DOUBLE THE DIMENSIONS

NUMBER • MEASUREMENT

- **Volume**
- **Surface area**
- **Spatial visualization**

Getting Ready

What You'll Need

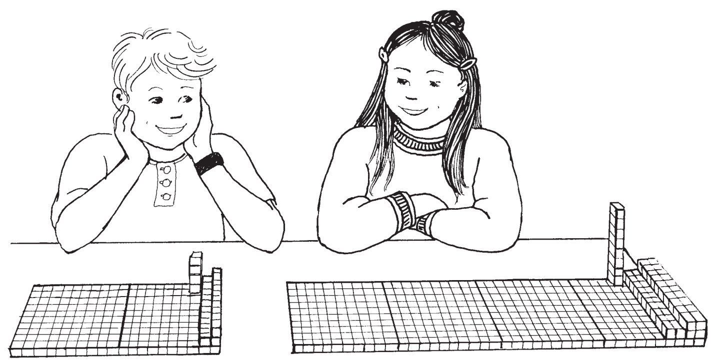
Base Ten Blocks (flats, longs, and units), 2 sets per pair

Base Ten Blocks (thousands cubes), 3 or more per pair (optional)

Overview

Children use Base Ten Blocks to design and build structures. They determine the volume and surface area of their structures and then predict how these will change when they "double" their structures. In this activity, children have the opportunity to:

- increase their spatial visualization skills
- determine and compare volume and surface area
- predict the volume and surface area of a "doubled" structure



The Activity

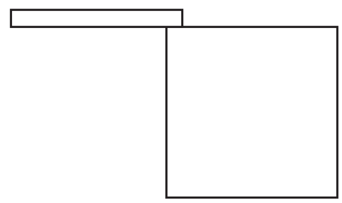
Introducing

- Have each child examine a base ten long as you ask the class to determine the volume of the long.
- Elicit that since *volume* can be represented by the number of cubic centimeters that make up a solid figure, the volume of a long is 10 cm^3.
- Challenge children to find the surface area of the long.
- Be sure children understand that since *surface area* is the total number of square units that completely cover the outside of a shape, the surface area of a long is 42 cm^2.
- Now have each child examine a flat and determine its volume and surface area.
- Establish that the volume of a flat is 100 cm^3 and that the surface area is 240 cm^2.
- Put a flat and a long together in any way so that at least 1 square centimeter of each touches the other. Identify this block arrangement as a "structure."
- Lead children to talk about how they could find the volume and surface area of this structure.

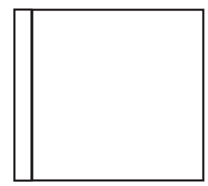
On Their Own

How can you use Base 10 Blocks to determine how the volume and surface area of a structure change when you double its dimensions?

- Work with a partner. Each of you build a structure using any Base 10 blocks according to these rules:
  - *All blocks must either lie down flat or stand up on end. They may not lean.*
  - *Each block must touch another block by at least 1 square centimeter.*
- Work together to find the volume and the surface area of each of your structures. Remember: Volume is the number of cubic units that make up a structure. Surface Area is the number of square units that completely cover a structure. Here are two examples:



Volume: 110 cm^3
Surface Area: 280 cm^2



Volume: 110 cm^3
Surface Area: 262 cm^2

- Now, think about how you can double your first structures. Predict what the volume and surface area of each doubled structure would be.
- Without taking your first structures apart, build new structures with doubled dimensions. Find and record the volume and surface area of each doubled structure.
- Be ready to tell how you doubled your structures and how the new dimensions compare to the old ones.

The Bigger Picture

Thinking and Sharing

Allow children to examine the structures built by various pairs, telling them to compare each first structure with its doubled version.

Use prompts like these to promote class discussion:

- How did you go about doubling your first structure?
- What did you do to find volume? What did you do to find surface area?
- Did you find any surprises when you doubled a structure? Explain.
- What changes in volume and surface area did you notice between a first structure and a doubled structure?
- Which way of doubling a structure do you think was best? Explain.

Writing

Have children describe why architects or designers might need to know the volume and/or surface area of a structure or container.

Teacher Talk

Where's the Mathematics?

For this activity, it is intended that children *not* be given a definition of "doubling." Part of their work is to decide this for themselves. Discussion between partners is likely to explore several ideas about doubling. Once a pair has decided how to double its structures, children are likely to predict that the volume and the surface area of their new structures will be double those of their first structures.

The first structures that children build may be simple, involving just a few blocks, or they may be complex, involving many blocks and reflecting elements of balance and design. Whatever the size and complexity of a structure, partners will determine that to find its *volume* they must find the total number of unit blocks that it contains. Children may notice that however they position each of the blocks in their structure, the volume remains unchanged.

To find the *surface area* of a structure, children may count or add to find the sum of all the unit squares visible on all the surfaces. Some pairs will realize that blocks "lose" surface area wherever they touch. They may then realize that the blocks used for a structure will necessarily have less surface area than they would if used individually.

At some point, children may decide to record the individual surface areas of the Base Ten Blocks for reference.

SURFACE AREA OF BLOCKS	
cube	600 cm^2
flat	240 cm^2
long	42 cm^2
unit	6 cm^2

Most children will realize that the surface area of a structure is equal to the total of the surface areas of all the blocks in the structure minus some number of square centimeters—those that are "lost" wherever blocks touch. Since each block loses at least 1 square centimeter at the point at which two blocks touch, 2 cm^2 are lost from the total "separate-block" surface area for every square centimeter joined. Children who come to this realization may, therefore, determine the surface areas of their structures by adding the individual surface areas of the blocks and then subtracting 2 cm^2 for every join at which two square centimeters touch.

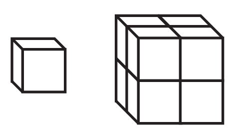
Some pairs may decide to build their new structures by doubling a single dimension of their first structures—length or width or height. Others may decide to make exact duplicates of their first structures and then join the duplicates together along a line of length, width, or height. Either of these methods should result in their new structures having twice as many blocks as their first structures. In contrast, other pairs may decide to start out by doubling the number of blocks used in their first structures and build the new structures without regard to doubling separate dimensions or replicating appearance. If they choose any of these ways, children are likely to notice that doubling a structure means doubling the volume but not the surface area.

When children add on Base Ten Blocks in their effort to double structures they are likely to see that surface area depends to a great extent on the kinds of joins they choose to make—that blocks joined by the least number of square centimeters will have the greatest surface areas, while the same blocks joined along many square centimeters will have the least surface areas.

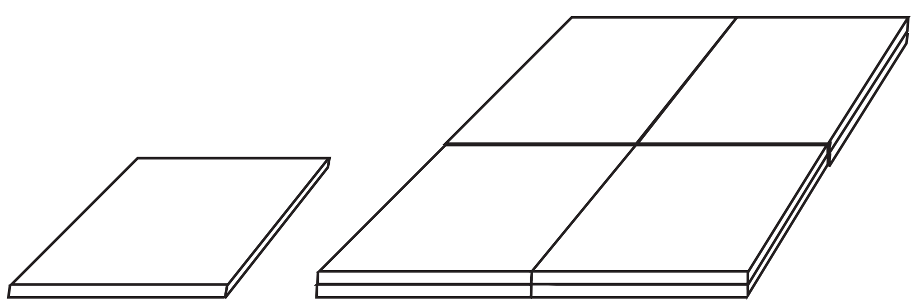
Some children may decide to build their doubled structure by doubling the "footprint," or base area, of their first. This may spark a discussion between partners about whether this means doubling the base area—for example, changing a 100-cm^2 footprint to 200 cm^2—or doubling the length and width of the footprint—for example, changing a 100-cm^2 footprint to 400 cm^2.

Similarly, partners may decide that building their doubled structure twice as high, twice as long, and twice as wide as their first is the only way to truly double the first. Children who do this will probably build the doubled structure beside the first structure. They may be surprised to realize that doubling each of the three dimensions exactly requires them to use, not twice as many blocks, but eight times as many! The doubled structure, then, will have a *volume that is eight times greater* than the first structure and a *surface area that is four times greater* than that of the first. For example:

> A $1 \times 1 \times 1$ cube is represented by just one unit block, but a $2 \times 2 \times 2$ cube requires eight unit blocks.



> A $10 \times 10 \times 1$ "floor" is represented by just one flat, but a $20 \times 20 \times 2$ floor requires eight flats.



When children understand these disparities, they may argue that what they have done is not really doubling. They are likely to conclude that doubling a structure is best shown by doubling volume exactly. While children may conclude that the directions for doubling need to be more specific, their various opinions of what doubling means can help them refine their own thinking.