

If you've mastered arithmetic, fractions, and the basic algebraic concepts illustrated in the problems below, you are ready for the Art of Problem Solving book **Introduction to Geometry**. Answers to these problems are on the following page. **Do not use a calculator**.

- 1. Solving linear equations. Sample questions:
 - (a) Find x: 31x + 24 = 365.
 - (b) Find n: 7n 4 = 2n + 16.
- 2. Simplifying fractions containing algebraic expressions. Reduce the following fractions: 3x + 6

(a)
$$\frac{3}{n(n-1)}$$

(b) $\frac{n(n-1)}{n(n+1)(r-1)}$

3. Addition and subtraction of quotients with different algebraic denominators. Write each of the following as a single fraction in simplest terms:

(a)
$$\frac{1}{mn} + \frac{1}{m(2n-2)}$$
.
(b) $\frac{r}{r-1} - \frac{r-1}{r}$.

4. Multiplication of polynomials and binomials. Expand each of the following:

(a)
$$(x+2)(x+3)$$
.
(b) $(x+y)(x^2+2xy+y^2)$.
(c) $(x-1)^4$. (Hint: $(x-1)^4 = (x-1)(x-1)^3$.)

- 5. Solving polynomial equations. Sample questions:
 - (a) Find $x: x^2 18x + 80 = 0.$
 - (b) Find x: $2x^2 + 5x + 2 = 0$.
 - (c) Find x: $x^4 13x^2 + 36 = 0$. (Hint: let $y = x^2$.)

6. Solving inequalities. Sample questions:

- (a) Find the solution set: $2x + 3 \le 5x 6$.
- (b) Find the solution set: |x 3| > 4.
- (c) Find the solution set: $|x 3| \le 4$.



Answers

1.

(a)
$$x = 11$$

(b) $n = 4$.

2.

(a)
$$x + 2$$
.
(b) $\frac{n-1}{(n+1)(r-1)}$ or $\frac{n-1}{nr+r-n-1}$.

3.

(a)
$$\frac{3n-2}{mn(2n-2)}$$
 or $\frac{3n-2}{2mn^2-2mn}$.
(b) $\frac{2r-1}{r(r-1)}$ or $\frac{2r-1}{r^2-r}$

4.

(a)
$$x^2 + 5x + 6$$
.
(b) $x^3 + 3x^2y + 3xy^2 + y^3$.
(c) $x^4 - 4x^3 + 6x^2 - 4x + 1$.

5.

(a)
$$x = 8, 10.$$

(b) $x = -2, \frac{-1}{2}.$
(c) $x = -3, -2, 2, 3$

6.

(a)
$$x \ge 3$$
.
(b) $x < -1$ or $x > 7$.
(c) $-1 \le x \le 7$.

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