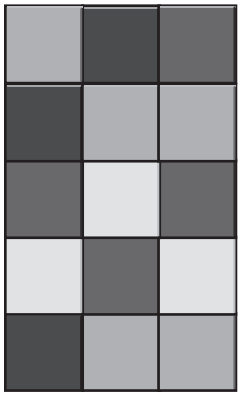
Use Color Tiles. Make each model. Fill in the blanks.

1. 

(Check students' work.)

2 + 2 + 2 + 2

4 rows of 2 tiles is 8 tiles.

2. 

3 + 3 + 3 + 3 + 3

5 rows of 3 tiles is 15 tiles.

Use Color Tiles. Make a 4 × 5 array. Draw the model. Fill in the blanks.

3.

5 + 5 + 5 + 5

4 rows of 5 tiles is 20 tiles.

Answer Key

Challenge! What repeated addition does 6×2 represent?

Challenge: 2 + 2 + 2 + 2 + 2 + 2 = 12