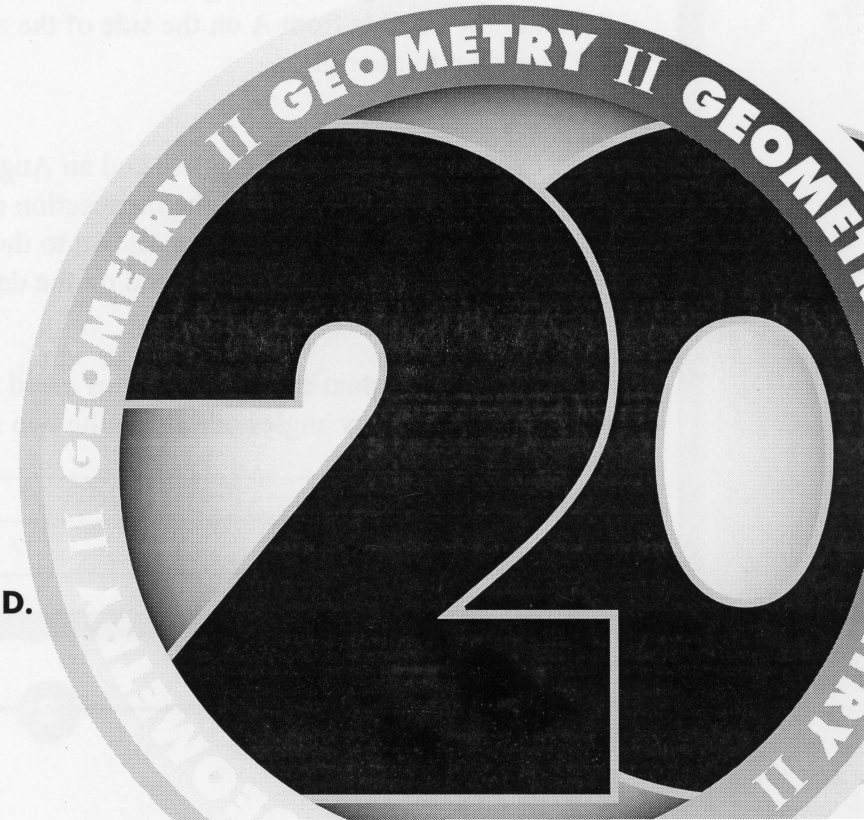
LEVEL TWENTY

GEOMETRY II

Developmental Mathematics



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Long Island University

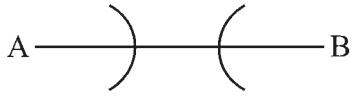
Date _____

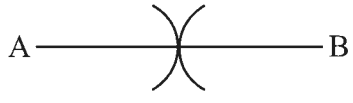
1 THE TRIANGLE

- To construct a triangle you need to locate three points. You will be able to locate two of these points if you are given a line segment which would be a side of the triangle (or the length measure of a side of the triangle). If $\overline{AB}$ is a side in the triangle, A and B are two vertices in the triangle.

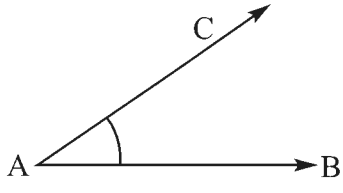
To locate the third point, more data is needed. For example, to find C, one of the following procedures would be performed:

a) You would be given the other two line segments which would be the two other sides of the triangles or their length measure. With A and B as centers you draw two circles of radii congruent to those of the two given line segments. The point of intersection of the two circles is the third Vertex C of the triangle. For the two circles to intersect there is one important condition that must be satisfied, the **sum** of the length measure of two sides of the triangle to be **larger** than the length measure of the third side. The following figures show two cases where the condition is not satisfied.

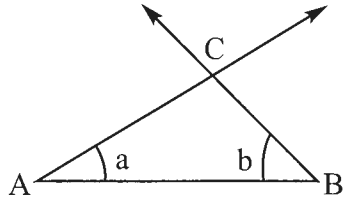




b) You would be given an Angle A of the triangle and how far C is from A on the side of the angle, as shown to the right.



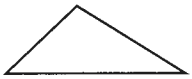
c) You would be given an Angle a and an Angle b of the triangle. C would be the intersection of the two sides of Angles a and b, as shown to the right. The procedure used would depend on the data given.



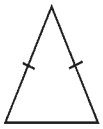
There is one important condition. Angles a and b must not be supplementary angles otherwise the two sides of the angles become _____.
Why? _____

_____

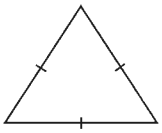
- It is obvious that all sides of a triangle may be congruent, two sides may be congruent or no sides are congruent. Names have been assigned to the different cases:



SCALENE
No congruent sides



ISOSCELES
Two congruent sides

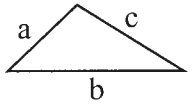


EQUILATERAL
Three congruent sides

- **The Perimeter:** The perimeter of any figure is the distance around the figure. Thus, the perimeter of a triangle is the **sum** of length measures of its sides.

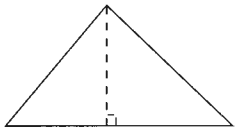
If the three sides are 9", 12" and 15"
the perimeter would be:

_____



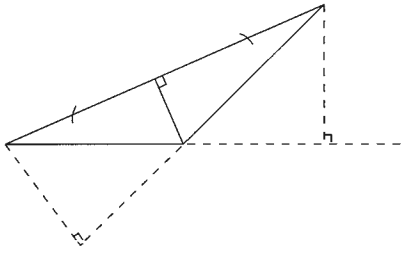
If the sides are a, b and c units of length, the perimeter would be _____ of the **same** units.

- **The altitude of a triangle:** If through a vertex of a triangle you construct a line perpendicular to the side opposite to the vertex, the line segment from the vertex to the opposite side is called the **altitude** of the triangle.



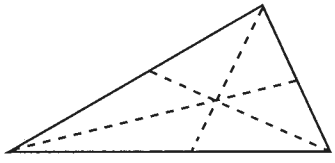
It is clear that any triangle has three altitudes.
Use the triangle to the right to illustrate this fact.

Sometimes you have to extend the side in order to construct a line that passes through the vertex perpendicular to the side, as shown to the right.



- **The median of a triangle:** The line segment from any vertex to the **midpoint** of the opposite side is called the **median** of the triangle.

It is clear that in any triangle there are three medians.
Use the triangle to the right to illustrate this.



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Date _____

APPLICATIONS

1. Find the perimeters of the triangles if the length of its sides are:

a) 9", 6", 7"

b) 4.2', 6.7', 9.3'

c) $6\frac{1}{2}$ yd., $8\frac{3}{4}$ yd., $4\frac{5}{8}$ yd.

d) $7\frac{1}{2}'$, $9\frac{2}{3}'$, $5\frac{5}{8}'$

2. Find the perimeter of an equilateral triangle if the length of its side is:

a) $3\frac{5}{6}$ yd.

b) $2\frac{3}{8}$"

c) 6.75 miles

d) 2.328 kilometers

3. Find the perimeter (P) of a triangle whose sides are a, b and c units of length.

P = _____

Find the numerical value of P when a is 5", b is 6.2" and c is 8.45".

P = _____

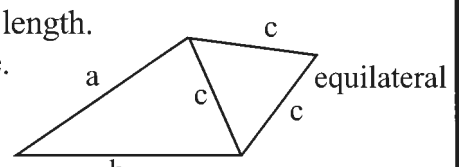
4. Find the perimeter (P) of an equilateral triangle if the length of its side is a units of length.

P = _____

Find the numerical value of p when a is 3 ft. 5 inches.

P = _____

5. The figure to the right is composed of a triangle whose sides are a, b and c units of length, and an equilateral triangle of c units of length. Find (in symbolic form) the perimeter (P) of the figure.



_____

Find the perimeter when a is $3\frac{1}{2}$ ft., b is $2\frac{1}{2}$ ft., and c is 50".

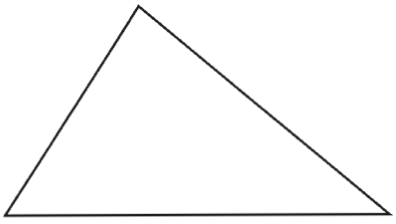
_____

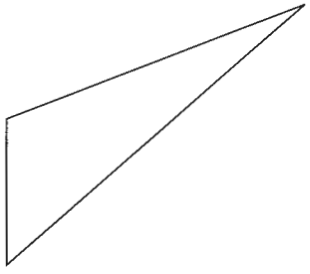
6. The perimeter of a triangle is 15 yds. The lengths of two sides are $6\frac{3}{4}$ yds. and $4\frac{1}{3}$ yds. Find the length of the third side.

7. In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$. If the perimeter is 19' and $m\overline{BC} = 4\frac{1}{2}$' find the length of $\overline{AB}$.

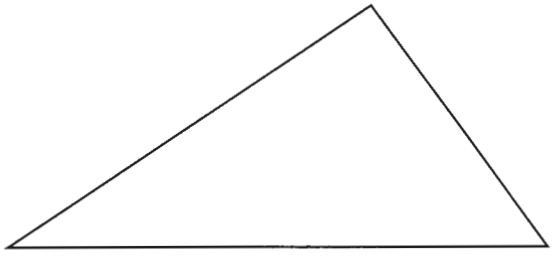
8. Find the length of the side of an equilateral triangle whose perimeter is:
   a) 70 miles
   b) $8\frac{1}{2}$yd.

9. Using your compass and straightedge only, construct the three altitudes of the triangles below.





10. Using your compass and straightedge only, construct the three medians of the triangle below.



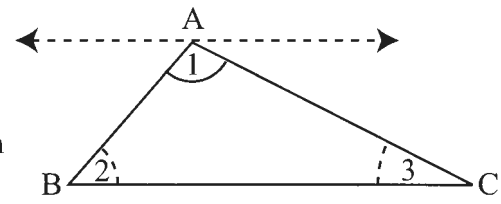
Date _____

2 HOW THE ANGLES OF A TRIANGLE ARE RELATED

- ABC is a triangle. We want to investigate how its three Angles 1, 2 and 3 are related. One way is to construct a line through A **parallel** to $\overline{BC}$.

Find an angle congruent to $\angle 3$ _____
Find an angle congruent to $\angle 2$ _____

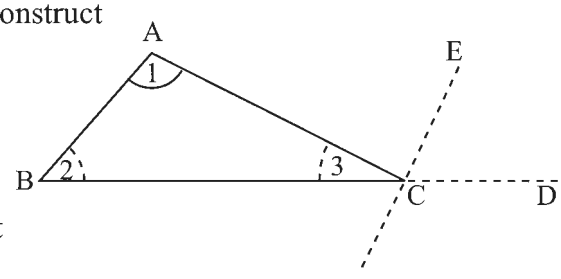
Now you have data that helps conclude that the sum of $\angle 1$, $\angle 2$, and $\angle 3$ is 180°.



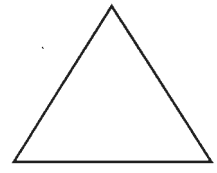
Another way to is to extend $\overline{BC}$ to D and then construct a Ray $\overrightarrow{CE}$ parallel to $\overline{BA}$.

Find an angle congruent to Angle 1 _____
Find an angle congruent to Angle 2 _____

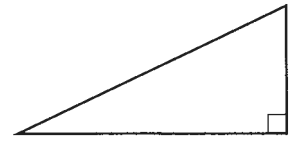
Now you have data that helps you conclude that the sum of $\angle 1$, $\angle 2$ and $\angle 3$ is 180°.



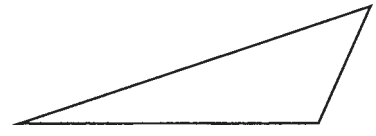
Since the sum of the three angles of a triangle is 180°, each angle may be less than 90°. In a triangle where the three angles are acute angles, the triangle is called an **acute triangle**.



If one of the three angles if a triangle is 90°, the sum of other two angles will be 90° which means that each of these two angles will be less than 90°. Thus, if one angle is a right angle and the other two are acute angles, the triangle is called a **right triangle**.

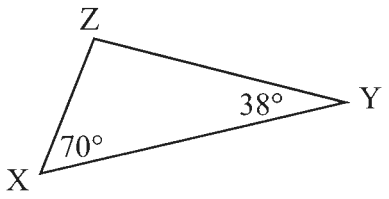


If one of the three angles of a triangle is an obtuse angle (more than 90°) the sum of the other two angles is **less** than 90°. Thus, if one angle is obtuse and the other two are acute, the triangle is called an **obtuse triangle**.



- **Examples:**

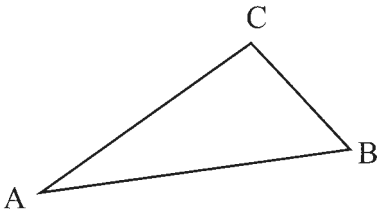
1. In △XYZ, m∠x = 70° and m∠y = 38°
   Find m∠Z.



Solution:

m∠x + m∠y + m∠z = _____
m∠x + m∠y = _____
Thus, m∠z = _____

2. In △ABC, m∠b = 2m∠a and m∠c = 3m∠a
   Find the measure of each angle.



Solution:

∠a + ∠b + ∠c = 180°

∠a + 2∠a + 3∠a = 6∠a = 180°

∠a = 180° √ 6 = 30°

Let the measure of ∠a be a number of degrees. Thus, the measure of ∠b is twice this number, and the measure of ∠c is 3 times the same number.

If we add all three angles the sum will be 6 times the number, but we know that the sum is 180°.

Ask yourself, "6 times a number is 180°. What is the number?"

The number is _____
Thus, m∠a = _____
m∠b = _____
m∠c = _____

Another solution:

Let the measure of ∠a be "a" degrees. Thus, the measure of ∠b = 2a degrees and the measure of ∠c = 3a degrees.

Equation:

a + 2a + 3a = 180°
thus, 6a = 180°
a = _____
Thus, m∠a = _____
m∠b = _____
m∠c = _____

3. In △MNO, the measures of Angles m, n and o are in the ratios 5:6:4. Find the measure of each angle.

Solution:

If ∠m is divided into 5 equal parts, ∠n will be 6 of these parts and ∠o will be 4 of these parts.

The three angles put together will be 15 of these parts. These 15 parts make the 180°.

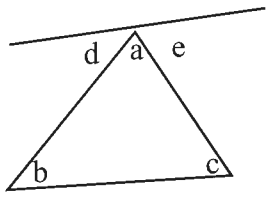
One part = _____
∠m = _____
∠n = _____
∠o = _____

A

Date _____

APPLICATIONS

1. Assuming the line in the picture is parallel to the bottom of the triangle, explain how to arrive at the conclusion that $m\angle a + m\angle b + m\angle c$ is 180°.



2. a) What is an acute triangle?

_____

b) What is a right triangle?

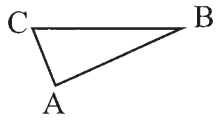
_____

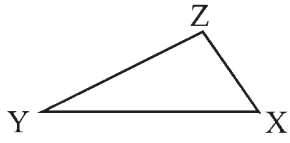
c) What is an obtuse triangle?

_____

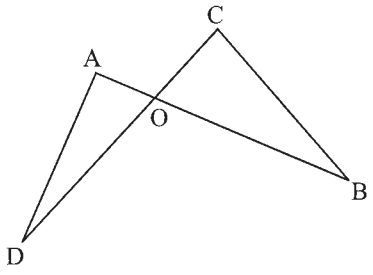
3. In $\triangle ABC$, $m\angle a = 50°$ and $m\angle b = 62°$.
Find $m\angle c$.

4. To the right, given $\angle a = \angle x$ and $\angle b = \angle y$, conclude that $\angle c = \angle z$.

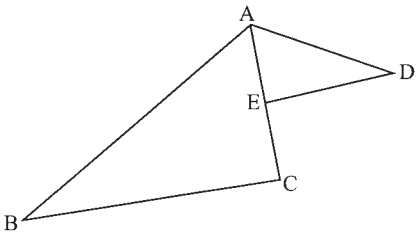




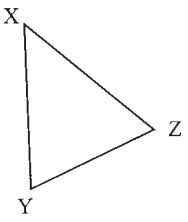
5. To the right, $\overline{AB}$ and $\overline{CD}$ intersect at O. Given that $\angle a$ and $\angle c$ are right angles, show that $\angle b = \angle d$



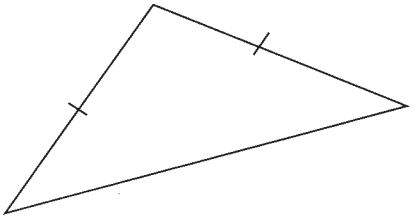
6. To the right, $\overline{AC}$ bisects $\angle BAD$.
$\overline{DE}$ is perpendicular to $\overline{AC}$ and
$\overline{BC}$ is perpendicular to $\overline{AC}$. Show that $\angle d = \angle b$.



7. In $\triangle XYZ$, $m\angle x = 35°$ and $\angle y = \angle z$. Find $m\angle y$.

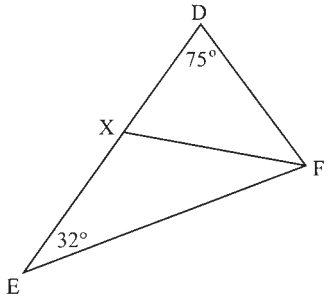


8. ABC is a right triangle in which $\angle b \cong \angle c$.
a) Which angle is a right angle?_____
b) Find $m\angle b$.



9. In $\triangle RST$, the measure of angles
r, s and t are in the ratio 2:1:5.
Find the measure of each angle.

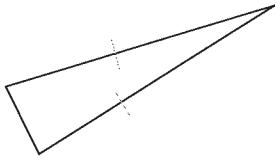
10. To the right, $\triangle DEF$ is a triangle in
which $m\angle d = 75°$ and $m\angle e = 32°$.
$\overrightarrow{FX}$ bisects $\triangle DFE$. Find $m\triangle DXF$.



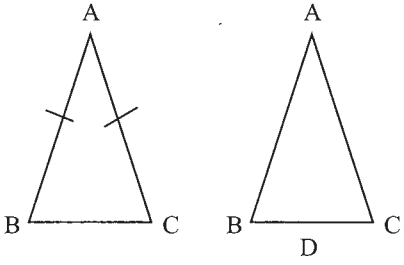
Date _____

3 THE ISOSCELES TRIANGLE

- An **Isosceles Triangle** is a triangle in which two sides are congruent. The third side is referred to as the base. The vertex opposite to the base is referred to as the vertex, and the angle opposite to the base as the vertex angle.



- Given an isosceles triangle, can we come to **specific** conclusions about its angles?



Let ABC be any isosceles triangle and we want to investigate its angles.

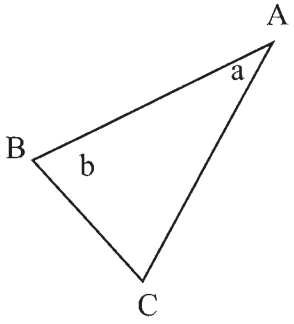
If we construct $\overline{AD}$ that bisects $\angle a$ and meets BC at D, triangle ABC is divided into the two triangles ABD and ACD.

Now you have enough data that make triangles ABD and ACD congruent.
Explain giving reasons:

As triangle ABD and ACD are congruent we can conclude that $\angle b \cong \angle c$.

- The same result applies to **any** isosceles triangle: If two sides of a triangle are congruent, the angles opposite those sides are congruent.
- As triangles ABD and ACD are congruent, we can also conclude that $\overline{BD} \cong \overline{CD}$, which means that the bisector of the vertex angle bisects the base.
- We can also conclude that $\angle ADB \cong \angle ADC$.

As the sum of these two angles is 180°, each angle is 90°, which means that $\overline{AD}$ is perpendicular to $\overline{BC}$.



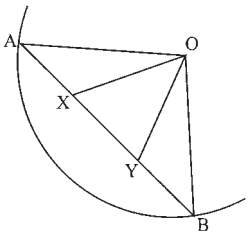
Example 1:
ABC is an isosceles triangle in which $\overline{AB} \cong \overline{AC}$. Given that $m\angle a \cong 35°$, find $m\angle b$.

Discussion:
ABC is an isosceles triangle in which $\overline{AB} \cong \overline{AC}$.
How are Angles b and c related? _____
Given that $m\angle a$ is 35°, what is the **sum** of $\angle b$ and $\angle c$? _____
Why? _____

Now you can conclude that $m\angle b$ is _____.

Example 2:

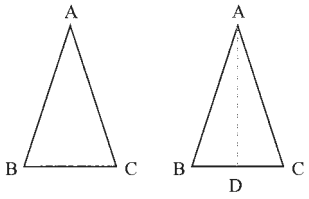
To the right, $\overline{AB}$ is a chord in a circle whose center is O.
Given that $\overline{AX} \cong \overline{BY}$, show that $\overline{OX} \cong \overline{OY}$.



Discussion:

$\overline{OX}$ and $\overline{OY}$ are two sides in the two triangles OXA and OYB.
Can you show that the two triangles are congruent?

- Suppose two angles in a triangle are congruent, how would the opposite sides be related?



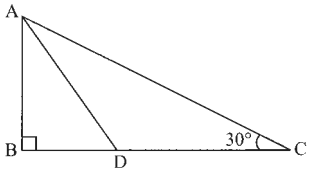
Let $\overline{AD}$ bisect $\angle a$ and meet $\overline{BC}$ at D.
You have data that make triangles
ABD and ACD congruent. Explain given reasons.

As triangles ABD and ACD are congruent, we can conclude that $\overline{AB} \cong \overline{AC}$.

- The same result applies to **any** triangle: If two angles of a triangle are congruent, the sides opposite to those congruent angles are congruent.

Example:

To the right: ABC is a right triangle in which $m\angle c \cong 30^\circ$
$\overline{AD}$ bisects $\angle BAC$. Show that $\overline{DA} \cong \overline{DC}$.



Discussion:

$\overline{DA}$ and $\overline{DC}$ are two sides in $\triangle DAC$.

To conclude that $\overline{DA} \cong \overline{DC}$, we first have to show that $\angle DAC$ and $\angle c$ are congruent.

We are given that $m\angle c \cong 30^\circ$. We have to find that $m\triangle DAC \cong 30^\circ$.

Solution:

In $\triangle ABC$, $m\angle b \cong 90^\circ$ and $m\angle c = 30^\circ$

Thus, $\triangle BAC \cong 180^\circ - 120^\circ \cong 60^\circ$

As $\overline{AD}$ bisects $\angle BAC$, $\angle DAC \cong 60^\circ \div 2 \cong 30^\circ$

In $\triangle DAC$, $\triangle DAC \cong \triangle DCA$

Thus, $\overline{DA} \cong \overline{DC}$.