# Expressions and Equations

In eighth grade, students continue to build versatility with expressions. They apply the properties of integer exponents to recognize, generate, and work flexibly with equivalent expressions. Through this work, they gain an appreciation for the notion that an exponent can be positive or negative (or zero), and they apply the idea in various useful ways. For example, students learn the use of positive and negative powers of 10 to express very large numbers, such as $2 \times 10^{12}$, and very small numbers, such as $2 \times 10^{-12}$. Students also work with radicals, such as square roots and cube roots.

Students build on previous concepts of unit rates and proportions and continue to develop understanding of how to use linear equations. For example, they will work with systems of linear equations to represent, analyze, and solve a variety of problems.

Additionally, students use similar triangles to explain why the slope $m$ is the same between any two points on a non-vertical line. They apply this and previous understandings to recognize that the only difference between the lines $y = mx$ and $y = mx + b$ is their $y$-intercepts.

**The Grade 8 Common Core State Standards for Expressions and Equations specify that students should–**

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

The following hands-on activities will help students develop a deeper understanding of expressions and equations. Teachers will want to encourage students to think, in particular, about situations in which inverse-related structures are applied, such as situations involving exponents and radicals. Precision is of special importance as students work with more complex ideas such as these. Teachers will want to quickly address misconceptions.

# Expressions and Equations

## Contents

LESSON 1

## Expressions and Equations

# Squares and Square Roots

### Objective

Find the square of a number and the square root of a perfect square.

### Common Core State Standards

- **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Building squares helps students identify the relationship between squares and square roots. Students who can find the square roots of perfect squares are better prepared to estimate the square roots of other numbers. Learning these skills enables students to understand and use the Pythagorean relationship, the quadratic formula, and the distance formula.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *Why is a number written with an exponent of 2 often read as that number squared?*
- **Ask:** *What is the relationship between a square and its square root?*
- **Ask:** *How can you find the square root of a number if you do not have tiles to make a square?*

## Solve It

Reread the problem with students. Discuss how the questions relate to finding squares and square roots. Have students write a paragraph using the terms *square* and *square root* to answer the Try It! questions.

## More Ideas

For other ways to teach the relationship between squares and square roots—

- Have students use Centimeter Cubes to make square arrays and record their observations in a table. One row of the table will be side length, or square root, and the second row will be the total number of cubes, or square. Encourage pairs to combine their cubes to make squares for larger numbers.
- Use Cuisenaire® Rods to have students model squares with side lengths up to 10 units. Have students group rods of equal length to make each square. The number of rods used equals the side length, or square root.
- Use Geoboards to make four of the square designs in the problem. Instruct students to consider the space between pegs as one unit when making squares.

## Formative Assessment

Have students try the following problem.

*What is the square root of 100?*

**A.** 1 **B.** 10 **C.** 1,000 **D.** 10,000

## Try It!

**25 minutes | Groups of 4**

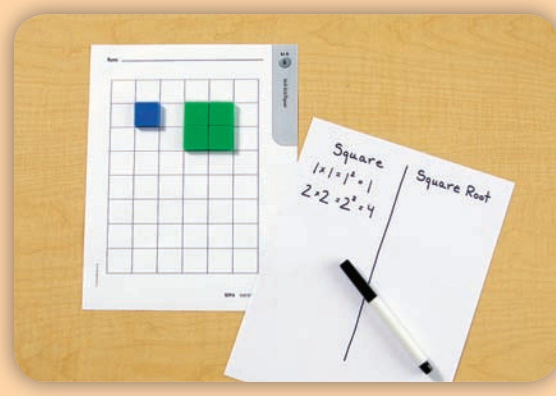
Here is a problem about squares and square roots.

*Kayla uses 91 square tiles to make 6 designs. Each design is a square and all are different sizes. How many tiles are used in each square? What is the length of the sides in each square?*

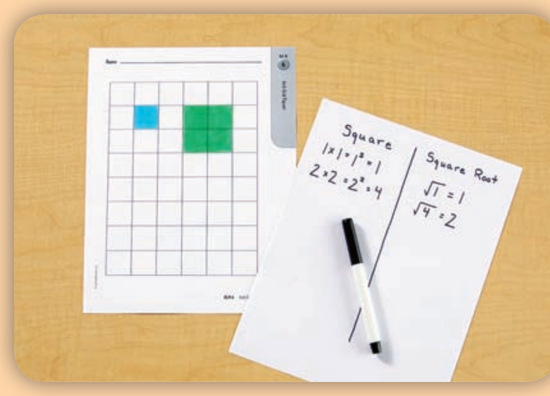
Introduce the problem. Then have students do the activity to solve the problem. Define the terms *square* and *square root.* Ask students to label one side of their papers "Square" and the other side "Square Root". Distribute the materials.

### Materials

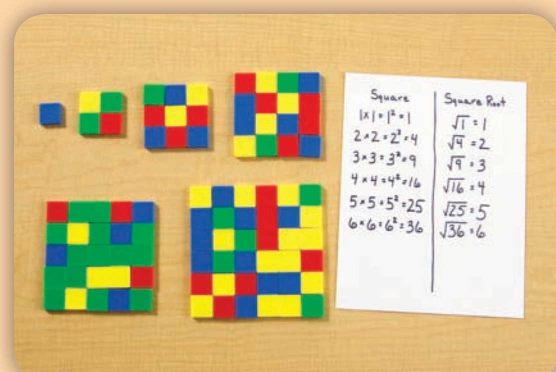
- Color Tiles (91 per group)
- Inch Grid Paper (BLM 1; 1 per group)
- paper (1 sheet per group)
- pencils (1 per group)
- colored pencils (2 per group)




**1.** On the grid paper, have students build a 1 × 1 and a 2 × 2 square. **Ask:** *How can you find the number of tiles in each square?* Write $1 \times 1 = 1^2 = ?$ and $2 \times 2 = 2^2 = ?$ on the board. Review the meaning of the exponent 2. Have students write equations to show each square number.




**2. Ask:** *Which part of the square represents the square root?* Write $\sqrt{4} = 2$ on the board. Explain that the radical sign means "find the square root." Have students copy the equation and write an equation showing the square root of 1 on their papers.



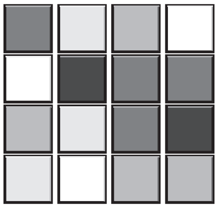

**3.** Have students use tiles to make the other squares. Remind them to include a one-tile square in their list and to use the appropriate symbols when writing equations for the square and the square root.

## ⚠ Look Out!

If students have difficulty finding the 6 smallest perfect squares, suggest that they start with one tile, add one tile to the row, and then add a tile to each column to make a square. Encourage them to continue this method to build larger squares. If students confuse the square and square root, point out that in mathematics *root* means "the answer." For a given number of tiles, the square root of that number gives the dimensions of the square that can be built with the tiles.

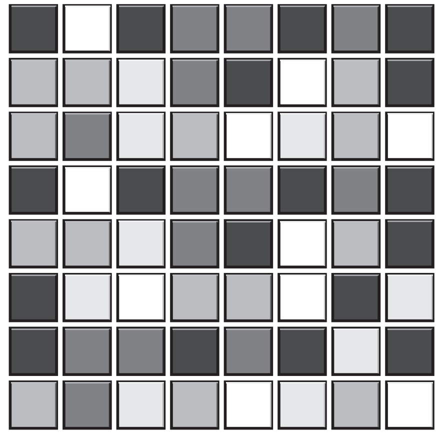
**Use Color Tiles to model each number. Write the perfect square under the radical symbol. Write the square root.** (Check students' work.)

**1.**



$\sqrt{16}$ = 4

**2.**



$\sqrt{64}$ = 8

**Using Color Tiles, model each number to determine if it is a perfect square. Sketch the model. Write the perfect square under the radical. Write the square root.**

(Check students' models.)

**3.** 36

$\sqrt{36}$ = 6

**4.** 100

$\sqrt{100}$ = 10

**Find each square root.**

**5.** $\sqrt{4}$ 2

**6.** $\sqrt{81}$ 9

**7.** $\sqrt{144}$ 12

**8.** $\sqrt{25}$ 5

**9.** $\sqrt{9}$ 3

**10.** $\sqrt{16}$ 4

**11.** $\sqrt{49}$ 7

**12.** $\sqrt{121}$ 11

**13.** $\sqrt{1}$ 1

## *Answer Key*

**Challenge!** Use the pattern that you noticed on the previous page to find the square root of 400. Explain.

Challenge: (Sample) Two identical numbers that can be multiplied to equal 400 are 20 and 20. So, $\sqrt{400} = 20$.

LESSON 2

## Objective

Evaluate cube roots.

## Common Core State Standards

- **8.EE.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Expressions and Equations

# Cube Roots

Students can build on their knowledge of square roots to understand cube roots. Geometrically, the square root of a number is the length of a side of the square with that area. Building on that concept, it follows that the cube root of a number is the length of an edge of a cube with that volume. By definition, if $r$ is the cube root of $x$, then $r^3 = x$.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *What is a cube?* Make sure students understand that the length, width, and height of a cube are the same.
- **Ask:** *What is the length of your cube? The width? The height? What is the volume?* Help students recognize that the volume is the number of cubes they started with and that the length of an edge is the cube root.
- Talk about the term *cube root*. Discuss that the length of the edge of a cube is the geometric representation of a cube root. **Ask:** *How could you write an equation to demonstrate the meaning of a cube root?* Students can write an equation such as $3 \times 3 \times 3 = 27$ or $3^3 = 27$ to show 3 is the cube root of 27, or $\sqrt[3]{27} = 3$.

## Solve It

Reread the problem with students. Have students draw a cube with a volume of 27. Have them label each edge 3 to demonstrate what they discovered when they modeled the problem. Finally, have them write the equation for the cube root under their drawing.

## More Ideas

For other ways to teach about cube roots—

- Have students use Snap Cubes® or Cuisenaire® Rods to build cubes of various sizes. Have students find the edge length and calculate for volume, and then use the edge length to state the cube root.
- To extend cube roots to decimals, ask students to locate cube roots on the Folding Number Line. They can use the number line to estimate irrational cube roots.

## Formative Assessment

Have students try the following problem.

*Lander is moving and needs to know if his sculpture will fit in a cubic box. He knows the volume is 1,000 cubic inches. What is the length of each of the box's edges?*

**A.** 8 in. **B.** 10 in. **C.** 100 in. **D.** 333 in.

## Try It! 15 minutes | Pairs

Here is a problem about cube roots.

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*A cubic storage shed has a volume of 27 cubic meters. How long is each side of the shed?*

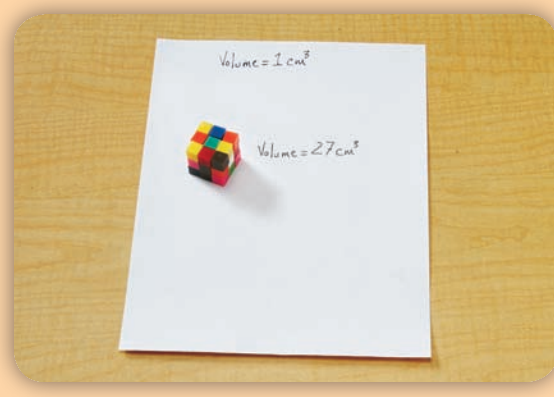
---

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

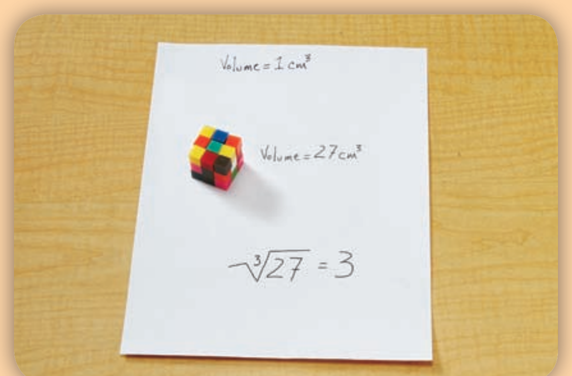
### Materials

- Centimeter Cubes (at least 27 per pair)
- paper (1 sheet per pair)
- pencils (1 per pair)


**1. Ask:** *What is* volume*?* Discuss with students how the Centimeter Cubes are related to volume. **Ask:** *What is the volume of a Centimeter Cube?* Explain that they will model the cubic meters in the problem using Centimeter Cubes.




**2. Ask:** *What is the volume of the storage shed?* Have students gather 27 Centimeter Cubes. **Ask:** *How can you build a cubic space with the cubes?* Prompt students to make a cube that includes all the Centimeter Cubes. **Ask:** *What does it mean to be* cubic*?* Elicit that the length, width, and height are all the same.




**3. Ask:** *What is the edge length of the cube?* Elicit that the edge length 3 is the cube root of 27. Ask students to write the equation for this cube root.

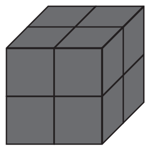
### ⚠ Look Out!

Students may build a rectangular prism instead of a cube. Remind them that all the edges should be the same in a cube.

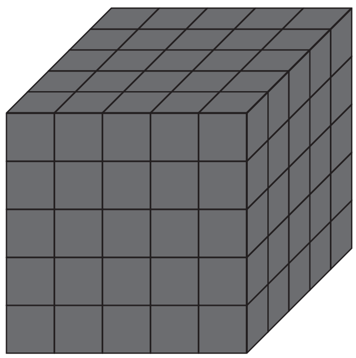
Answer Key

**Use Centimeter Cubes to build the model. Write the equation.** (Check students' work.)

**1.** Write an equation for the cube root. $\sqrt[3]{8} = 2$



**2.** Write an equation for the cube root. $\sqrt[3]{125} = 5$



**Using Centimeter Cubes, solve the problem.**

**3.** Leonard has a box with a volume of 1 cubic inch. What is the length of one edge? 1 inch

**4.** Paula tells her friend, "The cube root of 216 is the number of jobs I have lined up over summer break." How many jobs does she have? 6 jobs

**Complete the equation.**

**5.** $\sqrt[3]{1{,}728} =$ 12

**6.** $\sqrt[3]{729} =$ 9

## Answer Key

**Challenge!** You know that a cubic box holds 512 Centimeter Cubes. How can you use this information to find $\sqrt[3]{512}$?

Challenge: (Sample) I could line one edge of the box with Centimeter Cubes. The number of cubes is the cube root. In this case, the length, width, and height of the box are each 8 centimeters, so $\sqrt[3]{512} = 8$.

LESSON 3

## Expressions and Equations

# Slope as Rate of Change

### Objective

Determine the slope of a line when given a set of data points.

### Common Core State Standards

- **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

The students come to the concept of slope with an understanding of its "vertical rise to horizontal length," or simply "rise over run." In this activity, students will learn to formalize this understanding by working with rate of change and its impact on the slope of a line.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Say:** *Describe the slope of the line.*
- **Ask:** *Is the slope positive or negative? How do you get from one point on the line to the next?*
- **Ask:** *What would the line look like if the rate of change (slope) was up 3 and over (to the right) 1? Would this represent more money per car or less?*

## Solve It

Reread the problem with students. Ask students to describe the effect of rate of change on the slope of a line. Have them include sketches.

## More Ideas

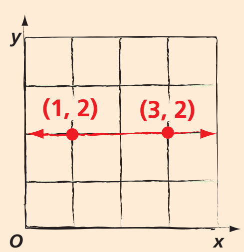
For other ways to teach about slope and rate of change—

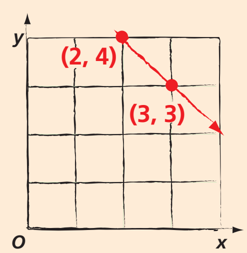
- Have students use their XY Coordinate Pegboards to create several lines with the same slope. **Ask:** *Will the lines be parallel to each other?*
- Have students use their XY Coordinate Pegboards to graph a line with a slope of 1/1 and another line with a slope of 2/1. **Say:** *Compare the two lines.* **Ask:** *What happens to the appearance (steepness) of the line as the slope is increased or decreased?*

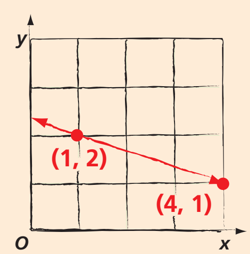
## Formative Assessment

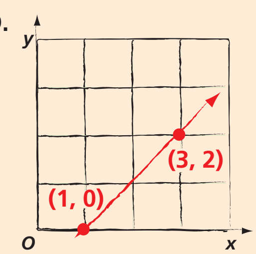
Have students try the following problem.

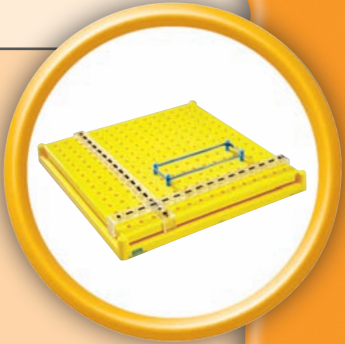
*Which graph shows a positive slope?*

A.



B.



C.



D.




## Try It! 25 minutes | Pairs

Here is a problem about rate of change.

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*The Math Club is raising money by holding a car wash. Graph the data shown in the table at right. Then examine the slope of the line to see the rate of change (slope). How much money will the club earn if they wash 6 cars?*

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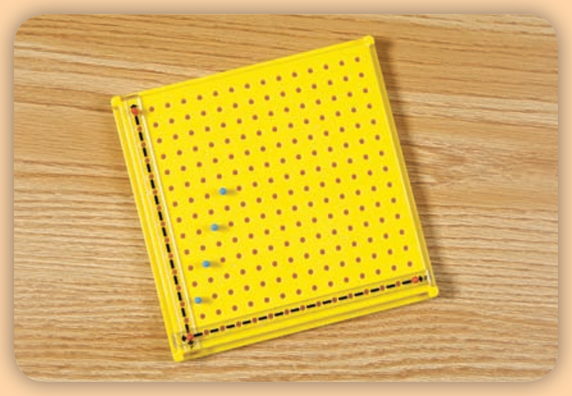
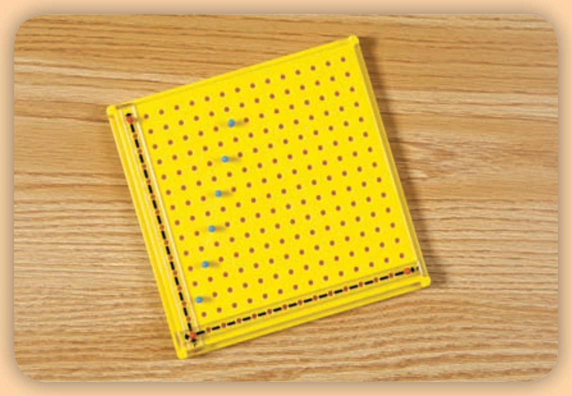
| Number of Cars Washed | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Money Earned | $2 | $4 | $6 | $8 |

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

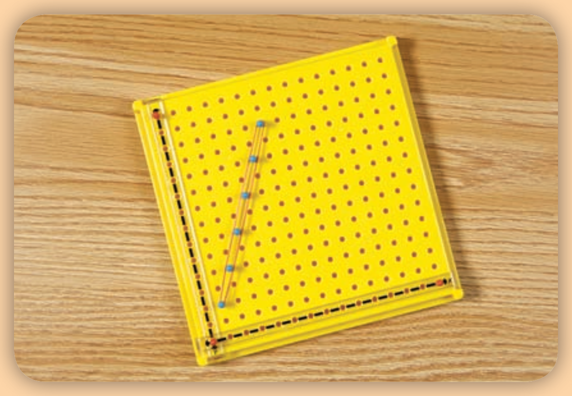
### Materials
- XY Coordinate Pegboard



**1.** Have students set up their pegboards for Quadrant I graphing. Then have them plot the values from the table.



**2. Ask:** *Do you see a pattern in the placement of the pegs?* Elicit from students that the pattern is "up 2 and over 1." Have students add two coordinates beyond those shown in the table.



### Look Out!

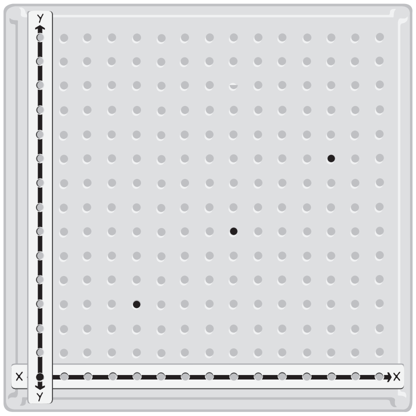
Make sure students recognize that the rate of change is the ratio of the vertical "rise" over the horizontal "run" and not vice-versa. Often students invert the slope.

**3.** Have students create a line by stretching a rubber band from (1, 2) to (6, 12). **Ask:** *As one value goes up, what happens to the other value?* **Say:** *If both values go up or both go down, the slope of the line is said to be positive. If one goes up while the other goes down, the slope is negative.* **Ask:** *Is the slope of this line positive or negative?* Discuss the rate of change (up 2 and over 1) between each of the data points with students. **Ask:** *How much will the Math Club earn if they wash 6 cars?*

**Answer Key**

**Use an XY Coordinate Pegboard to plot the ordered pairs. Complete the table. Find the slope, or rate of change, of the line.** (Check students' work.)

**1.**



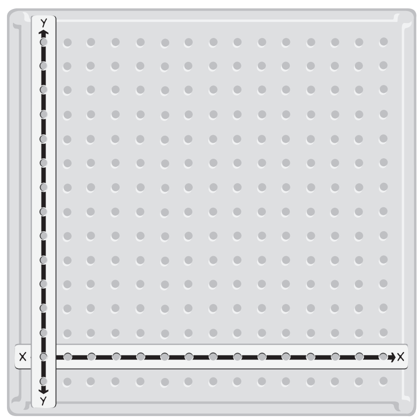
| *x* | *y* |
|---|---|
| 0 | 0 |
| 4 | 3 |
| 8 | 6 |
| 12 | 9 |

$\frac{3}{4}$

**Using an XY Coordinate Pegboard, model the ordered pairs. Sketch the graph. What is the slope of the line?** (Check students' models.)

**2.**

| *x* | *y* |
|---|---|
| 0 | 0 |
| 2 | 1 |
| 4 | 2 |
| 6 | 3 |
| 8 | 4 |
| 10 | 5 |



$\frac{1}{2}$

**Find the slope of a line that passes through each pair of points.**

**3.** (–1, 2) and (1, –2)

–2

**4.** (2, 2) and (1, 1)

1

**5.** (–7, 3) and (7, –3)

$-\frac{3}{7}$

**6.** (0, 0) and (5, 0)

0

**7.** (–1, –4) and (2, 8)

4

**8.** (4, 6) and (6, 9)

$\frac{3}{2}$

**9.** Which lines from Problems 3–8 have a negative slope?

Problems 3 and 5

## Answer Key

**Challenge!** Describe a line that has a positive slope. Describe a line that has a negative slope. What does a line with zero slope look like? Draw pictures to help.

Challenge: (Sample) If a line goes up to the right, then the line has a positive slope. If the line goes down to the right, then the line has a negative slope. A horizontal line has a slope of 0.

LESSON 4

Expressions and Equations

# Problem Solving with Rates of Change

## Objective

Graph lines using rates of change and determine the meaning of the point of intersection.

## Common Core State Standards

- **8.EE.8a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

As students increase their understanding of slope, *y*-intercept, and rate of change, it is important for them to test their knowledge with practical problems—those that they may encounter in the real world. This activity incorporates all of these concepts with a realistic problem about movie rentals.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *What is special about the y-intercept for Store B?*
- **Ask:** *On what day would Sara and Matt pay the same amount at either store? How can you tell from looking at the graphs?*
- **Ask:** *Suppose Sara and Matt wanted to rent the movie and to spend the least amount of money possible. What should they do? Explain.*

## Solve It

Reread the problem with students. Have students create a table of values that will represent the situation as stated in the story problem. Ask students to graph the values to determine the slopes and *y*-intercepts. Once their lines are formed, have them compare to see which store is the better value at $y = 10$.

## More Ideas

For another way to teach about rates of change—

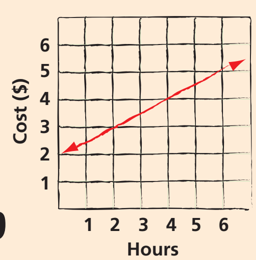
- Have students compare rental costs for various goods at local stores. Have them create a table of values and graph them on their XY Coordinate Pegboards. **Ask:** *Which stores offer better deals? Do some stores have better deals under certain conditions (as in the story problem on page 31)?*

## Formative Assessment

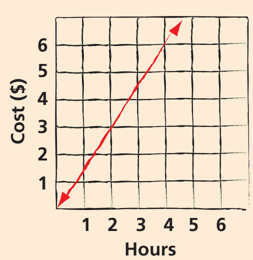
Have students try the following problem.

*Jim and Marty want to rent mountain bikes. The graphs show the pricing policy of four different outfitters. Which of the following graphs represents the best deal if they plan to use the bikes for 5 hours?*

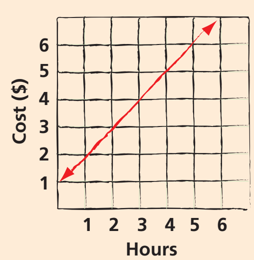
A.



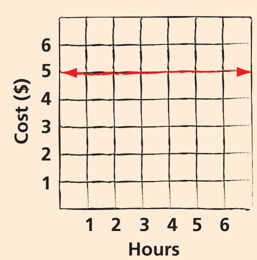

B.

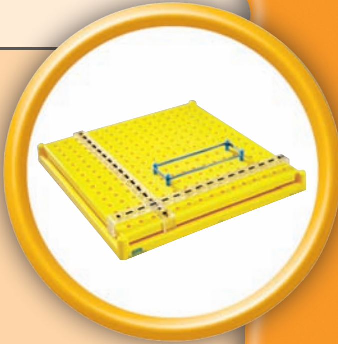



C.




D.





## Try It! 40 minutes | Pairs

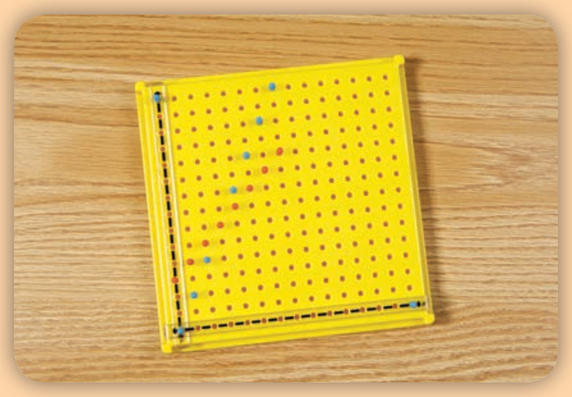
Here is a problem about rates of change.

*Sara and Matt are checking out movie rental stores for the best deal. Store A charges an initial fee of $3 and then $1 for each day the movie is rented. Store B charges no initial fee but charges $2 per day for rental. Sara and Matt want to keep the movie as long as possible, but they have only $10 to spend. Which store should they rent from?*

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

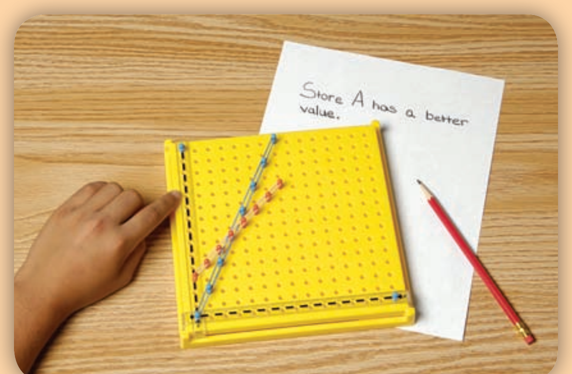
### Materials

- XY Coordinate Pegboard
- paper (2 sheets per pair)
- pencils (1 per pair)



**1.** Have students create a table of values for the two stores, listing seven values for each store, starting with $x$ = 1 (for Day 1). Ask students to plot the values, using red pegs for Store A and the blue pegs for Store B.


**2.** Have students use rubber bands to form the two lines. **Ask:** *What is the slope of each line? What is the* y*-intercept?* Have students write the equation for each line in the form of $y = mx + b$ on a sheet of paper. Elicit from them that $x$ is the number of days Sara and Matt can rent the movie and $y$ is the total cost of the rental.




**3.** Have students examine the lines to determine which store offers a better deal if Sara and Matt want to keep the movie for as long as possible, but have only $10 to spend.

### ⚠ Look Out!

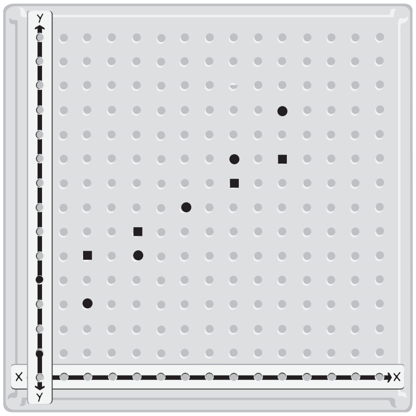
Some students may assume that the smaller the value of $x$, the better the deal they'll get. They need to understand that $x$ represents number of days, so, in the context of the story problem (Sara and Matt want to keep the movie as long as possible), more days is a better deal.

Watch for students who think that both lines should go through the origin of the graph. Remind them that one of the stores charges an initial fee of $3, which means that the $y$-intercept is 3.

**Answer Key**

**Use an XY Coordinate Pegboard to plot the ordered pairs shown. Draw each line and write its equation in the form $y = mx + b$. At what point do the lines intersect?** (Check students' work.)

**1.**




$y =$ $\frac{1}{2}x + 4$

$y =$ $x + 1$

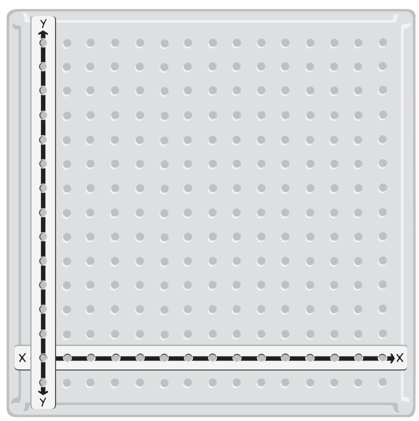
point of intersection: (6, 7)

**Using an XY Coordinate Pegboard, model the lines for the given equations. Name the slope and *y*-intercept for each line. Name the point of intersection of the lines.**

(Check students' models.)

**2.** $y = 3x$

$y = x + 6$



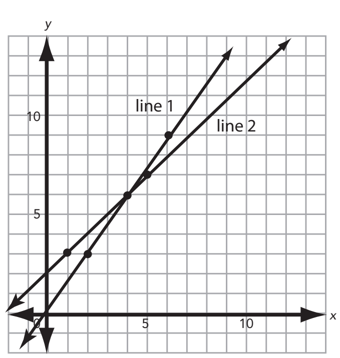

$m =$ 3, $b =$ 0

$m =$ 1, $b =$ 6

point of intersection: (3, 9)

**Name the slope and *y*-intercept of each line graphed. Name the point of intersection.**

**3.**

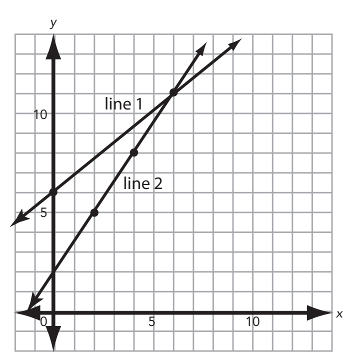



point of intersection: (4, 6)

line 1: $m = \frac{3}{2}$, $b =$ 0

line 2: $m =$ 1, $b =$ 2

**4.**




point of intersection: (6, 11)

line 1: $m = \frac{5}{6}$, $b =$ 6

line 2: $m = \frac{3}{2}$, $b =$ 2

## *Answer Key*

**Challenge!** What do you know about the ordered pair for the point of intersection of two lines in terms of their equations? What might it mean if the lines are parallel? Explain.

Challenge: (Sample) The ordered pair can be substituted into each equation and it will make a true statement. When lines are parallel, there is no ordered pair that will make both statements (equations) true.

LESSON

# 5

## Objective

Investigate slope and relate linear equations to proportional relationships.

## Common Core State Standards

- **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- **8.EE.6** Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.

Expressions and Equations

# Lines Through (0, 0) and (0, b)

Students know that corresponding angles in similar triangles are congruent. They also know that corresponding side lengths in similar triangles are proportional. Using similar triangles, students can visualize why the slope between any two points on a line is the same as the slope between any other two points on the same line. Students also can use this idea to see that a line described by $y = mx$ is the same as a line described by $y = mx + b$ for a given $m$, except for where it crosses the $y$-axis.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *How are the lines different? How are they the same?*
- **Ask:** *What do the similar triangles show us about the slope of a line?* **Say:** *They show us that the slope is the same between any two points on the line.*
- **Ask:** *Which line represents a proportional relationship?* **Say:** *The line for Caitlyn is a proportional relationship because it includes the origin.*
- **Ask:** *What kind of relationship does the line for JT express?* **Say:** *It is called a* linear relationship. *A proportion is just a specific type of linear relationship.*
- Have students compare and contrast the equations they wrote—$y = \frac{3}{5}x$ and $y = \frac{3}{5}x + 6$. Elicit that the only difference is JT's free points.

## Solve It

Reread the problem with the students. Have each student graph the lines on dot paper. Have them determine the unit rate for Caitlyn's line and the slope for JT's line. Ask students to write equations for the lines.

## More Ideas

For another way to teach about slope and relating linear equations to proportional relationships—

- Use masking tape to make a coordinate grid on the floor. Have students stand in the locations of the points in the problem. Use string to graph the lines. Make similar triangles with string to demonstrate the slopes of the lines and find equations for the lines.

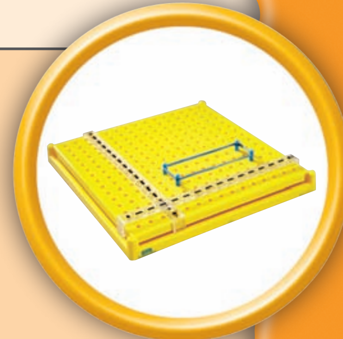
## Formative Assessment

Have students try the following problem.

*Lenny graphs the number of books he reads versus the time it takes him in weeks. He plots two points: (3, 6) and (6, 12). What is the equation of the line that passes through these two points?*

**A.** $y = 2x$ **B.** $y = 6x$ **C.** $y = 2x + 6$ **D.** $y = 6x + 12$

## Try It! 25 minutes | Groups of 3



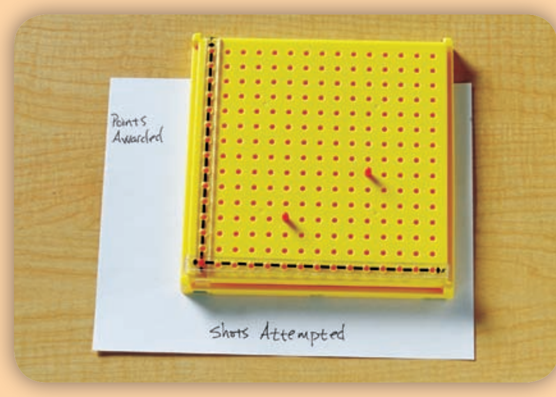
Here is a problem about slope and relating linear equations to proportional relationships.

*Caitlyn and her younger sister JT are shooting a basketball to see who can earn more points in 10 shots. One point will be awarded for each shot made, and Caitlyn has spotted JT 6 free points. Caitlyn makes 3 shots in 5 attempts and 6 shots in 10 attempts. JT makes 3 shots in 5 attempts and 6 shots in 10 attempts. Make a graph of points awarded versus shots attempted for the girls. Write the equation of the line that represents each girl's results.*

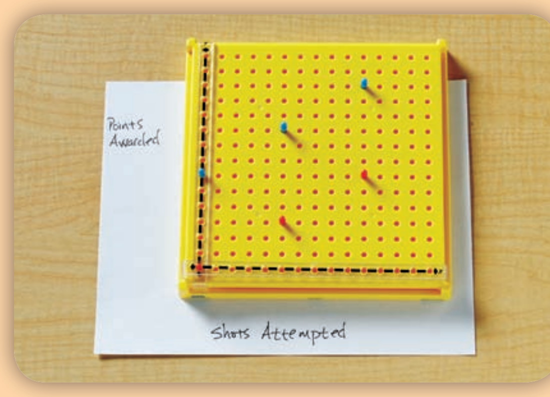
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

### Materials

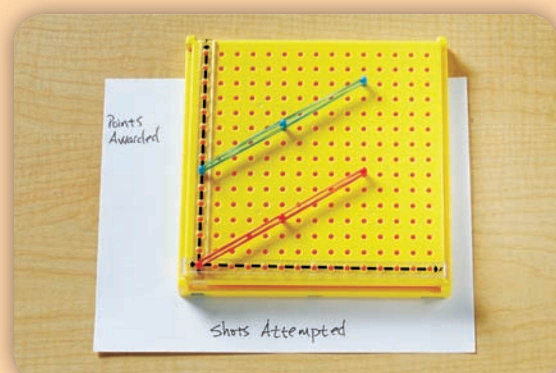
- XY Coordinate Pegboard
- Dot Paper (BLM 7; 1 per student)
- straightedge



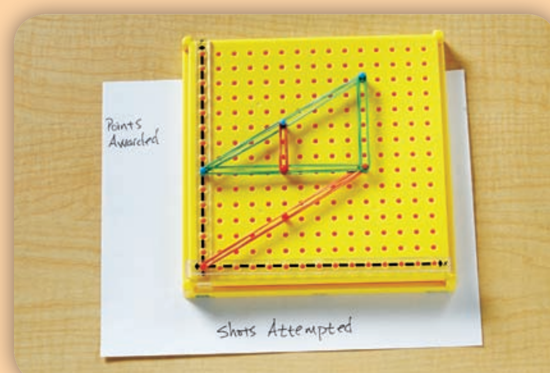

**1.** Have students prepare a pegboard for graphing in the first quadrant. Discuss what each axis will represent. **Ask:** *What points do we plot for Caitlyn?* Elicit from students that they should plot the points given in the problem—(5, 3) and (10, 6)—plus (0, 0), because Caitlyn did not have any points at the start.




**2. Ask:** *What do we plot for JT?* Elicit that the first point for JT is (0, 6). Elicit further that JT earned 3 more points in 5 shot attempts and 6 more points in 10 attempts, so the second point for JT is (5, 9) and the third point is (10, 12). Have students plot the points.




**3.** Have students stretch a rubber band around the pegs for each girl. Elicit that for each girl the pegs lie on a line. **Ask:** *Does either line represent a proportion? Which one?* **Say:** *Find the unit rate and write an equation for* y *versus* x.

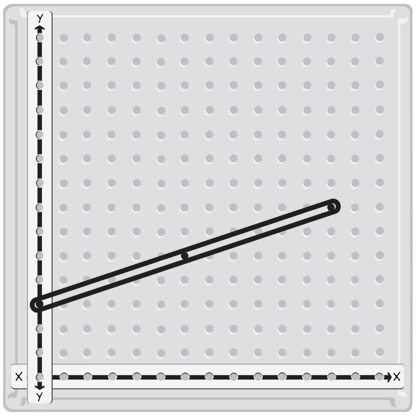



**4.** Have students add a peg at (5, 6) and make two triangles—one using (0, 6), (10, 6), and (10, 12), the other using (0, 6), (5, 6), and (5, 9). Have students see the triangles are similar, find the ratio of height to base length for each, and use the ratio to write an equation of the line for JT.

**Answer Key**

**Use an XY Coordinate Pegboard. Build the model and use it to answer the questions.** (Check students' work.)

**1.**




Louis is collecting silver dollars. In 6 months, he had 5 silver dollars in his collection. In 12 months, he had 7 silver dollars. Make a graph of dollars versus months. Assume the relationship is linear.

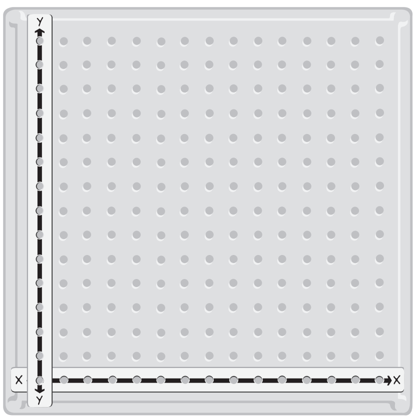
Does the line go through the origin? no

How many silver dollars did Louis start with? 3

Write the equation of the line. $y = \frac{1}{3}x + 3$

**Using an XY Coordinate Pegboard, model the problem. Draw the model and use it to answer the questions.** (Check students' models.)

**2.**




Jo paid $7 for 6 pounds of apples. She paid $14 for 12 pounds of apples. Make a graph of cost versus pounds. Assume the relationship is linear.

Does the line go through the origin? yes

Is the relationship a proportion? yes

Write the equation of the line. $y = \frac{7}{6}x$

**Use dot paper to graph the relationship. Answer the questions.**

**3.** At 8:00 a.m. on a cold Saturday, Juan decided to track the temperature on his patio. At 11:00 a.m., the temperature was 8°F. At 2:00 p.m., the temperature was 14°F. Make a graph of temperature versus elapsed time. Assume the relationship is linear.

Is the relationship a proportion? no

What was the temperature at 8:00 a.m.? 2°F

Write the equation of the line. $y = 2x + 2$

## *Answer Key*

**Challenge!** How do you know whether the equation of a line will be of the form $y = mx$ or $y = mx + b$? What is $b$?

Challenge: (Sample) If the line goes through the origin, it will be $y = mx$. If the line does not go through the origin, it will be $y = mx + b$. The constant $b$ is where the line crosses the $y$-axis or how far up it is translated from $y = mx$.

LESSON 6

## Objective

Identify linear equations that have one solution, no solution, and infinitely many solutions.

## Common Core State Standards

- **8.EE.7a** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).

Expressions and Equations

# One, No, or Infinitely Many Solutions

After achieving proficiency with solving linear equations, students can start to consider specific cases. They are familiar with linear equations with one solution but have not yet encountered equations with no solution or infinitely many solutions. By learning to analyze a solution of the form $x = a$, $a = a$, or $a = b$, where $a$ and $b$ are different numbers, students will be able to determine the number of solutions.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *How do you solve an equation? What are the steps?* Remind students that the goal is to isolate the variable. Discuss the process with students.
- **Ask:** *What is different about the equations in this problem, compared with the ones you saw when you first started learning to solve equations?*
- **Ask:** *When you solved the second equation, when did you recognize that the equation is an untrue statement? Did you recognize it before you reached 1 = 0 (or 4 = 3)?*
- **Ask:** *When you solved the third equation, when did you recognize that the value of* x *doesn't matter?*

## Solve It

Reread the problem with the students. Have each student use Algebra Tiles™ and the Algebra Tiles Equations Mat to solve all three equations. Compare the solutions. Discuss the forms of the solutions that give one solution, no solution, and infinitely many solutions.

## More Ideas

For another way to teach about solutions to equations—

- Solve the equations using Algeblocks® and the Equations Mat.

## Formative Assessment

Have students try the following problem.

*Henry solved an equation and found it had infinitely many solutions. Which equation did he solve?*

**A.** $14 = 4x + 8$ **B.** $20 = 5x$ **C.** $5(x + 1) = 5x + 5$ **D.** $4(x + 2) = 4x + 9$

## Try It! 25 minutes | Pairs

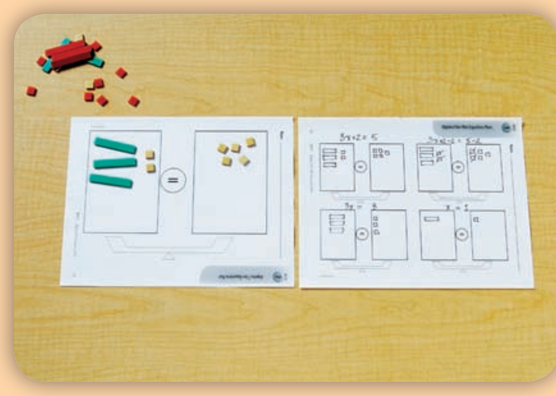
Here is a problem about linear equations with one solution, no solution, and infinitely many solutions.

*Kedrick wrote three equations to quiz his brother before a test. He wrote $3x + 2 = 5$, $3x + 4 = 3(x + 1)$, and $3(x + 2) = 3x + 6$. What is the solution to each equation?*

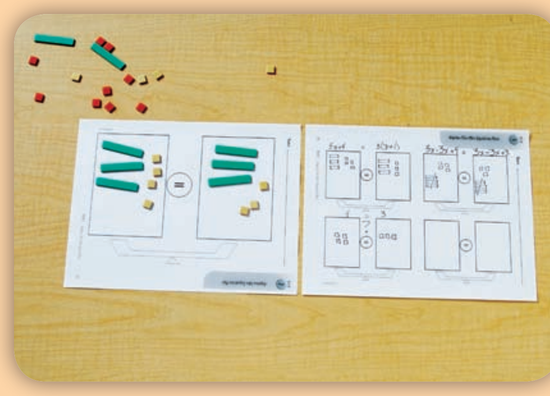
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

### Materials

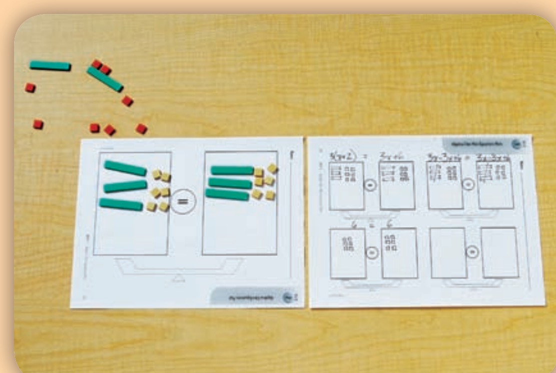
- Algebra Tiles™
- Algebra Tiles Equations Mat (BLM 2; 1 per pair)
- Algebra Tiles Mini Equations Mats (BLM 3; 3 per student)



**1.** Address the first equation. **Ask:** *How will you model the equation?* Elicit from students that they will use a green rectangle for each $x$ and a tan square for each unit. Have them model and solve the equation, recording each step on a Mini Equations Mat. **Ask:** *What is the solution? How many values of* x *satisfy the equation?* Elicit that the solution is $x = 1$. Have students record this result.



**2.** Have students model and solve the second equation, recording each step on a Mini Equations Mat. **Ask:** *What is the solution? Is anything wrong?* Elicit that the resulting equation, $1 = 0$ (or $4 = 3$), is not a true statement. Explain that this means there are no solutions. Have students record this result.



**3.** Have students model and solve the third equation, recording each step on a Mini Equations Mat. **Ask:** *What is the solution?* Elicit that the resulting equation, $0 = 0$, is always true. It doesn't matter what you substitute for $x$ in the original equation. Explain that this means there are infinitely many solutions. Have students record this result.

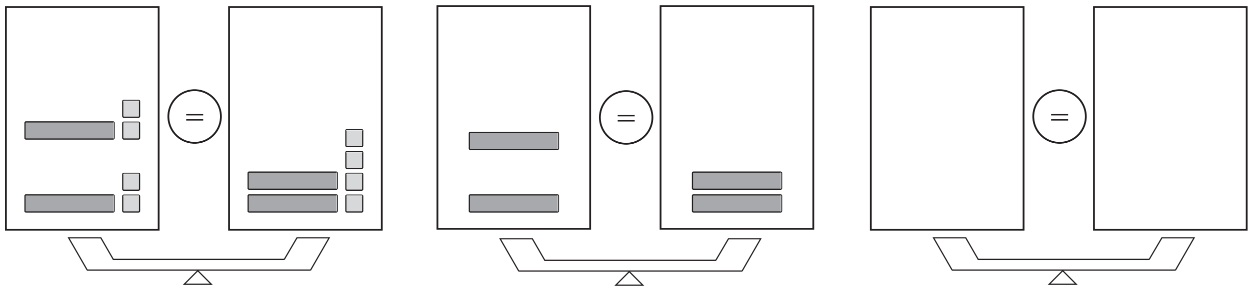
### ⚠ Look Out!

Students might confuse whether $a = a$ or $a = b$ means infinitely many solutions. For an $a = a$ case, have students go back a step in the solution. Students might feel more comfortable seeing the $x$ in the equation and recognizing the equality in that step.

**Answer Key**

**Use Algebra Tiles and the Algebra Tiles Equations Mat to model the equation shown and then solve it. Write the equation and the solution.** (Check students' work.)

**1.**

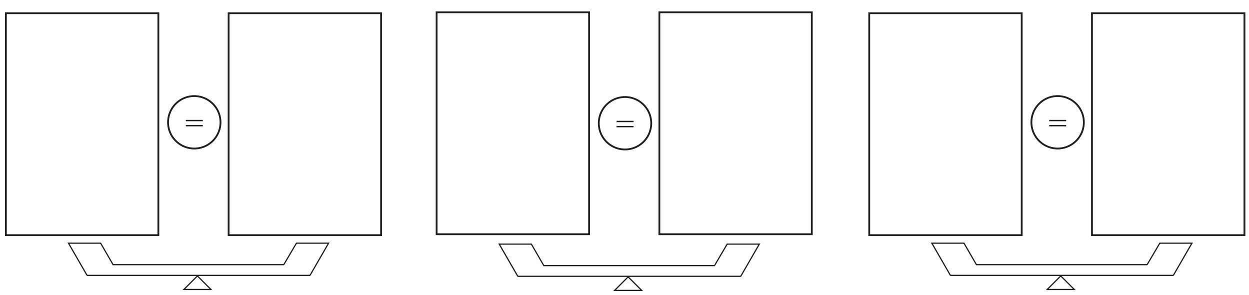


$2(x + 2) = 2x + 4$

infinitely many solutions

**Using Algebra Tiles and the Algebra Tiles Equations Mat, model the equation. Sketch the model and solution. Write the solution.**

**2.** $2x + 4 = 12$ $x = 4$ (Check students' models.)



**Solve the equation. Show your steps.**

**3.** $3(x + 4) = 3x + 12$

$0 = 0$, infinitely many solutions

**4.** $33 = 8x - 7$

$5 = x$

**5.** $5(x - 1) = 5x - 4$

$-5 = -4$ or $0 = 1$, no solutions

**6.** $7(x - 6) = 7x + 3$

$-42 = 3$ or $0 = 45$, no solutions

# Answer Key

**Challenge!** Give examples of equations and their solutions. Include equations with 1 solution, no solutions, and infinitely many solutions.

Challenge: Check solutions. Any solution of the form $x = a$ indicates one solution; $a = a$ indicates infinitely many solutions; $a = b$ indicates no solution.

LESSON 7

## Objective

Solve multi-step equations.

## Common Core State Standards

- **8.EE.7b** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Expressions and Equations

# Solving Multi-Step Equations

Once students have the foundations of solving equations in place, they will be able to begin to solve multi-step equations. They will combine the skills they have applied to solving one-step equations and their understanding of the order of operations to solve more complicated equations. Throughout algebra, students will solve multi-step equations when finding zeroes of functions.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *What is known? What is unknown? How is the perimeter of a triangle calculated?*
- **Ask:** *What equation did you write/model? How do you combine like terms using the Algebra Tiles™?*
- **Ask:** *Once you find the value of* x*, how do you find the lengths of the other sides? How can we be sure these sides make up the sides of a triangle?* (Triangle Inequality)

## Solve It

Read the problem with the students. Have students solve the multi-step equation. Ask students to show their work by creating a model with Algebra Tiles. Then have students sketch the model and record their work.

## More Ideas

For another way to teach about solving multi-step equations—

- Have students use Algeblocks® and the Algeblocks Sentences Mat to model each equation. Students can combine like terms and then reverse the order of operations to isolate the variable and find the answer.

## Formative Assessment

Have students try the following problem.

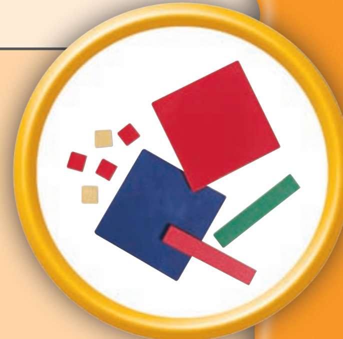
*A computer company charges a $75 diagnostic fee plus $32 per hour to fix any problem. If Marta pays $203 to have her computer repaired, how many hours did the company work on it?*

**A.** 2.7

**B.** 3

**C.** 3.1

**D.** 4



## Try It! **20 minutes | Groups of 3**

Here is a problem about solving multi-step equations.

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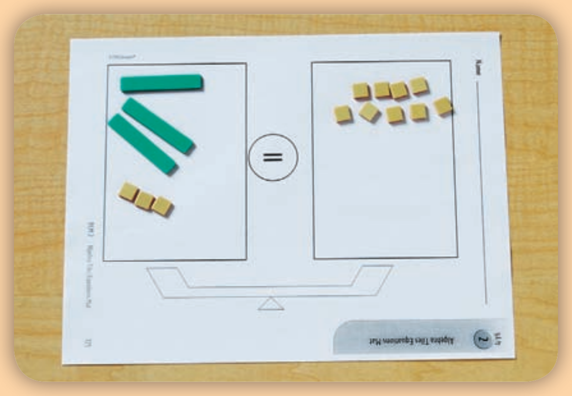
*The second side of a triangle is 2 times the length of the first. The third side is 3 units long. The perimeter of the triangle is 9. What is the length of the first side of the triangle?*

---

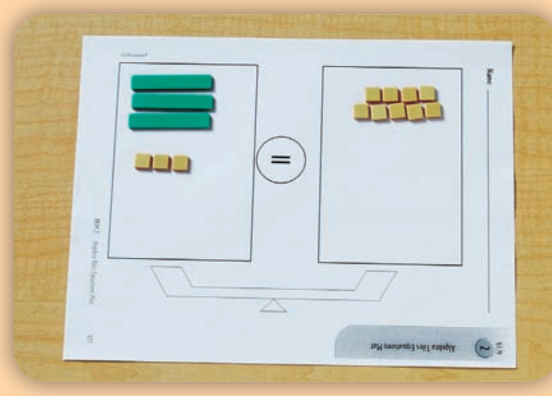
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

### Materials

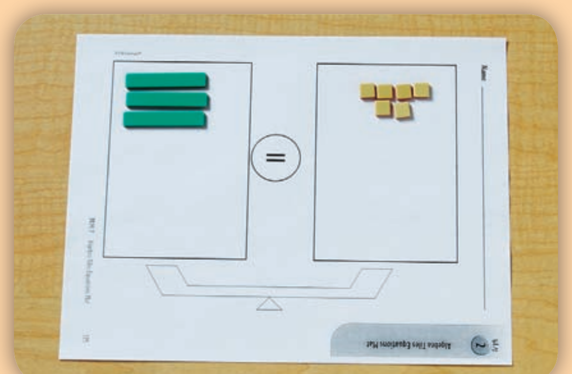
- Algebra Tiles™
- Algebra Tiles Equations Mat (BLM 2; 1 per group)
- Algebra Tiles Mini Equations Mats (BLM 3; 1 per student)



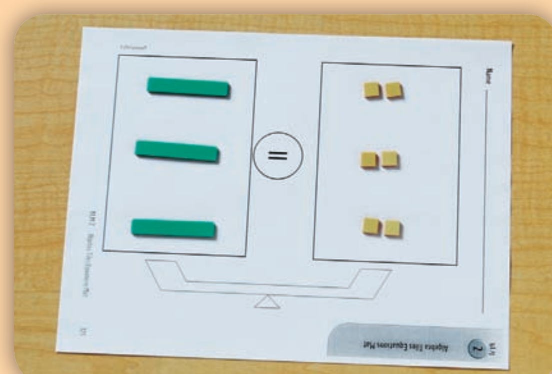
**1. Ask:** *What will the* x *Algebra Tile represent? What represents the length of the second side? The third side?* Have students use Algebra Tiles to model the triangle's perimeter and draw it on their Algebra Tiles Mini Equations Mats BLM 3. Students should write the equation. [$x + 2x + 3 = 9$]



**2. Ask:** *Can any of the groups be combined?* Have students combine like terms by joining the *x* Algebra Tiles into one group. Students should draw this arrangement on their mini mats and write the new equation, showing like terms combined. [$3x + 3 = 9$]



**3. Ask:** *How can we isolate the variable?* Have students remove 3 from each side. Then have students draw the Algebra Tiles on their mini mats. Students will write the changes to the equation, showing the steps and the results. [$3x = 6$]



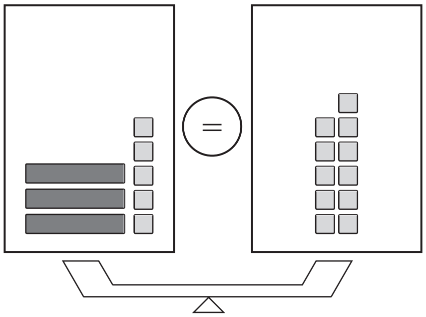
**4. Say:** *We need to find the value of one* x. Have students create 3 equal groups on each side, and draw the groups on their mini mats. **Say:** *Circle your groups. They will show that each side was divided by the same number and the resulting equation.* **Ask:** *What is the length of the first side?* [$x = 2$]

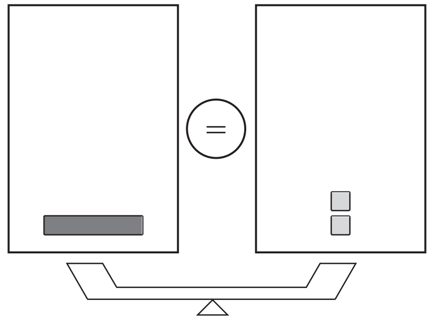
**Answer Key**

**Use Algebra Tiles and the Algebra Tiles Equations Mat to model the equation shown and then solve it. Write the equation and the solution.**

(Check students' work.)

**1.**





*x* + 5 = 11, *x* = 2

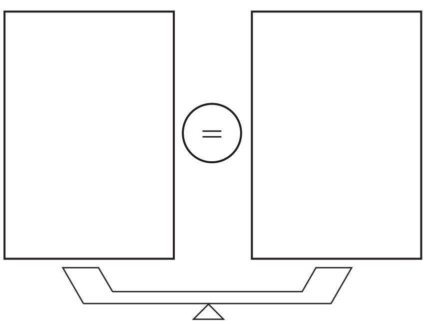
*x* = 2

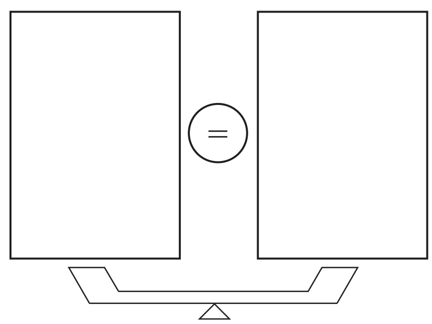
**Using Algebra Tiles and the Algebra Tiles Equations Mat, model the equation. Sketch the model. Solve the equation and write the solution.**

**2.** $8 = 2x + 2$ $x = 3$

**3.** $5x + 4 = 9$ $x = 1$

(Check students' models.)





**Find each solution.**

**4.** $6x + 12 = 18$

$x = 1$

**5.** $8x + 4 = 20$

$x = 2$

**6.** $16 = 10x + 6$

$x = 1$

**7.** $21 = 2x - 7$

$x = 14$

**8.** $3x + 9 = 51$

$x = 14$

**9.** $45 = 5x - 5$

$x = 10$

# Answer Key

**Challenge!** Explain the difference between solving equations like $2x = 14$ and $x + 6 = 14$ and solving $2x + 6 = 14$.

Challenge: (Sample) In the first 2 equations you're simply undoing one operation. In $2x + 6 = 14$, you have to undo two operations. So it's as if you're following the order of operations in reverse.

LESSON 8

# Objective

Solve equations with variables on both sides.

# Common Core State Standards

- **8.EE.7b** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.



# Solving Equations with Variables on Both Sides

Another form of multi-step equations involves variables on both sides of the equation. Students will need to be able to manipulate the variables as well as constants in an equation. By understanding how to isolate the variables on one side of the equation, students will be able to solve real-world problems in algebra that involve variables on both sides of the equation.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *What is known? What is unknown? What is the objective when solving an equation using Algebra Tiles™?*
- **Ask:** *How will you get all the variable terms to one side of the equation? Does it matter which side of the equation we decide to collect the terms with the variables? Explain.*
- **Ask:** *How do you make sure that the equation remains balanced?*

## Solve It

Read the problem with students. Have students solve the equation by creating a model with Algebra Tiles. Then have students sketch the model and record their work.

## More Ideas

For another way to teach about solving equations with variables on both sides—

- Have students use Algeblocks® and the Algeblocks Sentences Mat to model each equation. Have students isolate the variable by gathering all of the variable terms on one side of the equal sign and all terms that do not have the variable on the other side of the equal sign. Then, students should finish solving the equation.

## Formative Assessment

Have students try the following problem.

*Four more than three times a number is equal to two less than five times the number. What is the number?*

**A.** 0.25

**B.** 0.75

**C.** 3

**D.** 4

## Try It!

**20 minutes | Groups of 3**

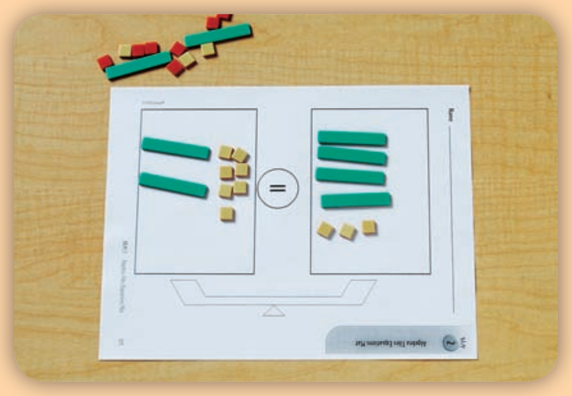
Here is a problem about solving equations with variables on both sides.

*Susan charges a $7 fee plus $2 per hour for tutoring. Joan charges a $3 fee plus $4 per hour. When will their tutoring fees be the same?*

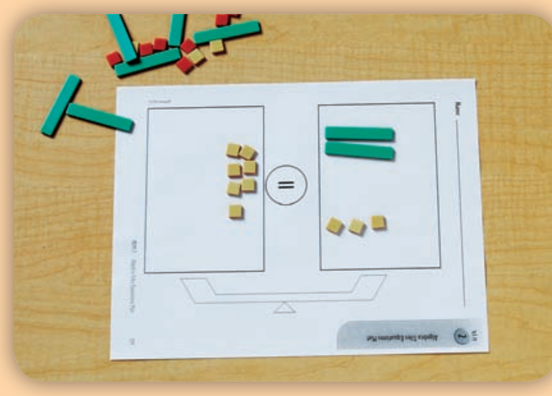
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

### Materials

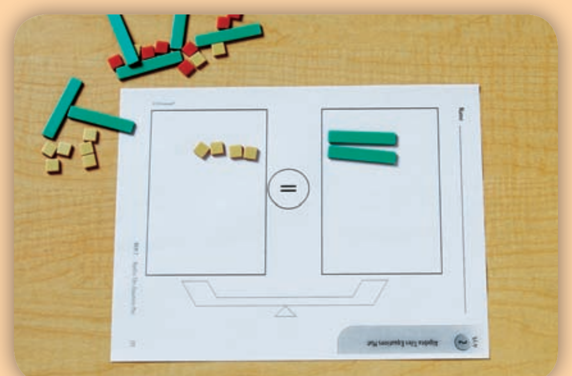
- Algebra Tiles™
- Algebra Tiles Equations Mat (BLM 2; 1 per group)
- Algebra Tiles Mini Equations Mats (BLM 3; 1 per student)



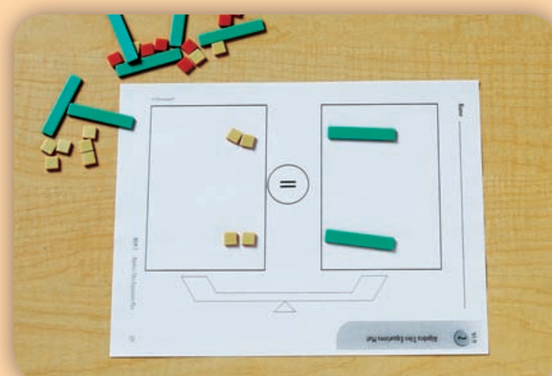
1. **Ask:** *What are we being asked to find?* Have students write what *x* represents. **Ask:** *What represents Susan's charge? Joan's charge?* Have students use tiles to model the charge for each. Have students draw the model on their Algebra Tiles Mini Equations Mats BLM 3 and write the equation. [$2x + 7 = 4x + 3$]



2. **Ask:** *Which side do we want to collect the variable on?* Have students remove 2 *x* Algebra Tiles from each side. Then, they cross out the tiles on their mini mats and write the new equation [$7 = 2x + 3$], showing the steps and results.



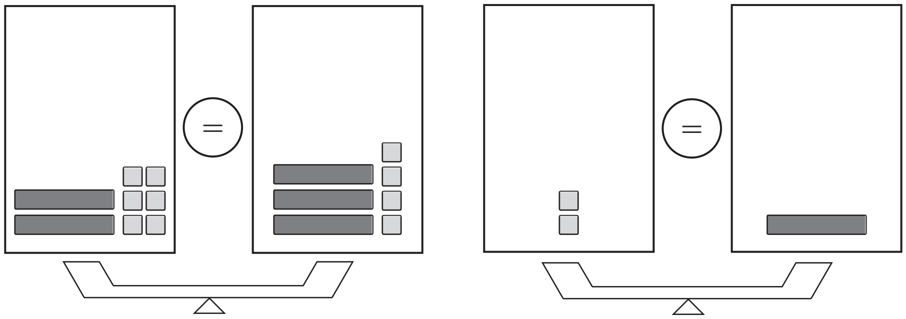
3. **Ask:** *How can we isolate the variable?* Have students remove 3 unit tiles from each side. Then, students should draw the tiles on their mini mats and write the steps they took and the resulting equation. [$4 = 2x$]



4. **Say:** *We need to find the value of one* x. Have students create 2 equal groups on each side, and draw the groups on their mini mats. **Say:** *Circle your groups.* Students will show the division and the resulting equation. [$4 \div 2 = x$] **Ask:** *When will their fees be the same?*

**Use Algebra Tiles and the Algebra Tiles Equations Mat to model the equation shown and then solve it. Write the equation and the solution.** (Check students' work.)

**1.**

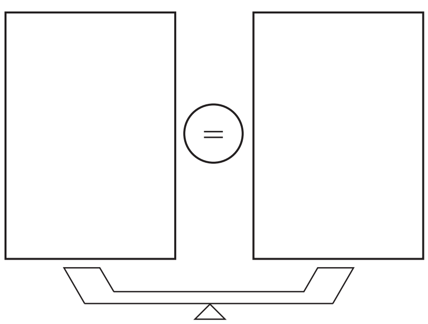


$2x + 6 = 3x + 4$

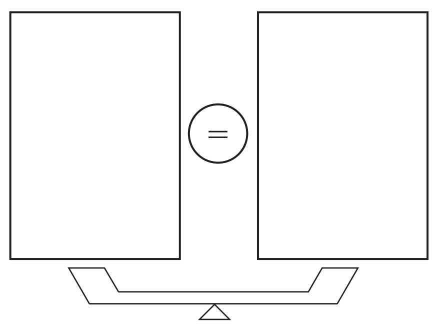
$2 = x$

**Using Algebra Tiles and the Algebra Tiles Equations Mat, model the equation. Sketch the model. Solve the equation and write the solution.** (Check students' models.)

**2.** $4x + 1 = x + 10$ $x = 3$



**3.** $x + 5 = 3x + 1$ $2 = x$



**Find each solution.**

**4.** $x + 10 = 2x$

$10 = x$

**5.** $7x - 4 = 6x + 12$

$x = 16$

**6.** $2x + 12 = 4x + 4$

$4 = x$

**7.** $6x + 9 = 5x + 15$

$x = 6$

**8.** $10x = 4x - 18$

$x = -3$

**9.** $9x + 18 = 11x$

$9 = x$

## *Answer Key*

**Challenge!** Explain how solving $2x + 16 = 14$ is different from solving $2x + 16 = x + 14$. How are they the same?

Challenge: (Sample) In $2x + 16 = 14$ there is only a variable on one side of the equal sign. But in $2x + 16 = x + 14$ there are variables on both sides. So you have to decide which side of the equal sign you want to isolate the variable on when solving $2x + 16 = x + 14$.

LESSON 9

## Objective

Solve systems of two linear equations in two variables.

## Common Core State Standards

- **8.EE.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
- **8.EE.8c** Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Expressions and Equations

# Solving Systems of Equations

This lesson introduces students to solving systems of equations in an informal manner so that they will have a basic understanding of what is usually thought to be a rather complex skill. In this activity, students solve the problem using a model and then transfer the information to paper in order to solve the problem algebraically.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- Students have expressed the number of girls in terms of boys (i.e., $2(7 - B) + 3B = 18$). **Ask:** *Can you solve the problem by expressing the number of boys in terms of girls?*
- **Ask:** *Why is it important to reread the problem after you've found the values of your unknown variables?*

## Solve It

Reread the problem with students. Discuss with them how to create a system of equations. Encourage them to be methodical as they analyze the information found in the story problem. Have students use substitution to solve the equation. Then have them substitute their results into the original problem in order to answer the question it poses.

## More Ideas

For another way to teach about solving systems of equations—

- Have students use the XY Coordinate Pegboard to solve this and similar problems. Students should create a table for both equations and then graph the results. The point of intersection of the two lines (3, 4) will be the solution to the problem.

$G + B = 7$

| $G$ | $B$ |
|---|---|
| 0 | 7 |
| 1 | 6 |
| 2 | 5 |
| 3 | 4 |
| 4 | 3 |
| 5 | 2 |
| 6 | 1 |
| 7 | 0 |

$2G + 3B = 18$

| $G$ | $B$ |
|---|---|
| 0 | 6 |
| 3 | 4 |
| 6 | 2 |
| 9 | 0 |

## Formative Assessment

Have students try the following problem.

*Steve has twice as many video games as David. Together, they have 15 video games. Which of the following systems of equations best represents this information?*

**A.** $S = 2D$, $S + D = 15$

**B.** $2S = D$, $S + D = 15$

**C.** $2S = D$, $S - D = 15$

**D.** $S = 2D$, $D = 15$

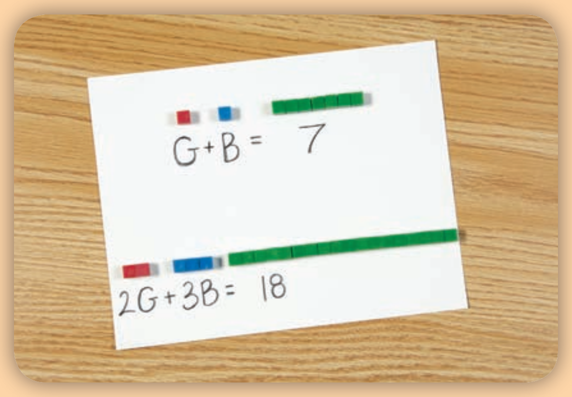
## Try It! 20 minutes | Pairs

Here is a problem about solving a system of equations.

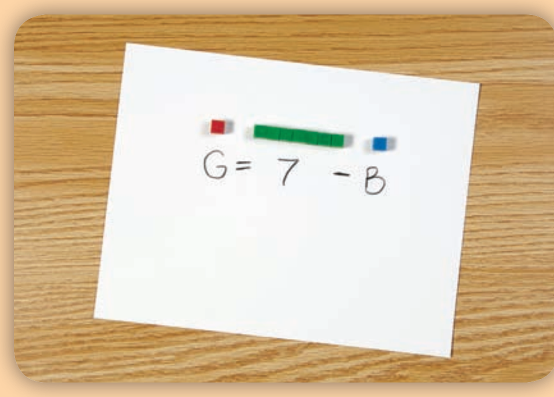
*There are 18 students in the Math Club. There are twice as many girls and three times as many boys as there are in the Jazz Club. The Jazz Club has 7 students in it. How many boys and girls are in the Math Club?*

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

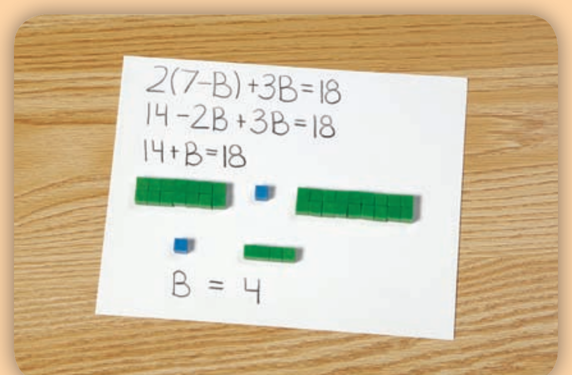
**Materials**
- Centimeter Cubes (5 red; 5 blue; 25 of any other color)
- paper (4 sheets per pair)
- pencils (1 per pair)



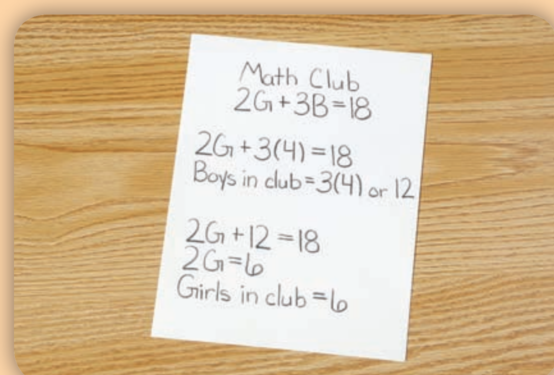

**1.** Have students use the red cubes to represent "girls," blue cubes to represent "boys," and cubes of another color to represent the number of students. Have students write their two equations and model them on a piece of paper.




**2.** Have students express the first equation as the number of girls in terms of boys. Have them write the new equation on their paper and model it with Centimeter Cubes.




**3.** Have students substitute the new equation into the second equation and use the Distributive Property to solve for boys.



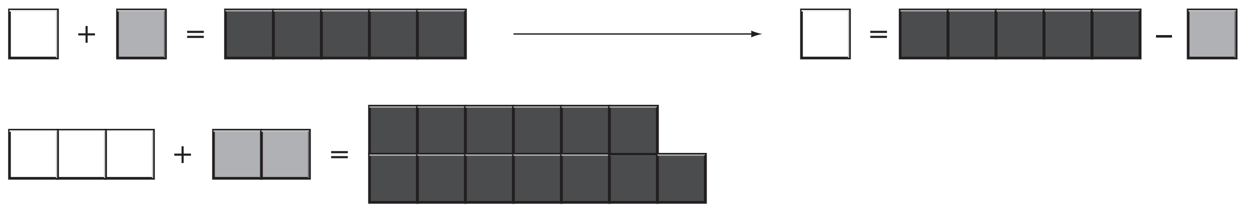

**4.** Use the value of *B* to solve for the number of boys and girls in the Math Club.

**Answer Key**

**Use Centimeter Cubes to model the system of equations shown. Write the system. Then use substitution to solve the system. Write all the steps to finding the solution.**

(Check students' work.)

**1.**



Equations in the system: $x + y = 5$

$3x + 2y = 13$

Solution: $x = 3, y = 2$

**Using Centimeter Cubes, model the system of equations and use substitution to find the solution. Sketch the models.**

**2.** $x + y = 5$ and $2x + y = 6$

(Check students' models.)

Solution: $x = 1, y = 4$

**Use substitution to solve each system of equations.**

**3.** $x + y = 10$

$x - 2y = 1$

$x = 7, y = 3$

**4.** $x + y = 7$

$x + 2y = 12$

$x = 2, y = 5$

**5.** $x + y = 4$

$4x - y = 1$

$x = 1, y = 3$

**6.** $x + y = 2$

$x + 5y = 2$

$x = 2, y = 0$

## Answer Key

**Challenge!** How many times do you use substitution to solve a system of linear equations? Explain when you use it.

Challenge: (Sample) Twice; In the beginning, you solve one of the equations for a variable, such as *x*. That expression is substituted into the other equation and the equation is solved. After you have that solution, you substitute that value into either equation to solve for the other variable.