AoPS Online
Art of Problem Solving

If you can solve nearly all of the following problems with little difficulty, then the Art of Problem Solving text **Intermediate Algebra** would only serve as a review for you.

1. Solve for real and complex solutions to each of the following equations:
   (a) $7x^2 - 17x = -101$
   (b) $\sqrt{x-5} + \sqrt{x+15} = 10$
   (c) $\sqrt[3]{x^2-1} + \dfrac{20}{\sqrt[3]{x^2-1}} = 12$
   (d) $x^6 = 1$
2. The sum of the roots to a certain quadratic equation is 20. The product of the roots is 91. What are the roots of the quadratic?
3. Find integers x and y ($x > y$) that satisfy $x + y + xy = 223$ and $x^2y + xy^2 = 5460$.
4. Simplify this expression: $\sqrt[4]{161 - 72\sqrt{5}}$
5. Factor completely $6x^8 - 25x^7 - 31x^6 + 140x^5 - 6x^3 + 25x^2 + 31x - 140$.
6. If a is an integer, what rational numbers could satisfy the equation $6x^3 - 17x^2 + ax = 35$?
7. Find integers a, b, and c such that the equation $x^4 + ax^3 + bx^2 + cx + 4 = 0$ has four distinct integer solutions.
8. For $x > 0$, find the minimum possible value of $4x + \dfrac{9}{x}$.
9. If $x + \dfrac{1}{x} = 5$, find the value of $x^5 + \dfrac{1}{x^5}$.
10. If $f(n)$ is a second degree polynomial such that $f(0) = 7$, $f(1) = 13$, and $f(2) = 23$, find $f(3)$.
11. What is the sum of the coefficients in the expansion of $(4x - 2y)^8$?
12. For how many of the first 500 natural numbers, n, will the equation
$$n = \lfloor 2x \rfloor + \lfloor 4x \rfloor + \lfloor 8x \rfloor + \lfloor 20x \rfloor$$
have solutions?
13. Find (x, y, z) such that
   i $x + y + z = 23$, and
   ii $xy + yz + zx = 144$, and
   iii $xyz = 252$, and
   iv $x > y > z$
14. If $P(x)$ denotes a fifth degree polynomial such that $P(k) = \dfrac{k}{k+1}$ for $k =$0, 1, 2, 3, 4, and 5, determine $P(6)$.
15. Find all functions that satisfy the identity $f(x + 5y) + f(x - 5y) = 2x^2 + 50y^2$.

16. Prove that there is no polynomial $P(x)$ with integer coefficients such that $P(1) = 2$, $P(2) = 3$, and $P(3) = 1$.

Don't look at the next page until you've attempted all the problems!

The answers to Do You Know **Intermediate Algebra** are below.

1. (a) $\frac{17\pm i\sqrt{2539}}{14}$
   (b) 21
   (c) ± 3, $\pm\sqrt{1001}$
   (d) ± 1, $\frac{1\pm i\sqrt{3}}{2}$, $\frac{-1\pm i\sqrt{3}}{2}$
2. 7 and 13
3. $x = 15$, $y = 13$
4. $\sqrt{5} - 2$
5. $(x-1)(x-4)(2x-5)(3x+7)(x^4+x^3+x^2+x+1)$ (Yes, there is a faster way than just plowing ahead with synthetic division.)
6. The 32 possible rational roots are all in the form $\pm\frac{m}{n}$ where m takes on each of the values 1, 5, 7, and 35, and where n takes on each of the values 1, 2, 3, and 6.
7. $a = 0$, $b = -5$, and $c = 0$
8. 12
9. 2525
10. 37
11. 256
12. 353
13. (14, 6, 3)
14. 1
15. $f(x) = x^2$
16. Consider the general polynomial $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$. Notice that $P(r) - P(s) = a_1(r-s) + a_2(r^2 - s^2) + \ldots + a_n(r^n - s^n) = (r-s)Q(r,s)$ for some integer $Q(r,s)$. This means that $2 - 3 = P(1) - P(2) = -Q(1,2)$, $3 - 1 = P(2) - P(3) = -Q(2,3)$, and $1 - 2 = P(3) - P(1) = 2Q(3,1)$. This gives a non-integer $Q(3,1)$, which is a contradiction; thus, no such polynomial $P(x)$ exists.