LESSON 5

Objective

Determine whether a spinner is fair by comparing its fractional parts.

Common Core State Standards

- **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- **7.SP.7b** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

Statistics and Probability

Probability and Fairness

Using an area model to determine theoretical probability involves the understanding that the sum of the fractional parts of a whole must be 1. Using an area model also facilitates an understanding of "fairness." This activity uses dartboards as a starting point for the investigation of these concepts.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What point values could you assign to each of the colored areas in order to make the second dartboard fair?*
- **Ask:** *In 20 tries, about how many times should you have hit red, blue, green, and yellow on the first dartboard? The second dartboard?*
- **Ask:** *What is the result if you add together the fractional parts of the first board? If you add together the fractional parts of the second board? In general, what should be the sum of the fractional parts of any board?*

Solve It

Reread the problem with students. Have students explain in writing how they determined whether the boards were fair or unfair.

More Ideas

For another way to teach about fair and unfair outcomes—

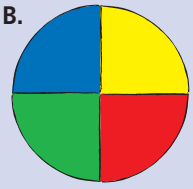
- Set up an experiment that will produce unfair outcomes. Place 2 yellow, 4 green, and 6 red Centimeter Cubes in a paper bag. Tell students not to look in the bag until the end of the activity. Have students pull out a cube, record its color, return the cube to the bag, and mix the cubes. Students should repeat the process 50 times. **Say:** *Examine the data from your experiment.* **Ask:** *What colors of cubes are there in the bag? Are the colors represented equally? If not, estimate the proportions that are represented.* When students have made and defended their predictions, have them open the bag and check their work.

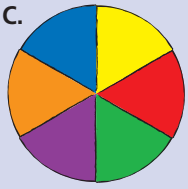
Formative Assessment

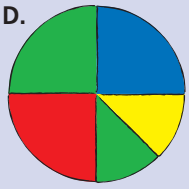
Have students try the following problem.

Which of the following dartboards is unfair?

A. 

B. 

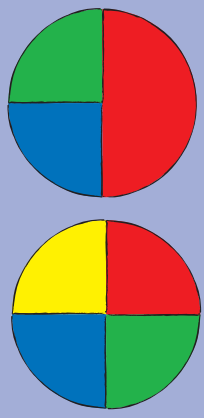
C. 

D. 

Try It! 30 minutes | Pairs

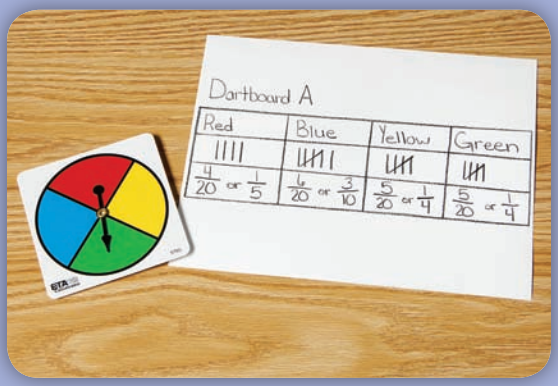
Here is a problem about determining whether a spinner is fair or unfair.

James and his friends play a magnetic dart game at their school's afternoon recreational program. Each player picks a different color as his or her target. The dartboard is mounted on a stand and spun. The players are blindfolded and take turns throwing darts at the spinning board, hoping to hit their target section. The players have two boards they can use. What is the probability of hitting each of the colors on the dartboards? Are the boards fair?

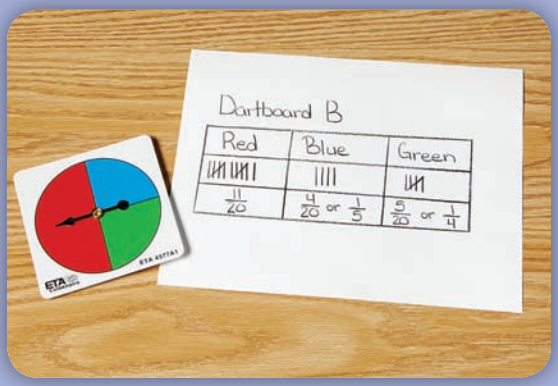


Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

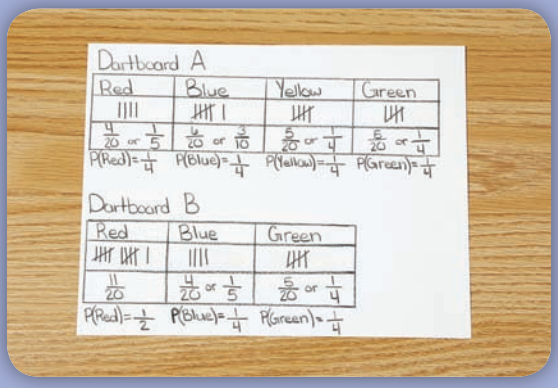
Materials
- Spinners



1. Have students spin the spinner, which is divided into four equal parts, 20 times and record the results on a tally chart. **Ask:** *Based on your results, what is the probability of hitting each of the colors on this spinner?*

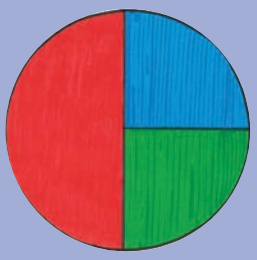


2. Now have students repeat the experiment with the spinner that is divided into one half and two quarters. **Ask:** *Based on your results, what is the probability of hitting each of the colors on this spinner?*



3. Have students compare the results of their experiments to the actual probabilities of hitting each color. Ask them to summarize their findings. **Ask:** *Are both of the dartboards fair? Explain.*

⚠ Look Out!

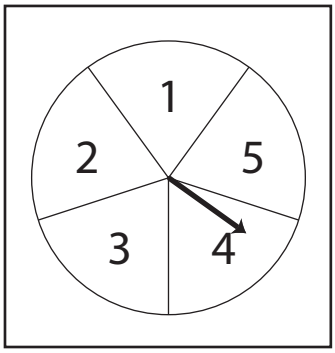


Some students may think that if an outcome is possible, that it is as likely to occur as any other possible outcome. Help students realize that the area a section covers influences the probability that the spinner will land there.

Answer Key

Use the spinner collection to model fair and unfair spinners. Find a spinner whose sections match each spinner below. Answer the questions.

1.



Find $P(1)$. $\frac{1}{5}$

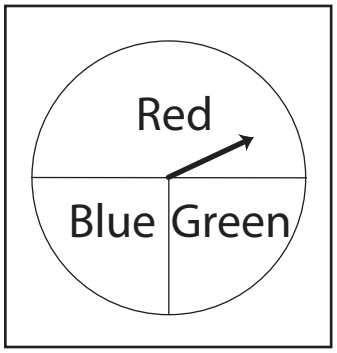
Find $P(3)$. $\frac{1}{5}$

Find $P(4)$. $\frac{1}{5}$

Is the spinner fair? yes

Why or why not? all sections have the same probability

2.



Find $P(\text{red})$. $\frac{1}{2}$

Find $P(\text{blue})$. $\frac{1}{4}$

Find $P(\text{green})$. $\frac{1}{4}$

Is the spinner fair? no

Why or why not? sections have different probabilities

Using the spinner collection, model a fair and an unfair spinner. Sketch the models. Answer the questions.

3. Sketch a fair spinner below.

Check students' models and explanations.

Why is the spinner fair? _____

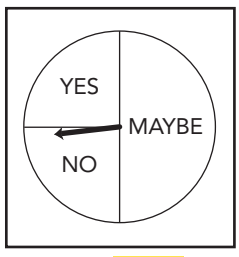
4. Sketch an unfair spinner below.

Check students' models and explanations.

Why is the spinner unfair? _____

Determine if each spinner is fair. Explain your answer.

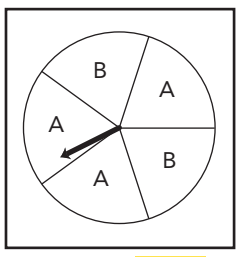
5.



no

the outcomes have different probabilities

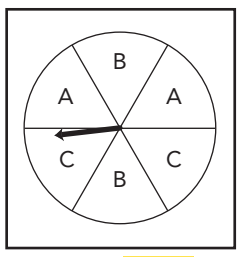
6.



no

the probability is greater for spinning A

7.



yes

the probability for each outcome is the same

Answer Key

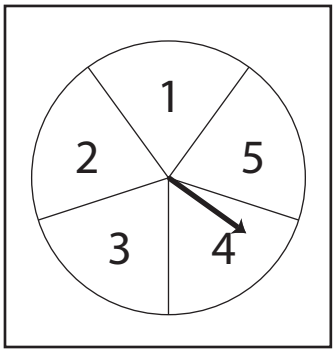
Challenge! When a spinner has an odd number of equal-sized sections and the sections are not uniquely labeled, how can you be certain that the spinner is not fair? Are there any odd numbers for which the spinner could be fair? Explain or draw an example.

Challenge: (Sample) The number and size of the sections with different labels have to be equal. If a spinner has 5 sections, you can meet that requirement if none of the sections has the same label. A spinner with 9 equal sections could be fair if the sections were marked with 3 different labels equally. This would be true for a spinner with 15 sections and 5 different labels. This type of pattern is true for spinners that have a number of labels that is a factor of the number of sections.

Name _____

Use the spinner collection to model fair and unfair spinners. Find a spinner whose sections match each spinner below. Answer the questions.

1.



Find $P(1)$. _____

Find $P(3)$. _____

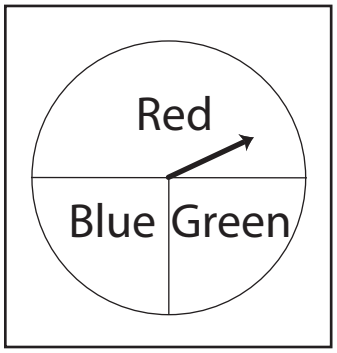
Find $P(4)$. _____

Is the spinner fair? _____

Why or why not? _____

_____

2.



Find $P(\text{red})$. _____

Find $P(\text{blue})$. _____

Find $P(\text{green})$. _____

Is the spinner fair? _____

Why or why not? _____

_____

Using the spinner collection, model a fair and an unfair spinner. Sketch the models. Answer the questions.

3. Sketch a fair spinner below.

Why is the spinner fair? _____

_____

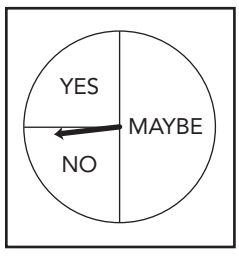
4. Sketch an unfair spinner below.

Why is the spinner unfair? _____

_____

Determine if each spinner is fair. Explain your answer.

5.



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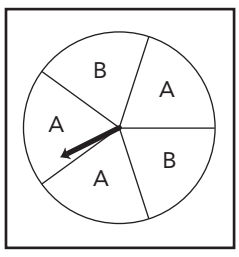
6.



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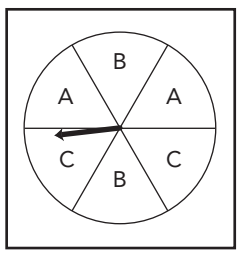
7.



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Name _____

Challenge! When a spinner has an odd number of equal-sized sections and the sections are not uniquely labeled, how can you be certain that the spinner is not fair? Are there any odd numbers for which the spinner could be fair? Explain or draw an example.