LEVEL FIFTEEN

FRACTIONS

Developmental Mathematics

ADVANCED SKILLS

15

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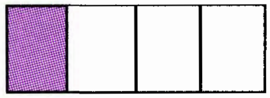
Date ______________

1 CHANGING INTO A SMALLER FRACTIONAL UNIT

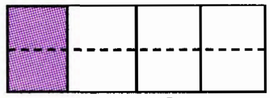
Example 1.

The figure to the right is divided into fourths.
One fourth is shaded.



- If each one fourth is divided into 2 equal parts, the figure becomes 8 eighths.

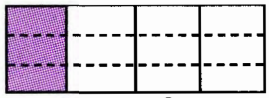
  1 fourth becomes 2 eighths.



$\frac{1}{4} = \frac{2}{8}$

- If each one fourth is divided into 3 equal parts, the figure becomes 12 twelfths.

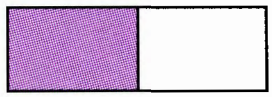
  1 fourth becomes 3 twelfths.



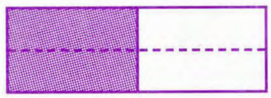
$\frac{1}{4} = \frac{3}{12}$

Example 2.

The figure is divided into halves.
One half is shaded.
Divide each one half into 2 equal parts.



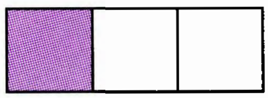
$\frac{1}{2} =$



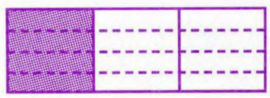
The whole becomes ______________
One-half becomes ______________

Example 3.

The figure is divided into thirds.
One third is shaded.
Divide each one third into 4 equal parts.



$\frac{1}{3} =$

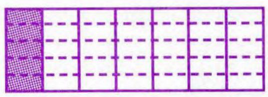


The whole becomes ______________
One third becomes ______________

Example 4.

The figure is divided into sevenths.
One seventh is shaded.
Divide each one seventh into 5 equal parts.

$\frac{1}{7} =$



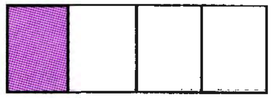
The whole becomes ______________
One seventh becomes ______________

APPLICATIONS

1. The figure is divided into fourths.
   1 fourth is shaded.

   Make the whole into 12 twelfths.

   1 fourth is made into ____________

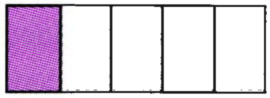


$\frac{1}{4} =$

2. The figure is divided into fifths.
   1 fifth is shaded.

   Make the whole into twentieths.

   1 fifth is made into ____________



$\frac{1}{5} =$

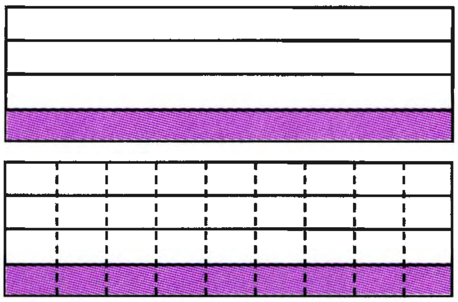
3. The figure is divided into thirds.
   1 third is shaded.

   Make the whole into eighteenths.

   1 third is made into ____________

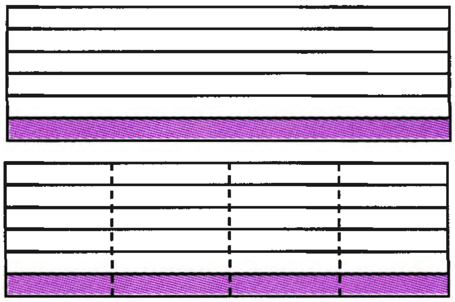
$\frac{1}{3} =$

4.



4 fourths become ________________
1 fourth becomes ________________

5.



6 sixths become ________________
1 sixth becomes________________

6. Write the missing number.

a. $\frac{1}{3} = \frac{}{12}$ b. $\frac{1}{5} = \frac{}{15}$ c. $\frac{1}{8} = \frac{}{56}$ d. $\frac{1}{9} = \frac{}{18}$

e. $\frac{1}{3} = \frac{}{15}$ f. $\frac{1}{6} = \frac{}{30}$ g. $\frac{1}{10} = \frac{}{40}$ h. $\frac{1}{7} = \frac{}{42}$

i. $\frac{1}{9} = \frac{}{45}$ j. $\frac{1}{8} = \frac{}{80}$ k. $\frac{1}{5} = \frac{}{85}$ l. $\frac{1}{12} = \frac{}{108}$

Date ______________

Example 5.

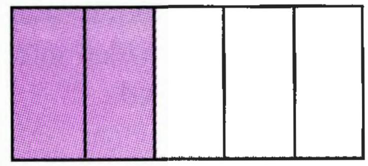
The figure is divided into fifths.

2 fifths are shaded.

Divide each one-fifth into 4 equal parts,

The whole becomes 20 twentieths.

2 fifths become 8 twentieths.



$\frac{2}{5} =$

Example 6.

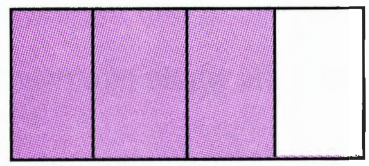
The figure is divided into fourths.

3 fourths are shaded.

Divide each one fourth into 5 equal parts,

The whole becomes 20 twentieths.

3 fourths become 15 twentieths.



$\frac{3}{4} =$

Example 7.

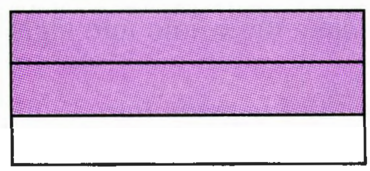
The figure is divided into thirds.

2 thirds are shaded.

Divide each one-third into 4 equal parts.

The whole becomes 12 twelfths.

2 thirds become _____ twelfths.



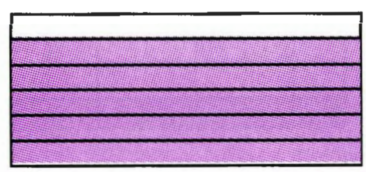
$\frac{2}{3} =$

Example 8.

The figure is divided into sixths.

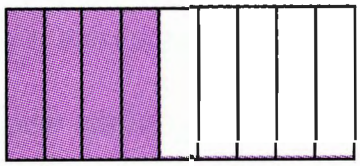
5 sixths are colored.

- If you divide each one-sixth into 2 equal parts,
  the whole becomes 12 twelfths.
  5 sixths become 10 twelfths.
- If you divide each one-sixth into 3 equal parts,
  the whole becomes 18 eighteenths.
  5 sixths become _______________
- If you divide each one-sixth into 5 equal parts,
  the whole becomes 30 thirtieths.
  5 sixths become _______________



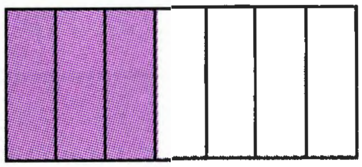
APPLICATIONS

1. The figure is divided into ninths.
   4 ninths are shaded.



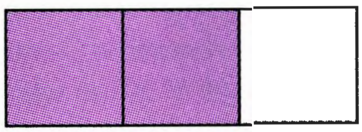
   You make the ninths into eighteenths.
   1 ninth is made into ____________ $\frac{1}{9} =$
   4 ninths are made into ____________ $\frac{4}{9} =$

2. The figure is divided into sevenths.
   3 sevenths are shaded.



   You make the sevenths into fourteenths.
   1 seventh is made into ____________ $\frac{1}{7} =$
   3 sevenths are made into ____________ $\frac{3}{7} =$

3. The figure is divided into thirds.
   2 thirds are shaded.



   - If you make the figure into sixths, what fraction of the figure would be shaded? ____________
   - If you make the figure into ninths, what fractions of the figure would be shaded? ____________
   - If you make the figure into twelfths, what fractions of the figure would be shaded? ____________
   - If you make the figure into fifteenths, what fraction of the figure would be shaded? ____________

4. Write the missing number:

a. $\frac{5}{8} = \frac{15}{24}$	b. $\frac{6}{7} = \frac{\quad}{35}$	c. $\frac{3}{10} = \frac{\quad}{30}$
d. $\frac{2}{7} = \frac{6}{21}$	e. $\frac{5}{11} = \frac{\quad}{55}$	f. $\frac{9}{20} = \frac{\quad}{100}$
g. $\frac{4}{5} = \frac{8}{10} = \frac{40}{50}$	h. $\frac{8}{12} = \frac{\quad}{60} = \frac{\quad}{24}$	i. $\frac{3}{8} = \frac{\quad}{64} = \frac{\quad}{40}$
j. $\frac{5}{6} = \frac{20}{24} = \frac{25}{30}$	k. $\frac{2}{9} = \frac{\quad}{45} = \frac{\quad}{36}$	l. $\frac{7}{10} = \frac{\quad}{100} = \frac{\quad}{1000}$

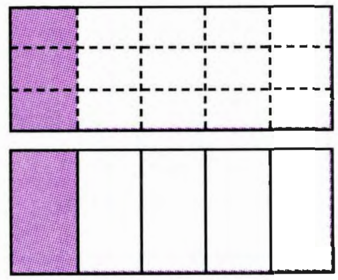
Date ________________

2 CHANGING INTO A LARGER FRACTIONAL UNIT

Example 1.

The figure to the right is divided into 15 equal parts.
Each part is $\frac{1}{15}$ of the whole.

We may group each 3 fifteenths.
The whole becomes 5 equal parts.
Each part is $\frac{1}{5}$ of the whole.
3 fifteenths become 1 fifth.

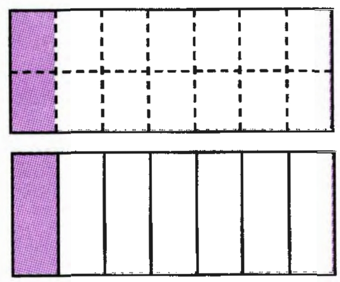


$\frac{3}{15} =$

Example 2.

The figure to the right is divided into 14 equal parts.
Each part is $\frac{1}{14}$ of the whole.

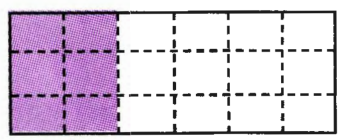
We may group each 2 fourteenths.
The whole becomes 7 equal parts.
Each part is $\frac{1}{7}$ of the whole.
2 fourteenths become 1 seventh.



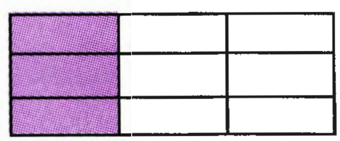
$\frac{2}{14} =$

Example 3.

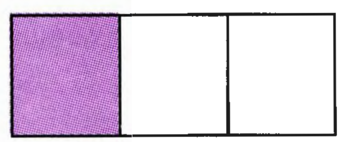
The figure to the right is divided into 18 equal parts.
Each part is $\frac{1}{18}$ of the whole.



- We may group each 2 eighteenths.
  The whole becomes 9 equal parts.
  Each part is $\frac{1}{9}$ of the whole.
  2 eighteenths become 1 ninth. $\frac{2}{18} =$



- We may group each 3 ninths.
  The whole becomes 3 equal parts.
  Each part is $\frac{1}{3}$ of the whole.
  3 ninths become 1 third. $\frac{3}{9} =$



- We may start by grouping each 6 eighteenths.
  The whole becomes 3 equal parts.
  Each part is $\frac{1}{3}$ of the whole.
  6 eighteenths become 1 third. $\frac{6}{18} =$

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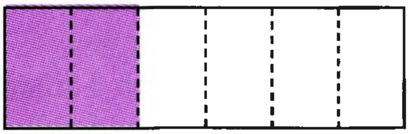
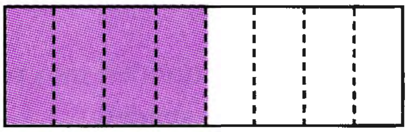
APPLICATIONS

1. The figure is divided into sixths.
   Group each 2 sixths.
   The whole becomes ____________
   Each part is _______ of the whole.



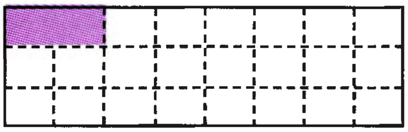
$\frac{2}{6} =$

2. The figure is divided into eighths.
   Group each 4 eighths.
   The whole becomes ____________
   Each part is _______ of the whole.



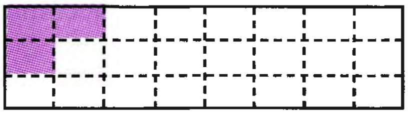
$\frac{4}{8} =$

3. The figure is divided into twenty-fourths.
   a. Group each 2 twenty-fourths.
      The whole becomes ____________
      Each part is _______ of the whole.



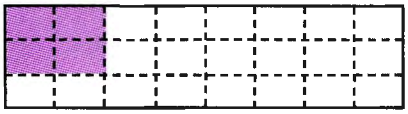
$\frac{2}{24} =$

   b. Group each 3 twenty-fourths.
      The whole becomes ____________
      Each part is _______ of the whole.



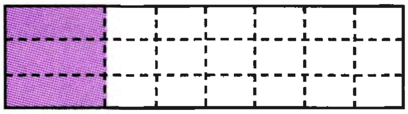
$\frac{3}{24} =$

   c. Group each 4 twenty-fourths
      The whole becomes ____________
      Each part is _______ of the whole.



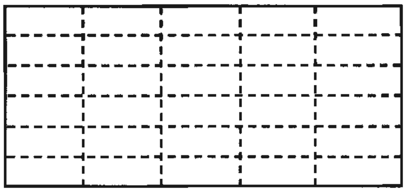
$\frac{4}{24} =$

   d. Group each 6 twenty-fourths.
      The whole becomes ____________
      Each part is _______ of the whole.



$\frac{6}{24} =$

4. The figure is divided into thirtieths.
   You may group the thirtieths in different ways.



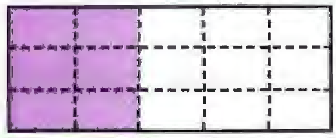
$\frac{2}{30} = \frac{1}{\quad}$ $\frac{3}{30} = \frac{1}{\quad}$ $\frac{10}{30} = \frac{1}{\quad}$

$\frac{5}{30} = \frac{1}{\quad}$ $\frac{6}{30} = \frac{1}{\quad}$ $\frac{15}{30} = \frac{1}{\quad}$

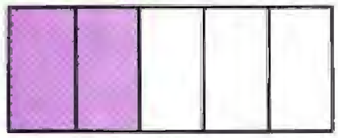
Date ________________

Example 4.

The figure is divided into fifteenths.
6 fifteenths are shaded.



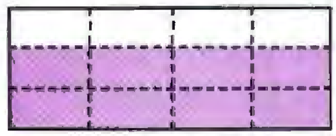
We may group each 3 fifteenths.
The whole becomes 5 fifths.
The shaded area becomes 2 fifths.



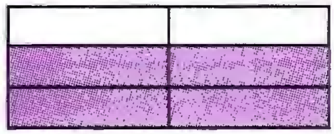
$\frac{6}{15} =$

Example 5.

The figure is divided into twelfths.
8 twelfths are shaded.

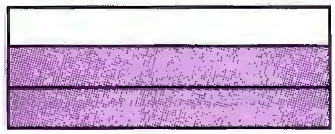


a. We may group each 2 twelfths.
The figure becomes 6 sixths.
The shaded area becomes 4 sixths.



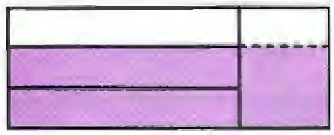
$\frac{8}{12} =$

b. We may group each 4 twelfths.
The figure becomes 3 thirds.
The shaded area becomes 2 thirds.



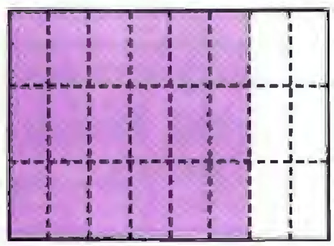
$\frac{8}{12} =$

c. We may group each 3 twelfths.
The figure becomes 4 fourths.
We are unable to decide how many fourths are shaded.



Example 6.

The figure is divided into twenty-fourths.
18 twenty-fourths are shaded.



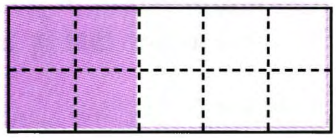
- If you group each 2 twenty-fourths,
the figure becomes 12 twelfths.
The shaded area becomes 9 twelfths. $\frac{18}{24} =$

- If you group each 3 twenty-fourths,
the figure becomes 8 eighths.
The shaded area becomes 6 eighths. $\frac{18}{24} =$

- If you group each 6 twenty-fourths, the figure becomes 4 fourths.
The shaded area becomes 3 fourths. $\frac{18}{24} =$

APPLICATIONS

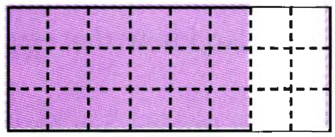
1. The figure is divided into tenths.
   4 tenths are shaded.
   Group each 2 tenths.
   The whole becomes ___________
   The shaded area becomes ___________



$\frac{4}{10} =$

2. The figure is divided into twenty-fourths.
   18 twenty-fourths are shaded.

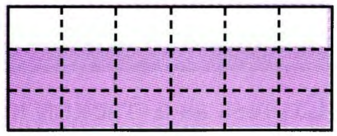
   Group each 3 twenty-fourths.
   The shaded area becomes ___________



$\frac{18}{24} =$

3. The figure is divided into eighteenths.
   12 eighteenths are shaded.

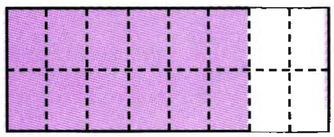
   Group each 2 eighteenths.
   The shaded area becomes ___________



$\frac{12}{18} =$

4. The figure is divided into sixteenths.
   12 sixteenths are shaded.

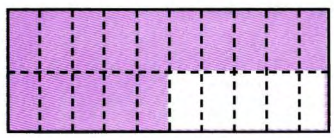
   Group each 4 sixteenths.
   The shaded area becomes ___________



$\frac{12}{16} =$

5. The figure is divided into twentieths.
   15 twentieths are shaded.

   Group each 5 twentieths.
   The shaded area becomes ___________



$\frac{15}{20} =$

6. Group as many units as you can, and then write the fraction you obtain:

a. $\frac{20}{30} =$ b. $\frac{20}{48} =$ c. $\frac{18}{27} =$

d. $\frac{8}{10} =$ e. $\frac{5}{100} =$ f. $\frac{75}{1000} =$

g. $\frac{42}{70} =$ h. $\frac{45}{60} =$ i. $\frac{48}{72} =$

A

Date ______________

3 EXPRESSING A DECIMAL AS A COMMON FRACTION AND VICE VERSA

- You know that .8 or 0.8 may be written $\frac{8}{10}$.
  You also know that $\frac{8}{10}$ may be changed into $\frac{4}{5}$.
  $.8 = \frac{8}{10} = \frac{4}{5}$

- You know that .75 or 0.75 may be written $\frac{75}{100}$.
  You also know that $\frac{75}{100} = \frac{15}{20} = \frac{3}{4}$.
  $.75 = \frac{75}{100} = \frac{15}{20} = \frac{3}{4}$

- You know that .025 or 0.025 may be written $\frac{25}{1000}$.
  You also know that $\frac{25}{1000} = \frac{5}{200} = \frac{1}{40}$.
  $.025 = \frac{25}{1000} = \frac{5}{200} = \frac{1}{40}$

In these examples, we have expressed each decimal fraction as a fraction with a numerator and denominator, and then changed the fraction into the largest unit.

EXERCISES

1. Express as a fraction in the largest unit.

a. 0.5 =

b. 0.05 =

c. 0.005 =

d. 0.0005 =

e. 0.2 =

f. 0.002 =

g. 0.6 =

h. 0.25 =

i. 0.125 =

j. 0.875 =

k. 0.625 =

l. 0.3125 =

2. Express the sum of .06 and 0.0025 as a fraction in the largest unit.

3. Express the difference between 12.7 and 12.675 as a fraction in the largest unit.

EXPRESSING A DECIMAL AS A COMMON FRACTION AND VICE VERSA

Example 1.

Change $\frac{1}{2}$ into a decimal fraction.

Discussion:

To change $\frac{1}{2}$ into a decimal fraction we have to change it into tenths, or hundredths, or thousandths.

$\frac{1}{2} = \frac{5}{10} = 0.5$

Example 2.

Change $\frac{1}{4}$ into a decimal fraction.

Discussion:

You cannot change $\frac{1}{4}$ into tenths, but you can change it into hundredths.

$\frac{1}{4} = \frac{25}{100} = .25$

Example 3.

Change $\frac{1}{8}$ into a decimal fraction.

Discussion:

You cannot change $\frac{1}{8}$ into tenths or hundredths, but you can change it into thousandths.

$\frac{1}{8} = \frac{125}{1000} = .125$

EXERCISES

1. Change into decimal fractions

a. $\frac{1}{5} =$ b. $\frac{3}{5} =$

c. $\frac{3}{4} =$ d. $\frac{1}{50} =$

e. $\frac{3}{50} =$ f. $\frac{37}{50} =$

g. $\frac{3}{25} =$ h. $\frac{6}{25} =$

i. $\frac{1}{20} =$ j. $\frac{3}{20} =$

2. Add 1.391 to $\frac{3}{4}$

3. Subtract $\frac{4}{5}$ from 0.912