MODELING MULTIPLICATION

NUMBER • MEASUREMENT

- Spatial visualization
- Properties of rectangles
- Area

Getting Ready

What You'll Need

Base Ten Blocks, 1 set per child

Base Ten Block Grid Paper, 1 sheet per child, page 93

Multiplication Spinner, 1 per group, page 91

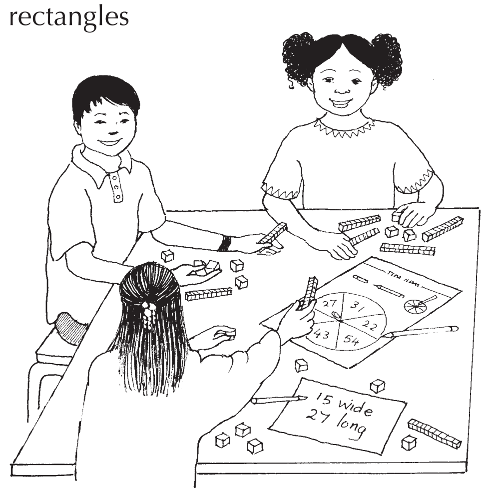
Overhead Base Ten Blocks and/or Base Ten Block Grid Paper transparency (optional)

Opaque Projector (optional)

Overview

Children use Base Ten Blocks to build rectangular arrays that model the multiplication of two 2-digit numbers. In this activity, children have the opportunity to:

- reinforce their understanding of the concept of area
- identify small rectangles contained in bigger rectangles
- determine the area of a rectangle as the sum of the areas of smaller rectangles



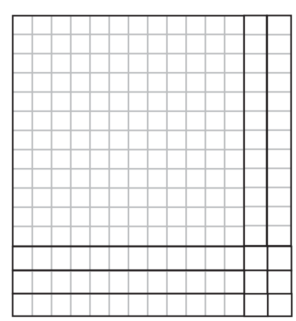
The Activity

To avoid possible confusion about the number of dimensions, you may wish to point out that in this activity, Base Ten Blocks are being used to model figures with two dimensions—length and width—even though the actual blocks have three dimensions, length and width and height.

Be sure children know that area *is the number of square units needed to completely cover a region.*

Introducing

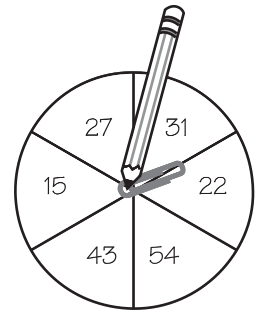
- Display a Base Ten flat and describe it as an "array" formed by rows and columns of squares.
- Call on a volunteer to explain how this array can be used to find the total number of squares in 10 rows of 10 columns.
- Use Base Ten Blocks to display a 12 × 13 array. Establish that the array is in the form of a rectangle with *dimensions* of 12 and 13.



- Ask children to identify four smaller, same-block rectangular regions within the 12 × 13 rectangle. Have them use crayons of four colors to record these regions on grid paper.
- Now have children find the *area* of each region.
- Elicit that the sum of the areas of the four smaller regions is equivalent to the area of the 12 × 13 rectangle.

On Their Own

How can you use Base 10 Blocks to model multiplication?



- Work with a group. Decide who will be first to spin a spinner like this.
- When it is your turn, spin twice. What numbers did you spin? Record your numbers. Think of them as the dimensions of a rectangle.
- Build a rectangle for your dimensions with the least number of flats, longs, and units possible.
- Estimate the area of your rectangle. Record your estimate.
- Now look for four smaller, same-block rectangles within your big rectangle. You may move the blocks apart slightly to show the smaller rectangles.
- Use the smaller rectangles to help you find the area of the big rectangle. Record your work.
- Keep on spinning, recording dimensions, and building rectangles until you are sure you have found the area of all the pairs of dimensions that it is possible to spin.
- Decide on a way to record your work.
- Leave your last rectangle where everyone can see it.

The Bigger Picture

Thinking and Sharing

Encourage class discussion about the groups' rectangles. Have groups describe how they recorded their work and invite comparisons. Post children's recordings or project them using an opaque projector, if one is available.

Use prompts like these to promote class discussion:

- How did you estimate the area of a big rectangle?
- What smaller rectangles did you find within a big rectangle?
- How did knowing the areas of smaller rectangles help you find the area of a big rectangle?
- How many different pairs of dimensions is it possible to spin with the spinner? Explain.
- How does this activity help explain why we can multiply like this to find the area of a rectangle? *(Write the following on the board.)*

$$\begin{array}{r} 43 \\ \underline{\times\ 22} \end{array}$$

Extending the Activity

1. Have a group put all its blocks together to make one huge rectangle, identifying four smaller rectangular regions within it and then estimating and determining its area.

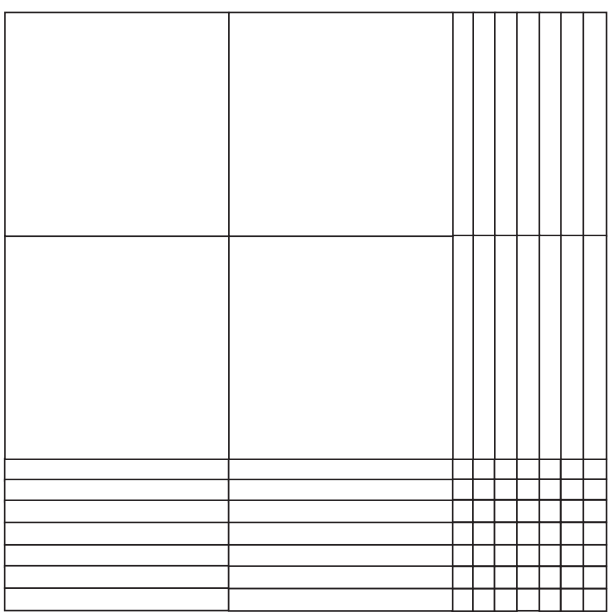
Teacher Talk

Where's the Mathematics?

Identifying and recording the small rectangular regions that lie within a 12 × 13 region prepares children for the kind of thinking that they will experience in *Modeling Multiplication.* Children are likely to identify smaller rectangles of these sizes: 10 × 10, 3 × 10, 2 × 10, and 3 × 2. After they have used crayons of different colors to record the small rectangles, it would be a good idea to have them outline the largest region with a dark-color crayon to reinforce the idea that the smaller rectangles are all contained within the larger one.

After children "spin dimensions," they are likely to begin making their rectangles by setting up blocks to represent one of the factors—first by using as many flats as necessary, then longs, and then units. However, as children place their blocks, some may find the process confusing and may have to be reminded to model just one dimension of a rectangle at a time.

Asking children to identify four rectangular, same-block regions within a larger region focuses their work on modeling the standard form of the algorithm for multiplying two 2-digit numbers. Asking them to use the least possible number of blocks in each array assures that their large arrays will contain four smaller, same-block rectangles—one made from flats alone, two made from longs alone, and one "corner" region made from units alone. (This assumes that children have a large enough supply of blocks so that they can build their models with the least number of blocks of each kind.)



In the course of their work, children may notice that when they spin "doubles" (15 × 15, 22 × 22, and so on), their arrays will form squares. They may also notice that the two rectangles contained within a larger square and made up of longs only are always the same size, although they are oriented differently.

Children's posted work should reflect the 21 two-number com-

2. Bisect a large rectangular array, from corner to corner. Challenge children to investigate how to find the area of each of the four triangles thus formed as well as the area of the large rectanglar array.

binations (including "doubles") that are possible with the Multiplication Spinner.

	FIRST NUMBER SPUN					
	31	**22**	**54**	**43**	**15**	**27**
SECOND NUMBER SPUN	22	54	43	15	27	27
	54	43	15	27	15	
	43	15	27	54		
	15	27	54			
	27	22				
	31					

Those children who look beyond the "same-block" and "4-rectangle" rules may also notice that many other patterns of smaller rectangles can be considered to make up total area. Sharing ideas about this observation can help children see that rectangular regions can show partial products in a variety of ways.

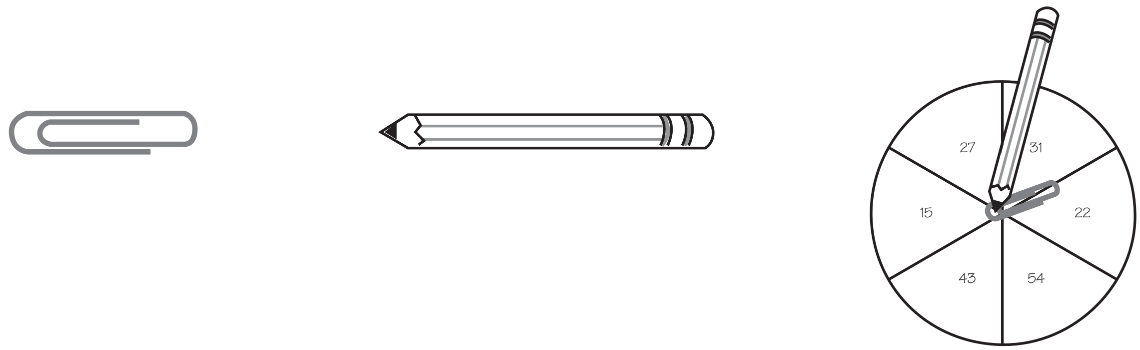
Many blocks are needed to make the largest rectangles. The 54 × 43 rectangle requires 20 flats, 31 longs, and 12 units. If blocks are in short supply, you may either have children work in pairs to build a single rectangle or have them share blocks among groups or you may choose to replace the numbers on the spinner with smaller numbers.

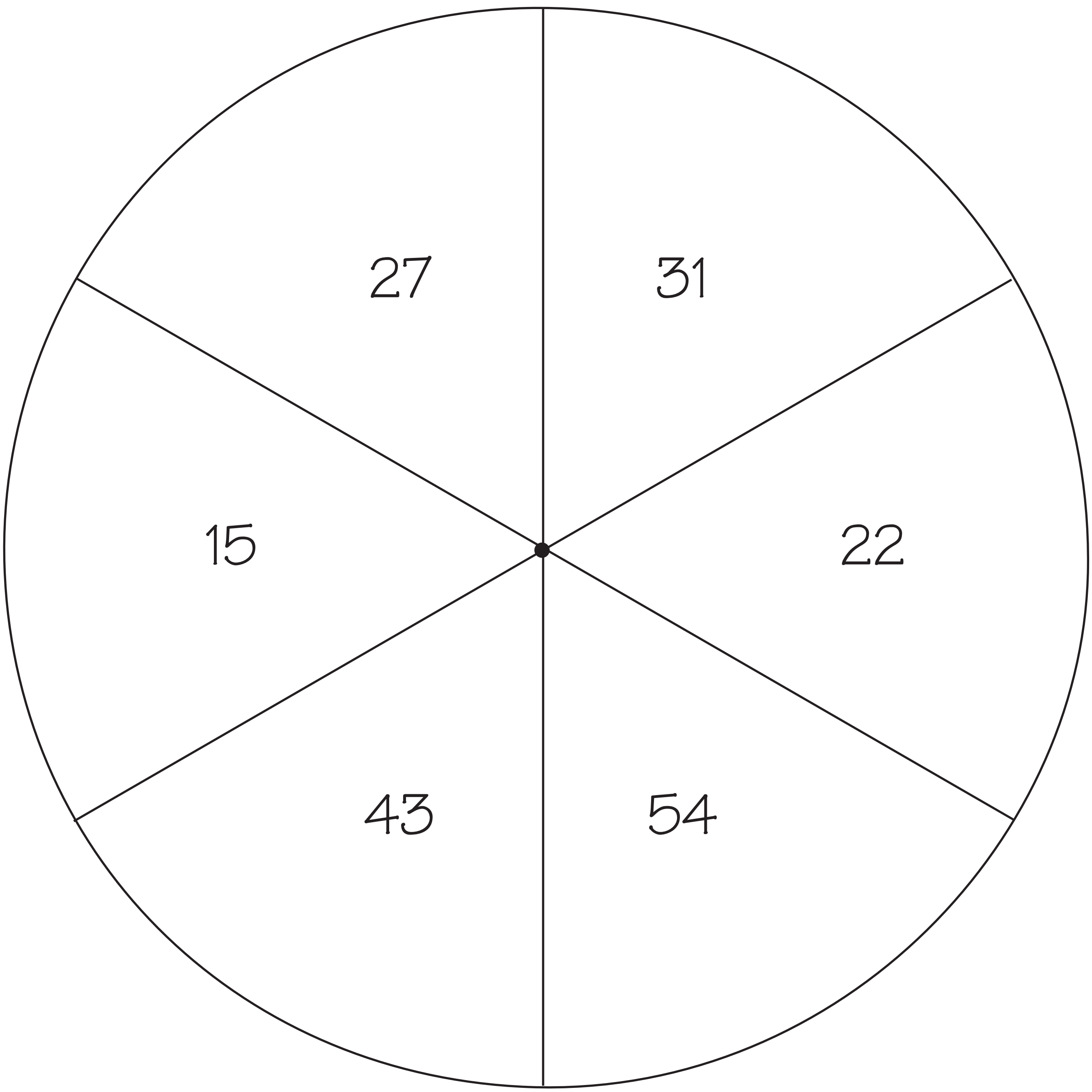
When using smaller rectangles to determine the area of a larger one, some children may count unit squares in each small array and add the results. Others will multiply the length and width of each smaller rectangle and add those products to find the total area. Some children may combine these methods. Whichever way children approach this activity, they may find it difficult to see how their rectangles relate to the paper-and-pencil multiplication algorithm. This is because the algorithm typically combines the four partial products that correspond to children's small rectangles into two. For example:

$$\begin{array}{rll}
43 & & \\
\underline{\times\ 22} & & \\
6 & (2 \times 3 = 6) & \\
80 & (2 \times 40 = 80) & [6 + 80 = 86] \\
60 & (20 \times 3 = 60) & \\
\underline{800} & (20 \times 40 = 800) & [60 + 800 = 860] \\
946 & & [86 + 860 = 946]
\end{array}$$

If asked how the partial products 86 and 860 in the example above can be seen in the 22 × 43 rectangle, children may say that they can model these amounts by combining the areas of two of their small rectangles and then combining the areas of the other two. In this way, children illustrate that multiplying whole dimensions is the same as multiplying parts and then combining those results—which suggests the *distributive law of multiplication* [*a* (*b* + *c*) = *ab* + *ac*].

MULTIPLICATION SPINNER





BASE TEN BLOCK GRID PAPER