Do You Know?

Precalculus, by R. Rusczyk

If you can solve nearly all of the following problems with little difficulty *without a calculator*, then the text **Precalculus** would only serve as a review for you.

1. What is the value of $\tan 75^\circ$?
2. Simplify $\cos(12^\circ)\cos(24^\circ)\cos(48^\circ)\cos(96^\circ)$.
3. Let $n \geq 3$ be a positive integer, and let $P_0P_1\cdots P_{n-1}$ be a regular n-gon inscribed in a circle with radius 1. Compute $P_0P_1 \cdot P_0P_2 \cdot P_0P_3 \cdots P_0P_{n-1}$ in terms of n.
4. What is the value of $\cos^2 10^\circ + \cos^2 50^\circ - \sin 40^\circ \sin 80^\circ$?
5. Find the roots of $z^6 + z^4 + z^2 + 1$.
6. Suppose $\dfrac{\cos 3x}{\cos x} = \dfrac{1}{3}$ for some angle x, $0 \leq x \leq \frac{\pi}{2}$. Determine $\dfrac{\sin 3x}{\sin x}$ for the same x.
7. Find the volume of the tetrahedron with vertices $(-1,3,0)$, $(2,1,7)$, $(-4,3,2)$, $(3,1,-2)$.
8. A sequence (a_1,b_1), (a_2,b_2), (a_3,b_3), $\ldots$ of points in the coordinate plane satisfies

   $$(a_{n+1},b_{n+1}) = (\sqrt{3}a_n - b_n, \sqrt{3}b_n + a_n)$$

   for all positive integers n. Suppose that $(a_{100},b_{100}) = (2,4)$. What is $a_1 + b_1$?
9. Find $\operatorname{Im}\left((\cos 12^\circ + i\sin 12^\circ + \cos 48^\circ + i\sin 48^\circ)^6\right)$.
10. Evaluate $\sin 10^\circ \sin 20^\circ \sin 30^\circ \cdots \sin 170^\circ$.

Don't look at the next page until you've attempted all the problems!

The answers to Do You Know **Precalculus** are below. (The solutions in the text and the solutions manual include detailed explanations, as opposed to the mere answers provided below.)

1. $2+\sqrt{3}$
2. $-\frac{1}{16}$
3. n
4. $\frac{3}{4}$
5. $\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i, i, -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i, -i, \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$
6. $\frac{7}{3}$
7. $25/3$
8. $1/2^{98}$
9. 0
10. $\dfrac{9}{65536}$