

EXERCISE 1.3.8.4C

Let \mathcal{A} and \mathcal{B} be $n \times n$ matrices. Prove that

$$\begin{aligned} \det(\mathcal{A} + \mathcal{B}) &= \det \mathcal{A} + \det \mathcal{B} \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \det(\mathcal{A} + \mathcal{B}_{ij}) \\ &\quad - \sum_{i=1}^n \sum_{j=1}^n \det(\mathcal{A} + \mathcal{B}_{ij}) \end{aligned}$$