Expressions and Equations

Algebra can be thought of as a "language" for generalizing arithmetic. When we take real-world situations and translate them into mathematical phrases or statements, we are *expressing* them in this language.

An algebraic **expression** is a combination of numbers, variables, and operations done with them. For example, $x + y + z$ and $2x - 9$ are algebraic expressions. An algebraic **equation** is a statement that declares equality between expressions, $x + y + z = 2x - 9$, for example.

Related to equations is a different kind of statement called an *inequality*. While an equation is a statement of comparison that says two expressions are equal, an inequality might say that one expression is less than another.

The Grade 6 Common Core State Standards for Expressions and Equations specify that students should–

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

At this level, students start learning how to work with expressions and equations to do a variety of useful things. Many students will feel a newfound sense of power in using expressions and equations to solve problems. Others will struggle with the new "language." The following hands-on activities will help students visualize these new concepts. The activities can help them develop deeper understanding and build on the algebraic thinking and reasoning skills learned in previous grades.

While working through the activities, teachers will want to encourage students to look closely for patterns in the structure of algebraic expressions and equations. Discerning relationships and patterns helps students expect outcomes and predict solutions, which they monitor and adjust accordingly. This process is important for becoming mathematically proficient.

Expressions and Equations

Contents

LESSON 1

Objective

Write and evaluate an expression with a variable.

Common Core State Standards

- **6.EE.2a** Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*
- **6.EE.2c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*
- **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Expressions with a Variable

As students become more fluent in computation, they begin to understand that mathematical relationships are not always static. For example, most students recognize that the total cost of movie tickets depends upon the number of tickets purchased. Using a variable as a placeholder for an unknown value allows them to communicate this relationship between cost and quantity. Students should recognize that variables are any letter or symbol used to represent a number.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How can the expressions* n × 5 *and* c × 5 *describe the same situation?*
- **Say:** *I have some green tiles and six red tiles in my hand.* **Ask:** *What expression describes how many tiles I have?* Have students suggest other situations that can be described using variable expressions. Elicit examples for addition, subtraction, multiplication, and division.
- Have students explain how to evaluate *x – 3* for *x = 10, 28,* and *52.*

Solve It

Reread the problem with students. Have them write a paragraph explaining the meaning of the expression *n "times" 5* and how they can use models and symbols to evaluate the expression for the sixth car.

More Ideas

For other ways to teach about expressions with variables —

- Write *a – 2, b + 2, 2c,* and *d ÷ 2* on the board. Have students give different ways to read each expression, such as *a minus 2* and *2 less than a* for *a – 2.* Have students use Two-Color Counters to model the four expressions when the variable in each equals 4 and when it equals 10. Discuss how to substitute numbers to evaluate expressions symbolically.
- Use polyhedral dice to provide students with more practice. Have students write an expression, state different ways to read the expression, and then roll the dice. They use the rolled number to evaluate the expression.

Formative Assessment

Have students try the following problem.

Evaluate 33 + z *for* z = 11.

A. 3 **B.** 11 **C.** 22 **D.** 44

Try It! 30 minutes | Groups of 4

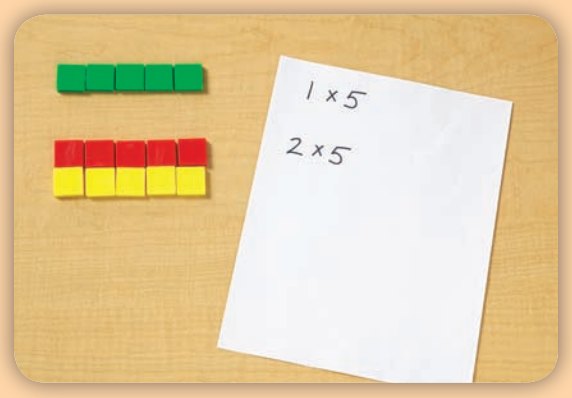
Here is a problem about expressions with a variable.

It takes 5 minutes to wash each car at Details Car Wash. The total time needed to get a car washed depends upon the car's position in line. Write an expression to show the number of minutes it will take Sally to get her car washed. Then evaluate the expression to determine how long it takes Sally to get her car washed if her car is the sixth car in line.

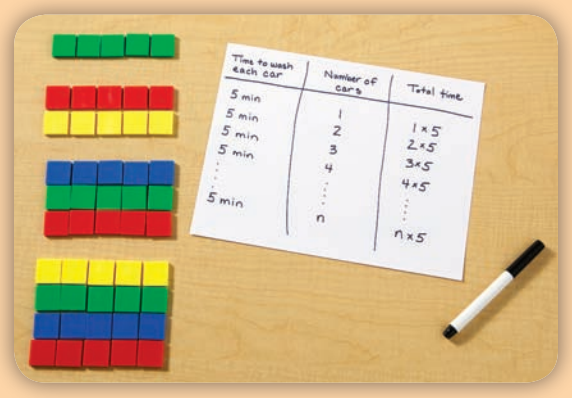
Introduce the problem. Then have students do the activity to solve the problem. Distribute Color Tiles, paper, and pencils to students. Write *1* × *2* and *n* × *2* on the board. Point out that each statement is an expression because it contains only numbers and/or symbols and operations (no equal sign).

Materials

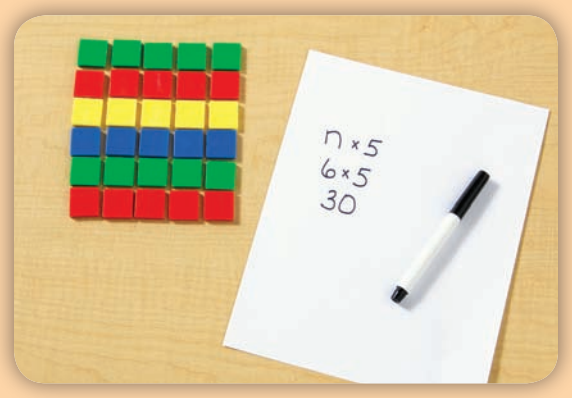
- Color Tiles (100 per group)
- paper (3 sheets per group)
- pencils (1 per group)



1. Say: *Let each tile represent one minute. Make an array to show how many minutes it will take Sally to get her car washed if she is the first car in line.* Have students write the expression shown by the array. Repeat for 2, 3, and 4 cars.



2. Have students create a table to organize the information shown by each array. **Ask:** *What changes in each expression?* Write *n* × *5* on the board. Introduce the term *variable*.



3. Say: *Find the length of the wait when n = 6.* Explain that finding the value of an expression is called *evaluating* the expression. Have students model the situation.

⚠ Look Out!

Students who have difficulty writing variable expressions may find it helpful to first think of the situation as static and use numbers to help them identify the relationship. Once they understand the relationship, they can replace the "made-up" number in the expression with a letter. Remind students that when they evaluate an expression, the operations and numbers (constants) in the expression do not change. Suggest that they think of the variable as the only amount that can vary, or change.

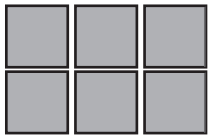
Answer Key

Use Color Tiles to model each term. Then use the rule to write an expression for any term, *n*.

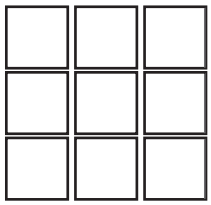
(Check students' work.)

1.

1 × 3



2 × 3

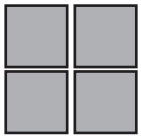


3 × 3

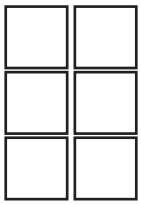
n × 3

2.

1 × 2



2 × 2



3 × 2

n × 2

Using Color Tiles, model the expression. Then evaluate the expression for the given values of *n*. Sketch the models.

3. 5 × *n*, when *n* = 1, 3, and 5

5; 15; 25

4. *n* + 2, when *n* = 2, 6, and 10

4; 8; 12

Evaluate each expression when *n* = 1, 4, 9, and 0.

5. 4 × *n*

4; 16; 36; 0

6. *n* + 10

11; 14; 19; 10

7. *n* × 6

6; 24; 54; 0

8. 1 × *n*

1; 4; 9; 0

9. *n* + 3

4; 7; 12; 3

10. 2*n*

2; 8; 18; 0

Answer Key

Challenge! How many times can you evaluate the expression $n \times 9$? How do you decide the value to use for n?

Challenge: (Sample) unlimited; There is an infinite number of numbers to use for the variable.

LESSON 2

Objective

Understand variables and the impact that a change in the value assigned to a variable can have on an algebraic relationship.

Common Core State Standards

- **6.EE.2c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*

Variables x, x^2, and Constants

As students advance through elementary school mathematics, they learn that variables are symbols that vary in value. Variables are usually represented by letters of the alphabet, such as x or y. Students also should understand that a constant does not vary in value. All numerals are constants, for example. At the middle school level, students must now expand their understanding to include the effect that a change in a variable will have on an algebraic relationship.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is the value of* x*? Why is the* x^2 *block commonly called the "*x *squared" block? What is its length? What is its width? What is its value if* x = 1*? If* x = 2*? If* x = 3*?*
- **Ask:** *How do you know you have correctly represented this expression?*
- **Ask:** *What would the value be if* x = 2*? If* x = 1*?*

Solve It

Reread the problem with students. Have students give another example of an algebraic expression. Ask them to change the value assigned to x. Have them write their understanding of how the value of the expression changes.

More Ideas

For other ways to teach about variables and constants—

- Have students set up the XY Coordinate Pegboard for graphing in Quadrants I and II. Have students plot x^2 using the blue pegs and $x^2 + 1$ using the red pegs. They should use positive and negative values for x. Discuss the effect the constant has on the graph.
- Have students create an input/output table of values for the equation $y = x^2 + x - 6$. Then have students peg the first few coordinate values on the XY Coordinate Pegboard. Then have students transfer the points to graph paper and continue plotting.
- Have students use Algeblocks to evaluate the expression $x^2 + 3x + 1$, when $x = 3$.

Formative Assessment

Have students try the following problem.

Find the value of the expression when x = 5.

$3x^2 + 4x - 6$

A. 239 **B.** 89 **C.** 54 **D.** 44

Try It! 20 minutes | Pairs

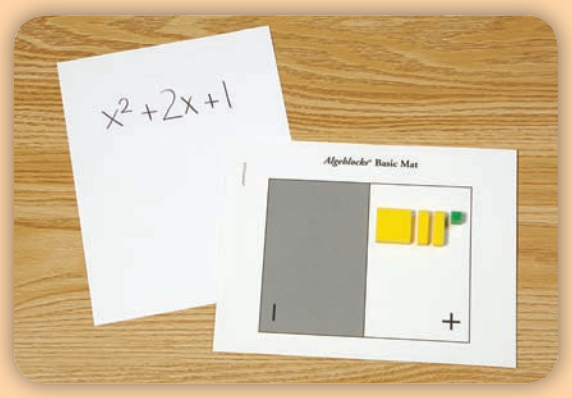
Here is a problem about variables and constants.

Kevin sells intricate glass mosaics at craft shows. He figures the price of each mosaic with the formula $x^2 + 2x + \$1$, where x represents the size of the mosaic in square inches. A customer has \$30 to spend. Will the customer have enough money to buy a mosaic that has an area of 4 square inches?

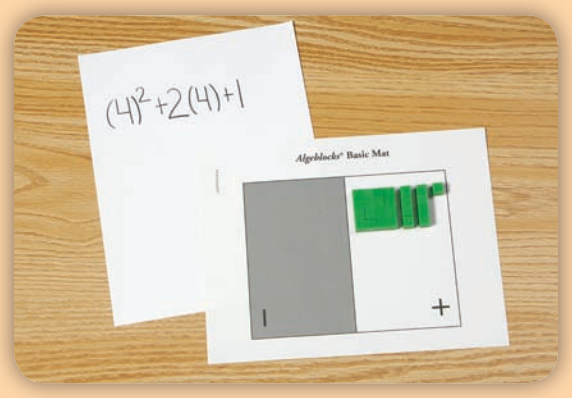
Introduce the problem. Then have students do the activity to solve the problem. Distribute Algeblocks and mats to students.

Materials

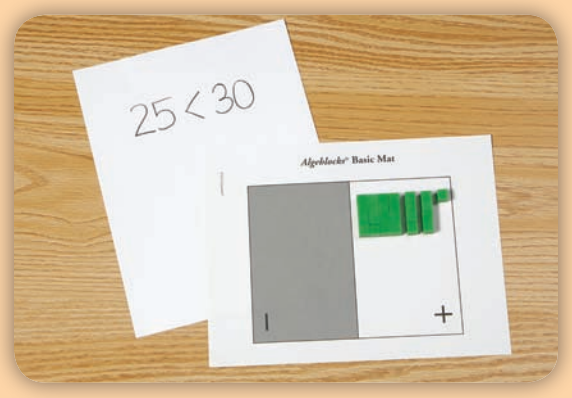
- Algeblocks® (x^2, x, and unit blocks)
- Algeblocks Basic Mat (BLM 6, 1 per pair)



1. Have students lay out the Algeblocks that represent the following algebraic expression: $x^2 + 2x + 1$ on their Basic Mat.



2. Have students exchange each variable for the appropriate value in unit cubes. Students count the unit cubes to determine the value of the expression when $x = 4$.



3. Ask: *How many unit cubes do you have in total? Is the number greater than or less than 30?*

⚠ Look Out!

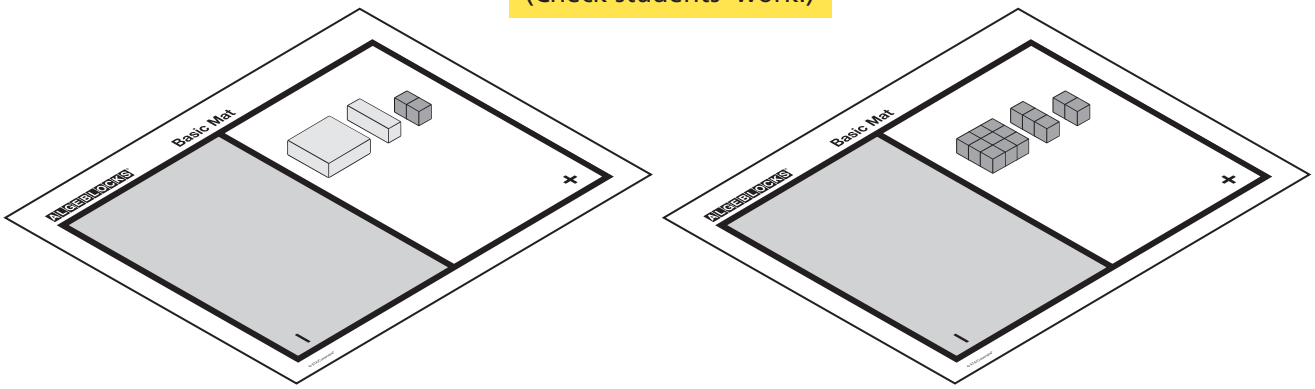
Make sure students do not forget to add the constant value after exchanging their variables for unit cubes. Explain that the constant unit cubes should be identical on both sides of the equation because constants never change value.

Answer Key

Use Algeblocks and an Algeblocks Basic Mat to model the expression. Evaluate the expression and complete each inequality.

1. $x^2 + x + 2$

(Check students' work.)



When $x = 3$, 14 (<) 28

When $x = 4$, 22 (>) 20

Using Algeblocks and an Algeblocks Basic Mat, model each expression and evaluate for the given values of *x*. Sketch the models.

2. $x^2 + 2x + 5$

When $x = 5$: 40

When $x = 10$: 125

3. $3x^2 + x + 1$

When $x = 1$: 5

When $x = 3$: 31

Evaluate each expression for the given values of *x*.

4. $2x^2 + 4x + 2$, when $x = 1$

8

5. $x^2 + x + 8$, when $x = 4$

28

6. $3x^2 + 2x + 4$, when $x = 2$

20

7. $x^2 + 5x$, when $x = 5$

50

Answer Key

Challenge! When you substitute a value into the expression $3x^2 + x + 11$, in what order do you simplify the operations? Choose a value for x and show the steps you use to find the value of the expression.

Challenge: (Sample) Use the order of operations. Simplify the exponents first, then multiply, and finally add.

LESSON 3

Objective

Simplify an expression by combining like terms.

Common Core State Standards

- **6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Expressions and Equations

Combining Like Terms I

Students learned how to evaluate an expression in an earlier lesson. Now they will learn to consider the other action that can be performed with expressions: rewriting an expression. Expressions that contain like terms can be rewritten in simplest form by combining those like terms, a process known as *simplifying.*

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *Use the Commutative Property of Addition to rearrange like terms next to each other. Combine terms in order,* x, y, *then constants.*
- **Ask:** *Why would we want to combine like terms?*

Solve It

Reread the problem with students. Have them create the block pattern with their Algeblocks. Then have them draw the pattern on a sheet of paper and label what each block represents. Have students arrange the blocks in order and combine like terms. Have students write their simplified answer on a sheet of paper.

More Ideas

For other ways to teach about combining like terms—

- Have students use Algeblocks® to create new block patterns, using several of each kind of block. Ask students to combine like terms to determine how many of each block they used.
- Have students use Algebra Tiles™ to make various patterns or expressions and combine like terms.
- Extend the activity to include negative coefficients. Have one student place 10 Algeblocks chosen at random on the positive side of the Basic Mat. Have a second student place 10 Algeblocks chosen at random on the negative side. Have students write a block-by-block expression of their results. Then have them group like blocks, remove zero pairs, and write the simplified expression.

Formative Assessment

Have students try the following problem.

Simplify the given expression.

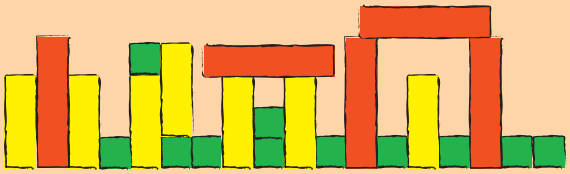
$$6x + 1 + x + 2y + 3 + 2x + y + 4$$

A. $8x + xy + 2y + 8$
B. $8x + 3y + 7$
C. $12xy + 8$
D. $9x + 3y + 8$

Try It! 20 minutes | Pairs

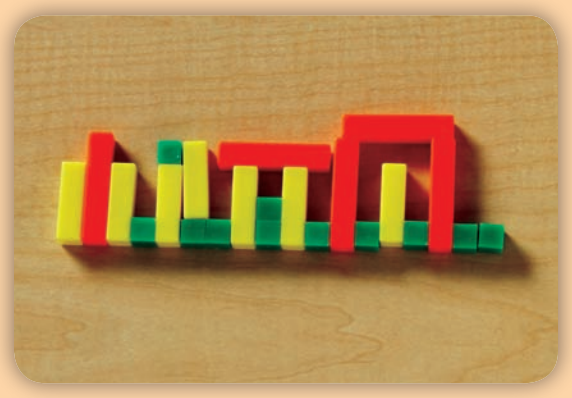
Here is a problem about combining like terms.

Ana has created the sculpture shown here for a modern art exhibit at her school. She wants to name the sculpture according to the blocks it contains. She has identified the blocks (roughly in order) as 1 + x + 1 + y + x + 1 + y + 3 + x + x + y + 1 + x + x + 1 + x + y + y + 3. She realizes, however, that she has to simplify the expression. What is the correct name of her sculpture?

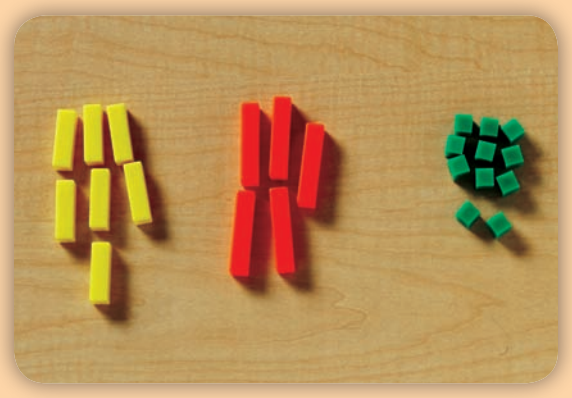


Introduce the problem. Then have students do the activity to solve the problem. Distribute Algeblocks to students.

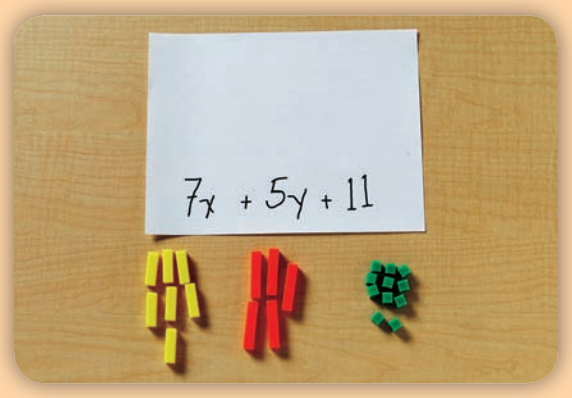
Materials
- Algeblocks® (*x*, *y*, and unit blocks)



1. Have students use Algeblocks to re-create Ana's block design.



2. Have students combine all similar blocks in order from left to right—*x*, *y*, then units.



3. Have students write the simplified expression on a sheet of paper next to the blocks.

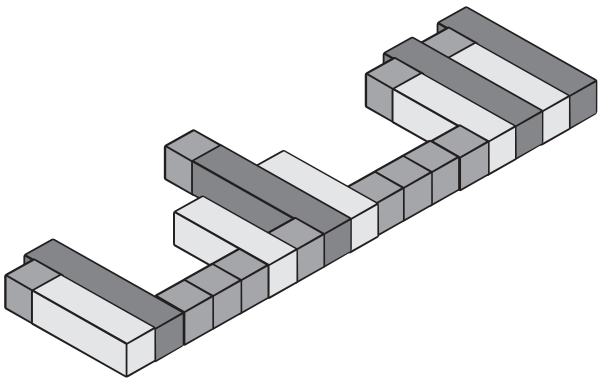
⚠ Look Out!

Watch for students who miscount terms or put the terms in the wrong order. Students may find it helpful to organize the Algeblocks groups in the correct order (*x*, *y*, and units). If they are using written notes, have them mark through the terms as they combine them so that they know what they have combined and what they have left to do.

Answer Key

Use Algeblocks to model the expression. Combine like terms. Write the terms. Then write the simplified expression. (Check students' work.)

1.



x terms: 5

y terms: 4

constant: 12

= $5x + 4y + 12$

Using Algeblocks, model the expression. Combine like terms. Write the simplified expression.

2. $x + 2y + 3x + 5 + 2x + 3y + 2y + 6 + 4y + 5x + y + 4x$

$15x + 12y + 11$

Combine like terms. Write each expression in simplified form.

3. $5x + 3y + 5 + 4x + 3y + y + 8$

$9x + 7y + 13$

4. $6y + 4x + 2x + 9y + 3y + 2$

$6x + 18y + 2$

5. $8y + x + 2 + 6y + 4y + y$

$x + 19y + 2$

6. $x + 3y + 1 + 2x + 3x + y + 1$

$6x + 4y + 2$

7. $x + y + 10 + x + 6x + 10$

$8x + y + 20$

8. $10y + 4y + 2 + 4 + 2y + y + 5$

$17y + 11$

Answer Key

Challenge! Are x and x^2 like terms? Explain.

Challenge: (Sample) no; The terms do not have the same exponent. They are not like terms.

LESSON 4

Objective

Simplify an expression by combining like terms.

Common Core State Standards

- **6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Expressions and Equations

Combining Like Terms II

The students' concept of perimeter can be linked to algebra. In this lesson, the link to algebra is established by using various Algeblocks to represent the sides of an irregular shape (rather than providing students with actual dimensions). Students apply the definition of perimeter to generate an expression that can be simplified by combining like terms.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *There are no dimensions given in this problem. How can we refer to each of the pieces that make up the floor plan?*
- **Say:** *Assign a value to* x *and then find the perimeter of Sara's shop.*

Solve It

Reread the problem with students. Ask them for at least two methods they can use to find the perimeter of any given shape. Invite students to create other floor plans for Sara's shop using the same Algeblocks pieces. Is the perimeter the same?

More Ideas

For another way to teach about combining like terms—

- Have students build shapes using Algebra Tiles™. For each shape, students can find an expression for the perimeter and simplify the expression as necessary.

Formative Assessment

Have students try the following problem.

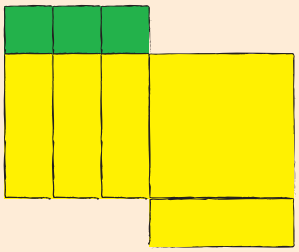
What is the perimeter of the figure shown here?

A. $3x + 6$

B. $4x + 3$

C. $4x + 6$

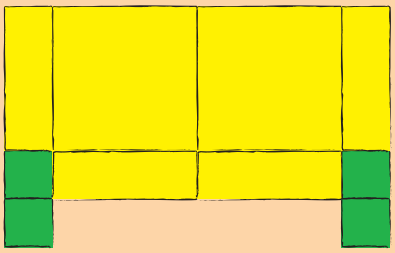
D. $4x + 10$



Try It! 15 minutes | Pairs

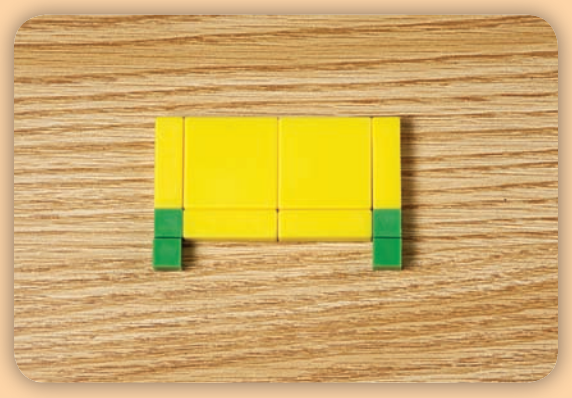
Here is a problem about combining like terms.

Here is the floor plan for Sara's bead shop. She has divided the space into a number of sections as shown. Now she wants to add a wallpaper border all around the walls near the ceiling. She needs to determine the perimeter of the shop so that she can buy enough wallpaper border. What is the perimeter of her shop?

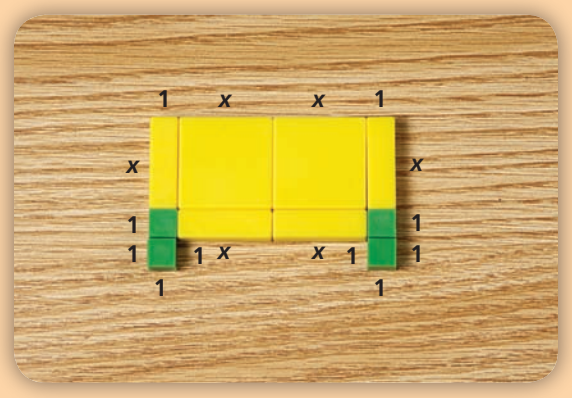


Introduce the problem. Then have students do the activity to solve the problem. Distribute Algeblocks to students.

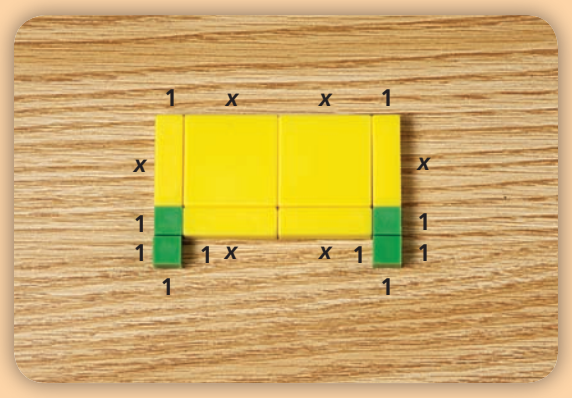
Materials
- Algeblocks® (x^2, x, and unit blocks)



1. Have students use the Algeblocks to build the irregular shape as shown.



2. Have students determine the dimensions of the figure by using x for the length of a yellow block and 1 for the end of a yellow block. The green block is 1 unit by 1 unit.



3. Have students combine like terms to determine the perimeter of the figure, $6x + 10$.

$P = 1 + 1 + x + x + 1 + 1 + 1 + 1 + x + 1 + x + x + 1 + x + 1 + 1$

$P = 6x + 10$

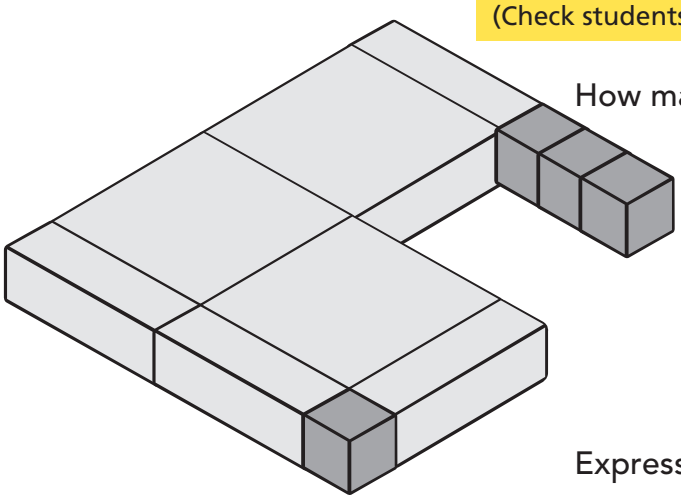
⚠ Look Out!

Students may be tempted to count the length of the x block as 3 units since it is close to 3 units long. Remind students as necessary that the x block represents x and not 3.

Answer Key

Use Algeblocks to model the irregular shape shown. Answer the questions. Write an expression for the perimeter of the shape in simplest form.

1.



(Check students' work.)

How many sides does the shape have? 8

What expressions represent the lengths of the sides?

(The side lengths are given starting with the side along the top of the shape and moving clockwise around the shape.)

Expression for perimeter: $2x + 2 + x + 3 + 1 + 3 + x + x + 1 + x + 1 + 2x + 1$; $8x + 12$

Using Algeblocks, build an irregular shape. Sketch the model. Write expressions for the lengths of each side. Write the expression for the perimeter of the shape.

2.

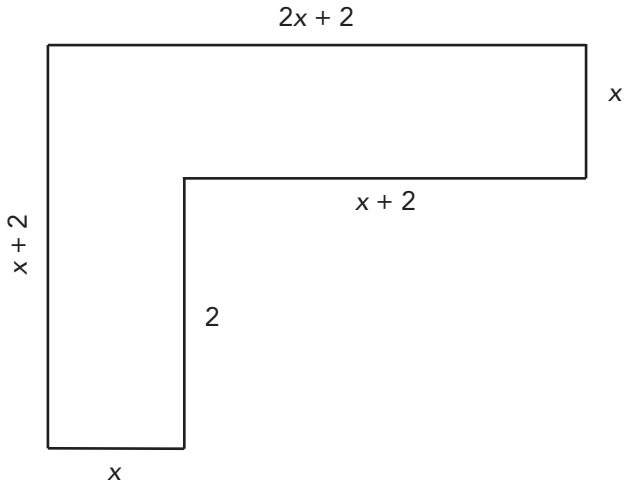
Check students' models; answers will vary depending on models.

Lengths of the sides:

Expression for perimeter:

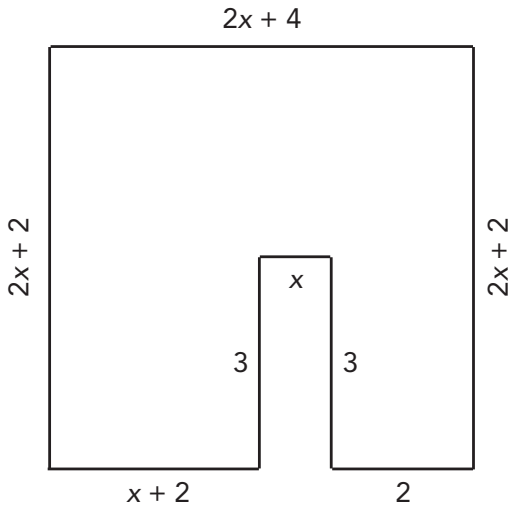
Write an expression for the perimeter of each shape.

3.



$6x + 8$

4.



$8x + 18$

Answer Key

Challenge! Write a description of how to find the perimeter of an irregular shape when the lengths of the sides are expressions that contain variable and constant terms. Use an example.

Challenge: (Sample) Add all of the like variable terms together and add the constant terms together.

LESSON 5

Objective

Use the Distributive Property to expand an algebraic expression.

Common Core State Standards

- **6.EE.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Expressions and Equations

Algebraic Equivalencies: Distributive Property

In previous grades, students have worked with the Distributive Property in multiplying numbers. Now they will learn to apply this property as they multiply a monomial by a binomial to find an equivalent algebraic expression.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is the Distributive Property? How can it be applied to this problem?*
- **Ask:** *What does it mean to "simplify" an expression?*
- **Ask:** *How do you know that both sides of the equation you wrote are equivalent?*

Solve It

Reread the problem with students. Review the formula for finding the area of a rectangle, if necessary. Ask them to use Algeblocks to represent the problem and its solution on the Quadrant Mat. Students should be able to refer to the visual model as a guide to writing the equivalent algebraic expression.

More Ideas

For other ways to teach about the Distributive Property—

- Extend the activity by having students insert numerical values for *x.* Have them exchange the variable Algeblocks® with unit blocks to determine a numerical answer.
- Have students grab a handful of *x* and unit blocks at random. Have them arrange these on the *x*-axis and *y*-axis of the mat as they please. They should then solve the multiplication problem they have formed and write the equation for the problem.

Formative Assessment

Have students try the following problem.

Which of the following is an example of the Distributive Property?

A. $4 + x = x + 4$
B. $3x + 2x + 1 = 5x + 1$
C. $(x + y) + 5 = x + (y + 5)$
D. $2(y + 3) = 2y + 6$

Try It! 30 minutes | Pairs

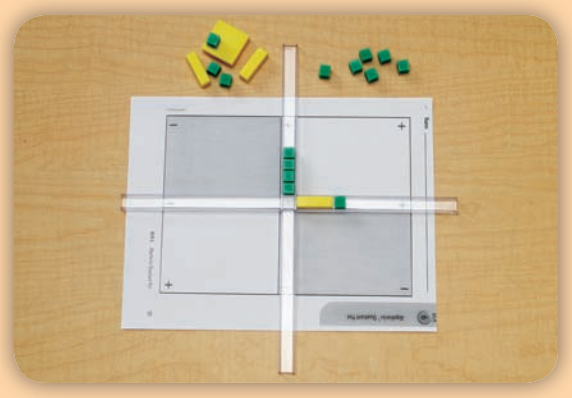
Here is a problem about using the Distributive Property.

Kelsey and Nick are trying to determine the area of their rectangular patio. Their father told them the width is 4 and the length is x + 1. What is the area of Kelsey and Nick's patio? Write an algebraic expression expressing the area.

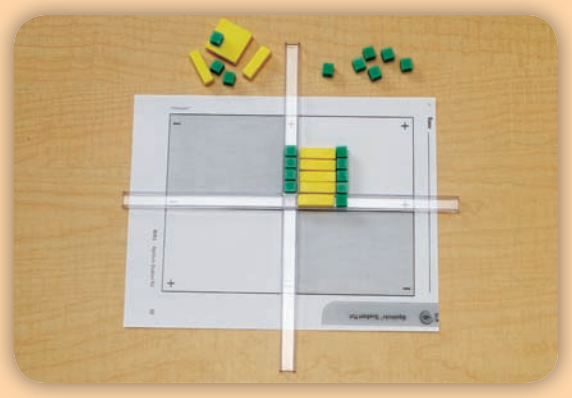
Introduce the problem. Then have students do the activity to solve the problem. Distribute Algeblocks, Factor Tracks, and mats to students.

Materials

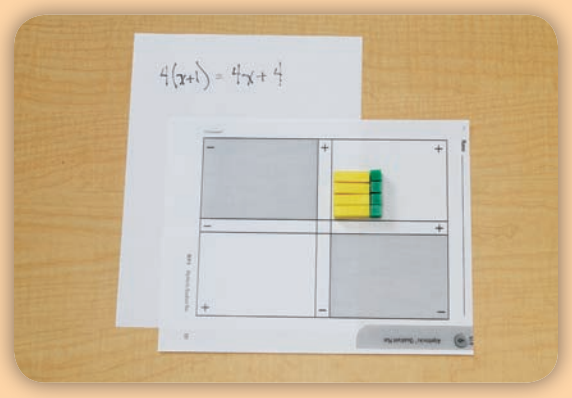
- Algeblocks® (*x* and unit blocks)
- Algeblocks Factor Track
- Algeblocks Quadrant Mat (BLM 7, 1 per pair)



1. Have students set up the algebraic expressions on their Quadrant Mat. Students should place the factors in the Factor Tracks.



2. Have students build a rectangle with the correct Algeblocks. **Say:** *Check your work to be sure that you have filled in the space with the correct blocks.* Review the value of the different Algeblocks, if necessary.



3. Have students remove the Factor Track and read the solution to the problem from the mat. They should then write out the equation: $4(x + 1) = 4x + 4$. **Ask:** *How did you use the Distributive Property to show that the left side of your equation is equivalent to the right side?*

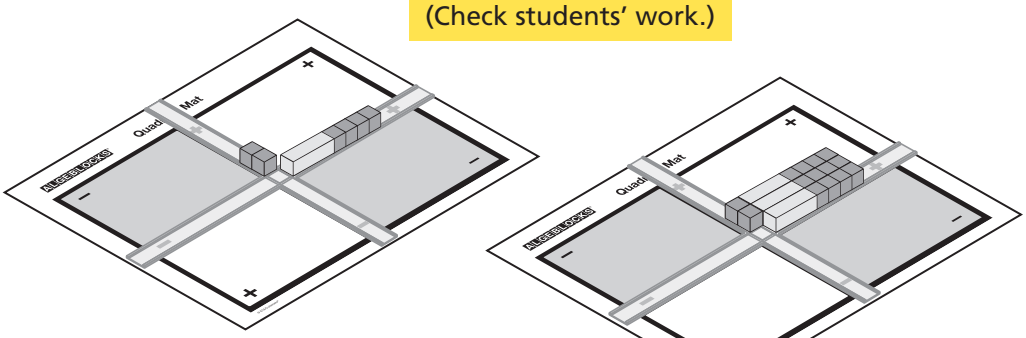
⚠ Look Out!

Some students may find it confusing that the answer to the question is not a number. Explain that this is because there is no assigned value for *x*. Give students a numeric value for *x* and have them solve the problem again.

Answer Key

Use Algeblocks, an Algeblocks Quadrant Mat, and a Factor Track to model the expression shown. Write the factors and their product.

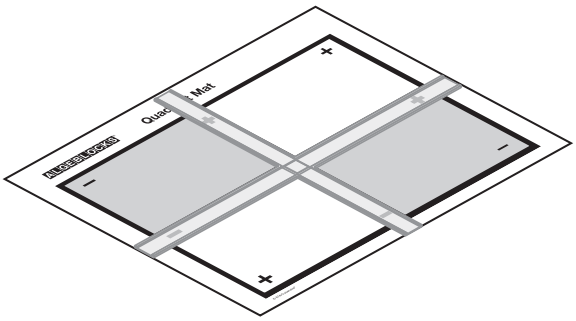
1. (Check students' work.)



2(x + 4) = 2x + 8

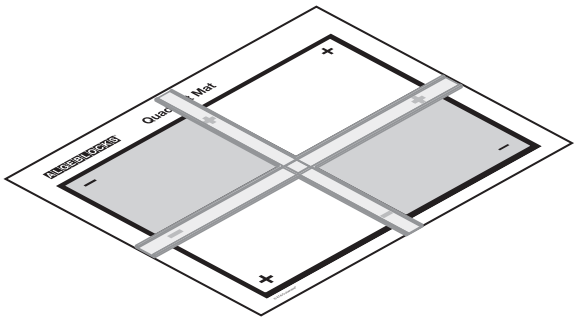
Using Algeblocks, an Algeblocks Quadrant Mat, and a Factor Track, model each pair of factors and their product. Sketch the model. Write each product.

2. 2(y + 3)



2y + 6

3. 2(2y + 1)



4y + 2

Find each product using the Distributive Property.

4. 2(x + 5)

2x + 10

5. 2(3y + 4)

6y + 8

6. 4(y + 1)

4y + 4

7. 2(4x + 1)

8x + 2

8. 5(x + 10)

5x + 50

9. 2(2y + 5)

4y + 10

Answer Key

Challenge! Will the Distributive Property work with subtraction? Use $3(x - 1)$ as an example. Explain.

Challenge: (Sample) yes; you can still distribute subtraction over multiplication. Multiply the number on the outside of the parentheses, 3, by each of the terms on the inside of the parentheses, the x and the 1. $3(x - 1) = 3x - 3$

LESSON 6

Expressions and Equations

Equations with a Variable

Students further develop their mathematical flexibility by using variable expressions to write equations. They use concrete models and mental math to solve for the unknown value in an equation. In the activity, the implicit use of inverse operations involving only basic facts lays the foundation for using symbolic representations to solve more complex equations.

Objective

Write and solve an equation with a variable.

Common Core State Standards

- **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How does mental math help you to solve the equations?*
- **Ask:** *How do you solve* 5 × b = 20 *using a related fact?*
- **Say:** *Tell a story that could be solved using the equation* 36 ÷ p = 4. *Explain how to solve the equation.*

Solve It

Reread the problem with students. Have them write a paragraph to explain how they use the models and mental math to solve the equations.

More Ideas

For other ways to teach about equations with a variable—

- Have students use Two-Color Counters to model the problem. They can start with 20 counters, all with the red side showing, to represent the apples that Bryce had after he went to the store. Students then turn over 6 of the counters to represent the apples that he bought at the store. Guide students to use the remaining red counters to solve the variable equation.
- Have groups of students make up simple word problems, use Centimeter Cubes to model the data, and write equations that can be used to solve the problem. Encourage students to use mental math to solve each problem.

Formative Assessment

Have students try the following problem.

There are 24 coins on Miguel's desk. Of these, 6 coins are quarters and the rest are dimes. Which equation could be used to find the number of dimes on Miguel's desk?

A. $6 \times d = 24$ **B.** $d - 24 = 6$ **C.** $d + 6 = 24$ **D.** $d \div 6 = 24$

Try It! 20 minutes | Groups of 4

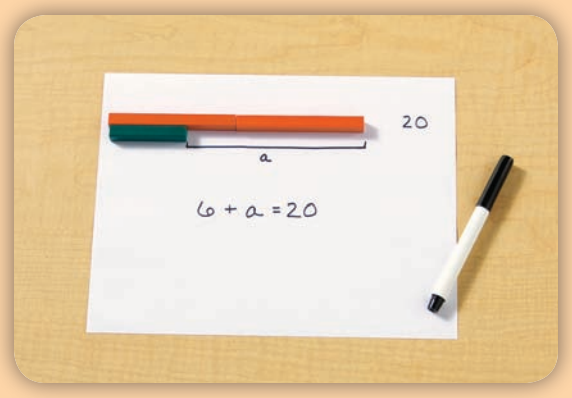
Here is a problem about equations with a variable.

Bryce has some apples. He buys 6 more at the store. Now he has 20 apples. Write an equation to determine how many apples Bryce had before he went to the store. He has a recipe that uses 5 apples to make one batch of applesauce. Write an equation to determine how many batches of applesauce he can make with 20 apples.

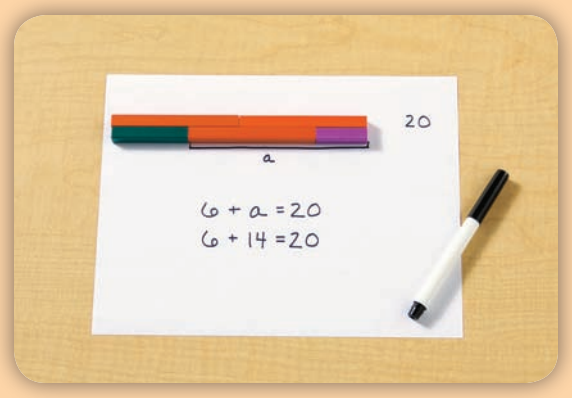
Introduce the problem. Then have students do the activity to solve the problem. Distribute the Cuisenaire Rods, paper, and pencils to students. Explain that an equation is a statement that two quantities are equal.

Materials

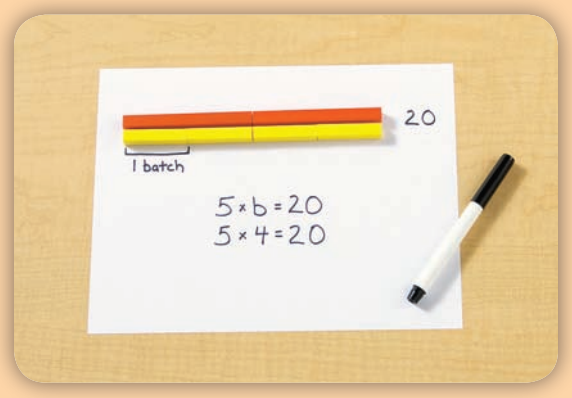
- Cuisenaire® Rods (1 set per group)
- paper (2 sheets per group)
- pencils (1 per group)



1. Have students state what is unknown. Using rods, have students model the facts of the first question. **Ask:** *What operation does buying more apples suggest?* Guide students to write an addition equation using a variable to represent the unknown number of apples. Write *6 + a = 20* on the board.



2. Have students complete the model by making the two rows equal. **Say:** *Think "Six plus what number equals 20?"* Encourage students to use mental math or related facts when they solve the equation.



3. Say: *Bryce uses 5 apples to make one batch of applesauce. What equation would you use to determine how many batches of applesauce he can make with 20 apples?* Have students model the problem and write the multiplication equation.

⚠ Look Out!

Students who cannot write equations may need to list the known facts and the information needed to solve the problem. Review key words that may help students recognize the operation needed to solve the problem. Reviewing and modeling fact families to see the relationships between operations may benefit the students.

Answer Key

Use Cuisenaire Rods to model each equation containing the variable *b* and the operation stated. Solve the equation.

1. addition

(Check students' work.)

$5 + b = 18$; $b = 13$

2. multiplication

$b \times 4 = 16$; $b = 4$

Using Cuisenaire Rods, model the given equation. Sketch the model. Solve the equation.

3. $a + 8 = 21$

$a = 13$

4. $3 \times d = 15$

$d = 5$

Solve each equation.

5. $4 + f = 19$

$f = 15$

6. $3 \times g = 15$

$g = 5$

7. $22 = s + 9$

$s = 13$

8. $13 + j = 27$

$j = 14$

9. $7 \times k = 28$

$k = 4$

10. $9 \times p = 45$

$p = 5$

Answer Key

Challenge! How can you use related facts to help you solve the equations in this lesson? Give an example.

Challenge: (Sample) I used the two numbers that were in the equation and the operation that is the opposite of the operation in the equation. For the equation $3 \times g = 15$, I used $15 \div 3 = g$. Division is the opposite operation of multiplication.

LESSON 7

Objective

Solve addition and subtraction equations.

Common Core State Standards

- **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Addition and Subtraction Equations

Students who first solve simple, intuitive equations, such as $x + 1 = 4$, and analyze how they find the solution, tend to understand the concepts (for example, inverse operations) used in solving equations. Applying these methods to slightly more difficult equations helps students to expand and generalize their understanding.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How does subtraction help you solve the problem?*
- **Say:** *Choose a number. Add 6 to the number; then subtract 6 from the sum.* **Ask:** *What number is left?* Explain that *adding 6 and subtracting 6* are inverse operations since one action undoes the other. Have students give other examples.

Solve It

Reread the problem with students. Have them explain how to use inverse operations to solve the equation. Then have students write the solution to the addition equation in the proper form.

More Ideas

For other ways to teach about solving addition and subtraction equations—

- Use Two-Color Counters to model the problem. Have students fold a sheet of paper in half with each half representing one side of the equation. Have them add or subtract counters on one side to make the two sides equal. Discuss how to use inverse relationships to find the solution.
- Have each student make up two addition and two subtraction equations that each contain one variable. Students then use Snap Cubes® to model each equation. Have them write one related subtraction equation for each addition equation and one related addition equation for each subtraction equation.

Formative Assessment

Have students try the following problem.

Which equation expresses the solution to k – 8 = 16*?*

A. 16 – 8 = 8

B. 16 + 8 = 24

C. 16 ÷ 2 = 8

D. 2 × 8 = 16

Try It! 20 minutes | Groups of 4

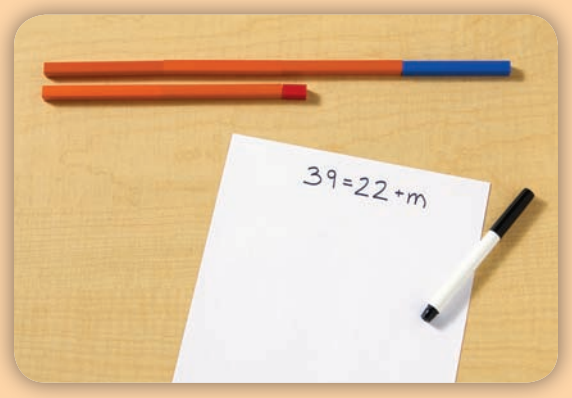
Here is a problem about solving addition and subtraction equations.

Randy spent 39 minutes on his computer. He spent the first 22 minutes downloading music and the remaining time messaging friends. Write and solve an addition equation to determine how many minutes Randy spent messaging his friends.

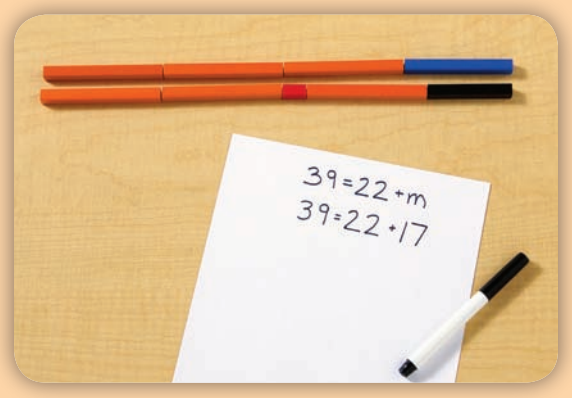
Introduce the problem. Then have students do the activity to solve the problem. Distribute Cuisenaire Rods, paper, and pencils to students. Suggest that *m* represents the number of minutes that Randy spent messaging his friends.

Materials

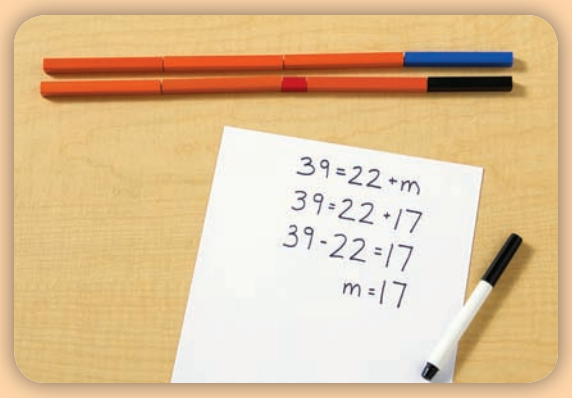
- Cuisenaire® Rods (2 sets per group)
- paper (1 sheet per group)
- pencils (1 per group)



1. Write *39 = 22 + m* on the board. **Ask:** *How does this equation represent the situation?* Have students use Cuisenaire Rods to model the equation.



2. Say: *To solve this equation, you can think "39 equals 22 plus what number?"* Have students complete the model and write the equation that represents the model.



3. Have students refer to the model to visualize the inverse relationship between addition and subtraction. Elicit that students can subtract 22 from 39 to solve for *m*. **Say:** *Write a subtraction equation that expresses the information in the problem.*

⚠ Look Out!

Be sure students left-align the rods in their models to help them recognize the relationship between the numbers.

Answer Key

Use Cuisenaire Rods to model each equation. Write an addition equation using the variable *n*. Use the Rods to solve the equation.

(Check students' work.)

1.

$6 + n = 20$; $n = 14$

2.

$2 + 10 + n = 28$; $n = 16$

Using Cuisenaire Rods, model the given equation. Sketch the model. Solve the equation.

3. $18 = 2 + q$

$q = 16$

4. $30 = 23 + r$

$r = 7$

Solve each equation.

5. $12 + w = 31$

$w = 19$

6. $35 = t + 14$

$t = 21$

7. $42 = u + 28$

$u = 14$

8. $11 + v = 29$

$v = 18$

9. $7 + c = 19$

$c = 12$

10. $19 + x = 25$

$x = 6$

Answer Key

Challenge! How can you check your answer after you have solved an equation?

Challenge: (Sample) I can put the number in the equation and then do the operations. I should get a true statement.

LESSON 8

Objective

Solve multiplication and division equations.

Common Core State Standards

- **6.EE.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **6.EE.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Multiplication and Division Equations

Students use variables as placeholders for missing numbers in equations. Arrays demonstrate the inverse relationship between multiplication and division and provide a foundation for the use of symbolic representations to solve equations. To be fluent, students must recognize that a relationship may be expressed in different ways.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Why can you rewrite a multiplication equation as a division equation?*
- **Ask:** *What does it mean when we say that division is the inverse of multiplication?*
- **Ask:** *How can you solve* p ÷ 7 = 8 *using multiplication?*

Solve It

Reread the problem with students. Ask them to explain in writing how they used the relationship between multiplication and division to solve the problem. Have students use the models to help them. Remind students to write the solution to the multiplication equation.

More Ideas

For other ways to teach solving multiplication and division equations—

- Use Color Tiles to make arrays to solve the problem. Some students may prefer to organize their work by counting out 32 tiles and placing each individual tile in the array.
- Use Two-Color Counters to model related multiplication and division equations. Provide students with equations such as $6 \times a = 66$, $b \times 4 = 56$, $d \div 2 = 14$, and $48 \div e = 16$. Have students make stacks of counters to show the multiplication and division equations. Discuss how to use inverse relationships to find the solutions to the equations.

Formative Assessment

Have students try the following problem.

Which equation expresses the solution to n × 5 = 60*?*

A. 60 – 5 = 55

B. 5 + 60 = 65

C. 5 × 60 = 300

D. 60 ÷ 5 = 12

Try It! 20 minutes | Groups of 4

Here is a problem about solving multiplication and division equations.

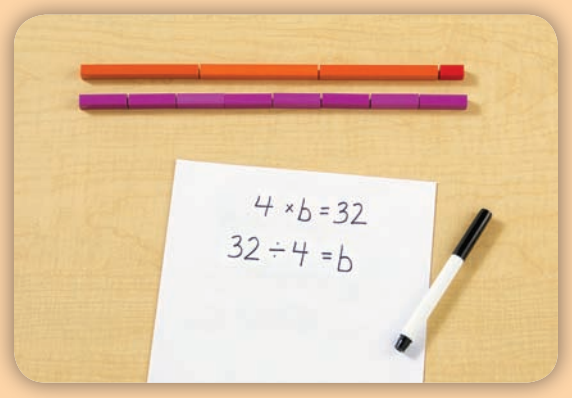
Zoe has 4 bookshelves. If each shelf has the same number of books, and there are 32 books in all, how many books are on each shelf? Write a multiplication equation to represent the number of books on each shelf. Use division to solve the equation.

Introduce the problem. Then have students do the activity to solve the problem. Distribute Cuisenaire Rods, paper, and pencils to students. Suggest that *b* represent the number of books on each shelf.

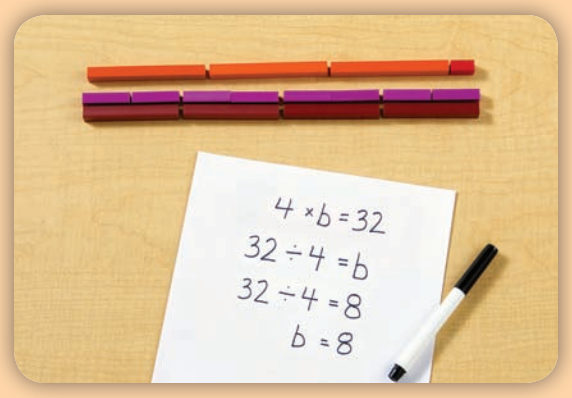
Materials

- Cuisenaire® Rods (2 sets per group)
- paper (1 sheet per group)
- pencils (1 per group)

1. Write *4 × b = 32* on the board and have students discuss how the equation is related to the problem. Have students use rods to model the equation.



2. Ask: *How can you solve the problem using division? What division equation is shown by the model?* Write *32 ÷ 4 = b* on the board.



3. Have students solve for *b.* Have them discuss what each purple rod represents. Tell students to use brown rods to make an alternate version of the model, and ask them to discuss what each brown rod represents.

⚠ Look Out!

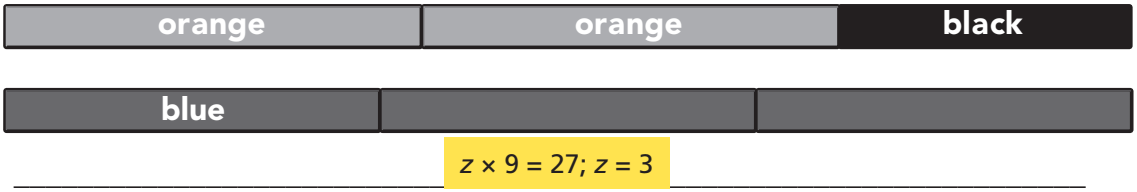
If students have trouble determining the division equation, they may find it helpful to review multiplication and division fact families. Have them write the four facts that use the numbers *3, 4,* and *12.* Then have them compare the four facts to see how they are alike and how they are different. Have students make a 3 × 4 array and explain how each fact is shown by the array.

Answer Key

Use Cuisenaire Rods to model the equation. Write a multiplication equation with the variable *z*. Use multiplication or division to solve the equation.

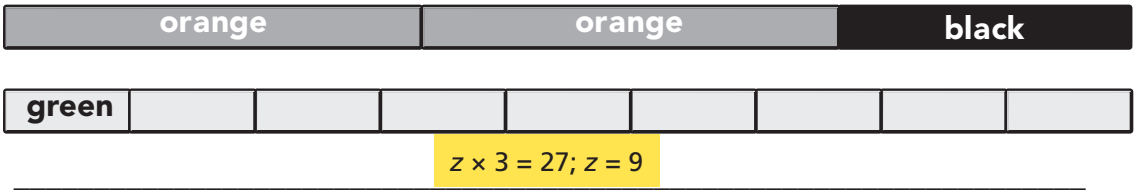
(Check students' work.)

1.



$z \times 9 = 27$; $z = 3$

2.



$z \times 3 = 27$; $z = 9$

Using Cuisenaire Rods, model the given equation. Sketch the model. Solve the equation.

3. $24 = 8 \times a$

$a = 3$

4. $35 = b \times 5$

$b = 7$

Solve each equation.

5. $12 \times d = 36$

$d = 3$

6. $55 = m \times 11$

$m = 5$

7. $40 = f \times 8$

$f = 5$

8. $9 \times g = 72$

$g = 8$

9. $6 \times h = 54$

$h = 9$

10. $5 \times k = 25$

$k = 5$

Answer Key

Challenge! How are multiplication and division related? How are the operations used to solve equations in this lesson? Show an example and draw a picture to help.

Challenge: (Sample) Multiplication and division are opposite operations. They "undo" each other when solving equations.

LESSON 9

Objective

Use patterns and function tables to solve problems.

Common Core State Standards

- **6.EE.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Expressions and Equations

Patterns and Function Tables

The ability to recognize and use patterns to solve problems forms the basis of algebraic thinking. Representing patterns in function tables allows students to discover patterns more easily and to see how *x*-values relate to *y*-values. These relationships can then be used to make predictions. Function tables will help students to graph relationships between *x* and *y*.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What does the variable* x *in the function table represent? What does the variable* y *represent?*
- **Ask:** *What words can you use to describe the pattern in the function table? How does the function table help you see the pattern?*
- **Ask:** *Why should you test the function rule for all of the values in the table?*

Solve It

Reread the problem with students. After they build a model for 4 square stones, students can use the pattern in the table to predict the number of triangle stones for 6 square stones. Have students complete the function table to check their predictions.

More Ideas

For other ways to teach about using patterns and function tables—

- Have students use triangle Pattern Blocks to find the perimeter of one triangle, then two, three, and four triangles placed side-to-side. They record the number of triangles as *x* in the function table and the perimeter as *y*. Have them predict perimeters for 6, 7, and 8 triangles placed side-to-side.
- Have students use Centimeter Cubes to build other patterns, such as triangular numbers. Guide them to use 1, 3, 6, and 10 Centimeter Cubes to build triangles. Have them use the pattern in the function table to predict the number of cubes used in the fifth figure.

Formative Assessment

Have students try the following problem.

Raul is using blue and gold tiles to create a mosaic pattern. If x *represents blue tiles and* y *represents gold tiles, how many gold tiles does Raul use if he uses 8 blue tiles?*

A. 11 **C.** 21

B. 18 **D.** 24

x	y
3	9
4	12
5	15

Try It! 20 minutes | Pairs

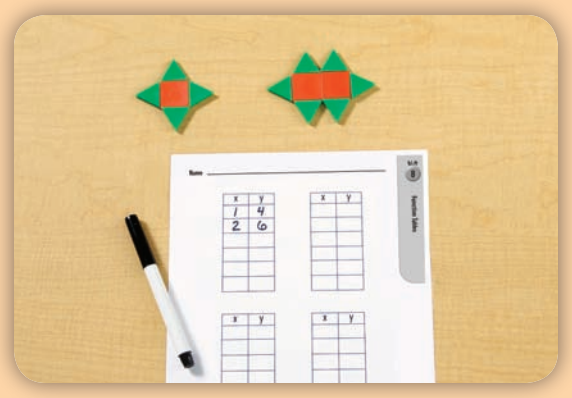
Here is a problem about patterns and function tables.

Lee Ann will use square and triangular stones to make a path in her garden. Each side of a triangular stone is the same length as each side of a square stone. The triangles will be used as a border around a row of squares. For example, she would use four triangles for a border around one square and six triangles for a border around two squares. How many triangles will she need for a border around six squares?

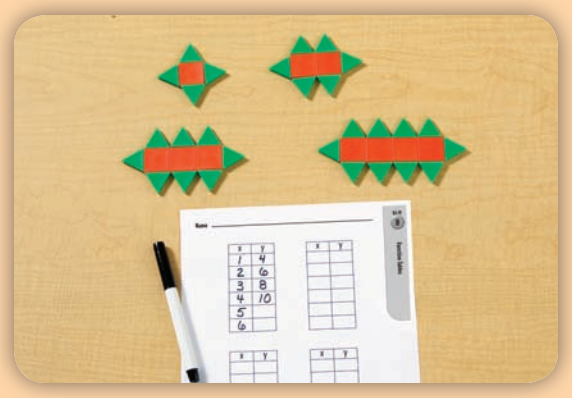
Introduce the problem. Then have students do the activity to solve the problem. Distribute Pattern Blocks, function tables, and pencils to students.

Materials

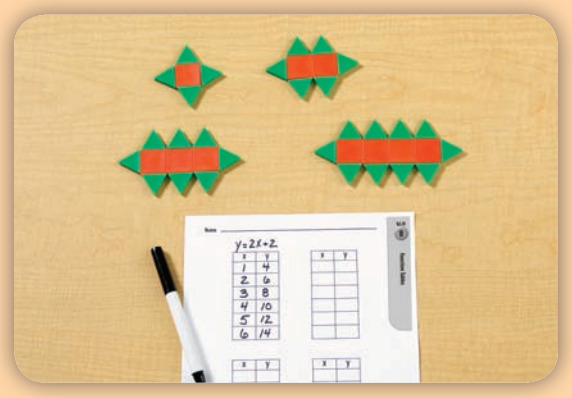
- Pattern Blocks (25 per pair)
- Function Tables (BLM 8; 1 per pair)
- pencils (1 per pair)



1. Say: *Use a square Pattern Block to represent each square stone. Use a triangle to represent each triangular stone.* Have students create models for paths with one and two squares. Then for each path have them record the number of squares in the *x* column and the number of triangles in the *y* column.



2. Say: *Use more squares to model paths with three and four square stones. Add triangles and then fill in the next two rows of the table.* Have students identify the pattern and use it to fill in the table for 5 and 6 square stones.



3. Ask: *What operations can you perform on the x-values in the table to get the y-values?* Guide students to see that they can multiply *x* by 2 and add 2 to get *y*. Have them write the equation as $y = 2x + 2$ and then check that the equation works for all of the values in the table.

⚠ Look Out!

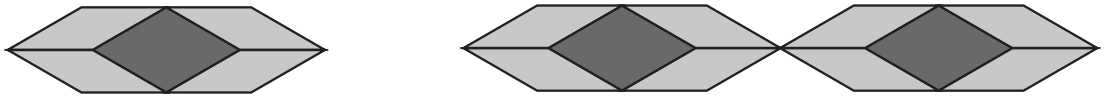
Some students may have difficulty finding a rule to represent the pattern in the function table. Have them look at the model for two squares placed end-to-end. Point out that the number of triangles on each side of a square is twice the number of squares since there are two sides to each square and in addition there is a triangle at each end. Have them check that this pattern holds for each number of square stones.

Answer Key

Use Pattern Blocks to model the pattern. Complete the table to show the relationship in the pattern. Write a rule for the pattern.

(Check students' work.)

1.



Let x = number of ◆. Let y = number of ▰.

x	1	2	3	4	5	6
y	4	8				

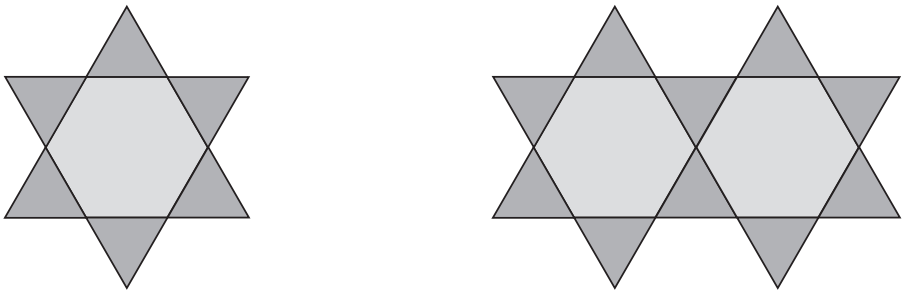
Rule: _____

3	4	5	6
12	16	20	24

$y = 4x$

Using Pattern Blocks, model the figure shown. Then create a pattern. Sketch the pattern. Make a table that shows the relationship in the pattern. Write the rule.

2.



x = the number of hexagons;
y = the number of triangles;

x	1	2	3	4	5
y	6	10	14	18	22

$y = 4x + 2$

Make a table for each function rule.

3. $y = x + 5$

x	1	2	3	4	5
y	6	7	8	9	10

4. $y = 4x$

x	1	2	3	4	5
y	4	8	12	16	20

5. $y = 7 + x$

x	1	2	3	4	5
y	8	9	10	11	12

Answer Key

Challenge! Design your own pattern using Pattern Blocks. Then make a table to show the relationship of the numbers of each block as the pattern grows. Write the rule.

Challenge: Answers will vary.