Operations and Algebraic Thinking

Students in third grade develop an understanding of the meaning of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models. *Multiplication* is finding an unknown product, and *division* is finding an unknown factor. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size.

Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. Most third graders will be able to memorize all of the products of two one-digit numbers.

The Grade 3 Common Core State Standards for Operations and Algebraic Thinking specify that students should–

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

The following hands-on activities will help students model problem-solving situations involving multiplication and division. The activities will help students gain understanding of the relationship between multiplication and division and why their related algorithms make sense.

Mathematically proficient third graders will look to discover patterns and structure as they develop and apply mathematical thinking. They may use properties of operations as strategies to multiply and divide. Similarly, they notice repetitive actions in computation and look for shortcut methods. For example, students may use the distributive property as a strategy for using products they know to find products they don't know.

Operations and Algebraic Thinking

Contents

LESSON 1

Operations and Algebraic Thinking

Multiplying with Arrays

Arrays are arrangements of equal groups that can be used to show repeated addition and multiplication. Arrays can be made from Color Tiles or units of Base Ten Blocks but are also found in everyday objects, such as the arrangement of cans in a six-pack or eggs in a carton. They can be especially helpful as students learn multiplication facts.

Objective

Relate arrays and repeated addition to multiplication.

Common Core State Standards

- **3.OA.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How did the array help you solve the problem?* Discuss with students how they used the array and repeated addition to solve the problem.
- **Ask:** *If you made 4 rows of 8 and added 8 + 8 + 8 + 8, would you get the same answer?* Have students model 4 × 8 and compare the 2 arrays side by side.
- **Ask:** *If you wanted to find 8 × 3 instead of 8 × 4, how would you change the array you made?*

Solve It

With students, reread the problem. Have students draw a picture of the array of chairs. Then have them write the numerical representation as both repeated addition and multiplication.

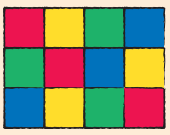
More Ideas

For other ways to teach about relating arrays and repeated addition to multiplication—

- Have students work in pairs. One partner will create an array with Color Tiles. The other partner must then create an equation using repeated addition to represent the array. Both students should count the tiles to check the equation.
- Have students use Color Tiles to create an array using prime numbers. For example, ask students to make an array using 29 tiles. Challenge them to explain why this array can only be made as a straight line rather than a square or rectangle.
- Have students use Color Tiles to explore arrays of perfect squares. Ask students to explain why they can make a square array for 16 but not for 18.

Formative Assessment

Have students try the following problem.



Which number sentence describes the array?

A. 3 × 4 **C.** 4 + 4 + 4 + 4

B. 3 + 3 + 3 **D.** 4 + 3

Try It! 20 minutes | Pairs

Here is a problem about relating arrays and repeated addition to multiplication.

Mr. Booth asked a police officer to speak to his class and another third-grade class about summer safety. To make room for the other students, Mr. Booth arranged the chairs in his classroom into 8 rows and put 4 chairs in each row. How many chairs were there in all?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Color Tiles, paper, and pencils to students.

Materials

- Color Tiles (40 per pair)
- paper (1 sheet per student)
- pencils (1 per student)

1. Tell students that one way to solve the problem is by using an array. **Say:** *Use the tiles to show 8 rows of 4 tiles.* Emphasize rows and columns in the array.

2. Ask: *How could we find the number of tiles in the array?* Point out that students can use the array they made to add 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4.

3. Have students calculate the answer to the problem by adding. **Ask:** *What other ways can we find the answer?* Demonstrate that 8 × 4 is the same as 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4.

⚠ Look Out!

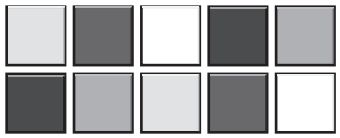
Make sure students are aware of the difference between a row and a column. Additionally, students may not understand that they need to have the same number of tiles in each row. Reinforce that in order to use repeated addition for multiplication, they need the same number in each group. To increase students' understanding of the number sense behind multiplication, encourage them to use skip-counting for repeated addition before they start to memorize multiplication facts. This will give them a deeper understanding of what multiplication is, rather than merely encouraging rote memorization.

Answer Key

Use Color Tiles to build each array.
Write the multiplication sentence for each array.

(Check students' work.)

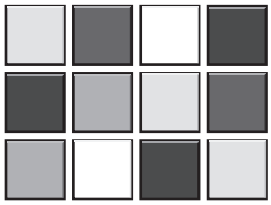
1.



2 rows of 5 tiles

2 × 5 = 10

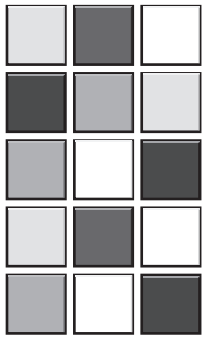
2.



3 rows of 4 tiles

3 × 4 = 12

3.



5 rows of 3 tiles

5 × 3 = 15

Build each array using Color Tiles. Then sketch the model below.
Write each multiplication sentence.

(Check students' models.)

4. 2 rows of 9 tiles

2 × 9 = 18

5. 7 rows of 4 tiles

7 × 4 = 28

6. 5 rows of 6 tiles

5 × 6 = 30

Find the answer to each multiplication problem.

7. 8 × 5 = 40

8. 2 × 7 = 14

9. 4 × 4 = 16

10. 6 × 3 = 18

11. 3 × 5 = 15

12. 7 × 6 = 42

Answer Key

Challenge! Which two problems from the previous page can be used to demonstrate the Commutative Property of Multiplication? Model the arrays that show both multiplication expressions.

Challenge: (Sample) Problems 3 and 11; Both have factors of 3 and 5. In Problem 3, there are 5 rows of 3 tiles for a total of 15 tiles. The multiplication sentence is $5 \times 3 = 15$. In Problem 11, the array is 3 rows of 5 tiles for a total of 15 tiles. The multiplication sentence is $3 \times 5 = 15$. These arrays show that the order of the factors does not change the product.

LESSON 2

Objective

Multiply a number by 5 using grouping and skip-counting.

Common Core State Standards

- **3.OA.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Operations and Algebraic Thinking

Multiplying by 5

Students may already be familiar with counting by 5s from counting their fingers to make 10 and 20. Also, students are familiar with grouping manipulatives to multiply by single-digit numbers. Multiplying by 5 and skip-counting teach students that patterns can help them remember number facts.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Did you get the same answer for 4 × 5 when you used the Two-Color Counters to multiply as you did when you skip-counted on the Hundred Chart?*
- **Ask:** *Was it easier to count all the counters one by one to solve 4 × 5, or to skip-count by 5s?*
- **Ask:** *What do you notice about your answers to all of the multiplication facts of 5? Do you think the answer would still end in 5 or 0 if we multiplied a larger number, like 5 × 17? What about 2,378 × 5? Can we use this pattern to help us solve any multiplication problem that uses 5?*

Solve It

With students, reread the problem. Have students write a short paragraph summarizing how they know 4 groups of 5 paintbrushes will be a number ending in 5 or 0. Then have them explain how they can solve 4 × 5 as proof. Encourage them to describe using a counter array or hundred chart as support.

More Ideas

For other ways to teach about multiplying by 5—

- Have students work in groups to create number lines from 1 to 50 on paper. Have students use Centimeter Cubes to show skip-counting by 5s on the number line and explain how it relates to multiplication facts involving 5.
- Challenge groups of students to model multiples of 5 through 5 × 20 using Color Tiles to build arrays and skip-count. Have students record the algorithms for these facts and discuss the patterns involved.

Formative Assessment

Have students try the following problem.

In gym class, there are 5 teams with 5 players on each team. How many students are in the gym?

A. 15 **B.** 20 **C.** 25 **D.** 30

Try It! 20 minutes | Pairs

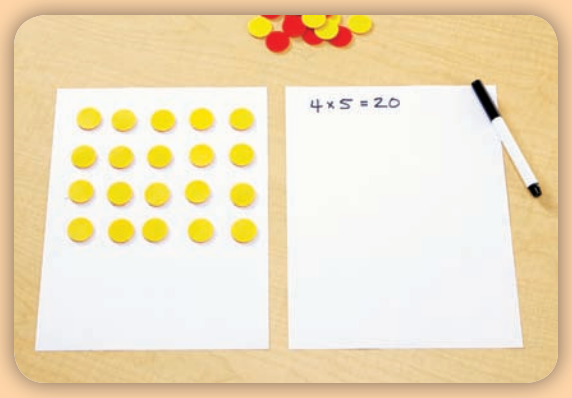
Here is a problem about multiplying by 5.

The art teacher divided the class into 4 equal-sized groups. Each group needs 5 paintbrushes. How many paintbrushes are needed in all?

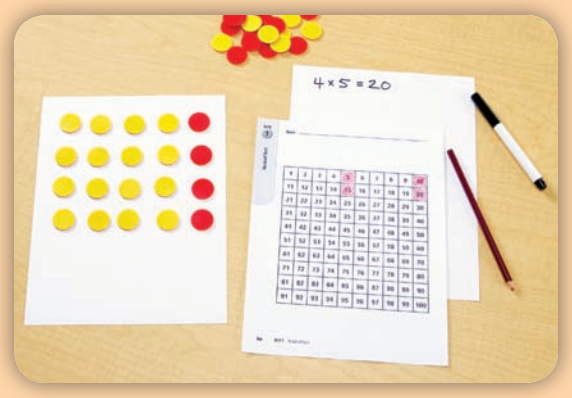
Introduce the problem. Then have students do the activity to solve the problem. Distribute Two-Color Counters, Hundred Chart (BLM 1), pencils, and paper to each pair of students.

Materials

- Two-Color Counters (50 per pair)
- Hundred Chart (BLM 1; 1 per pair)
- pencils (1 per pair)
- paper (2 sheets per pair)



1. Direct students to divide counters into 4 rows of 5 counters, all yellow-side up. Have students represent the model by writing the multiplication sentence 4 × 5. Have students solve the problem 4 × 5 by counting each individual counter (1 to 20).



2. Have students repeat counting and flip over every fifth counter to red as they count. Then have students transfer the value the red counters represent to their Hundred Charts by circling or shading appropriate values (5, 10, 15, 20). Discuss the pattern created on the Hundred Chart and its link to the array of counters. **Ask:** *How many rows are in the counter array? How many numbers did we shade on our Hundred Chart? How many counters are in each row? How much does the value of each shaded number on the chart increase each time?*

3. Have students continue skip-counting on their Hundred Charts to complete multiples of 5 through 50. Discuss the pattern of numbers when skip-counting or multiplying by 5s (the place value of ones is 5 or 0). Challenge students to use their completed charts to create multiplication sentences for facts up to 5 × 10.

⚠ Look Out!

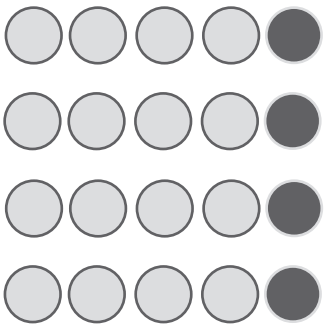
Watch out for students who can complete multiplication facts involving 5, but cannot represent multiples of 5 with an array or a hundred chart. This is an indication that rote memory has been used to solve the facts, but that the solid number sense driving the algorithms is not yet understood. Provide guided assistance to make sure students can model multiplication facts as well as solve their algorithms.

Answer Key

Use Two-Color Counters to build the multiples of 5 shown. Write the multiplication fact for each multiple of 5 modeled by the darker counters.

(Check students' work.)

1.



1 × 5 = 5

2 × 5 = 10

3 × 5 = 15

4 × 5 = 20

Build each multiplication fact using Two-Color Counters. Then sketch the model and use a Hundred Chart to find the next two multiples of 5.

(Check students' models.)

2. 4 × 5 = 20

5 × 5 = 25 6 × 5 = 30

3. 8 × 5 = 40

9 × 5 = 45 10 × 5 = 50

Write the next four multiples of 5.

4. 2 × 5 = 10

3 × 5 = 15

4 × 5 = 20

5 × 5 = 25

6 × 5 = 30

5. 5 × 5 = 25

6 × 5 = 30

7 × 5 = 35

8 × 5 = 40

9 × 5 = 45

6. 7 × 5 =35

8 × 5 = 40

9 × 5 = 45

10 × 5 = 50

11 × 5 = 55

Answer Key

Challenge! When you use Two-Color Counters and a Hundred Chart to find all the multiples of 5 less than or equal to 100, how many numbers do you color? Describe the pattern of the colored numbers.

Challenge: (Sample) 20; Every fifth number is colored red. The multiples of 5 to 100 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, and 100.

LESSON 3

Objective

Explore the meaning of division.

Common Core State Standards

- **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Exploring Division

Exploring ideas visually and kinesthetically helps students learn new concepts. When students reach grade 3, the emphasis on operations switches from addition and subtraction to multiplication and division. Using concrete models to solve division problems allows students to see the meaning of the different parts of the division problem and how the numbers are tied together.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How many groups of Two-Color Counters did you get when you divided 24 into equal groups of 4? How many groups were there when you divided 24 counters into equal groups of 8?*
- **Ask:** *What do you notice about the division sentences you wrote when you divided 24 into equal groups of 2 and equal groups of 12? How can you use what you know about multiplication to help you solve a division problem?*
- **Ask:** *Will you always have equal parts when you divide something? Why or why not?*

Solve It

With students, reread the problem. Have students write about how they can use what they know about arrays and multiplication to solve division problems.

More Ideas

For other ways to teach about exploring division—

- Sort Centimeter Cubes and paper cups into evenly divisible groups for a variety of division problems. Present a problem to students, such as *21 divided into equal groups of 3.* Then have students model the problem by placing the correct number of cubes into each cup.
- Have students work in pairs using Color Tiles. One student will make an array using the tiles, and the other student must come up with a multiplication sentence and a division sentence that matches the array. Students take turns creating arrays and multiplication and division sentences.

Formative Assessment

Have students try the following problem.

Which grouping of tally marks shows 24 ÷ 3?

A. 

B. 

C. 

D. 

Try It! 30 minutes | Pairs

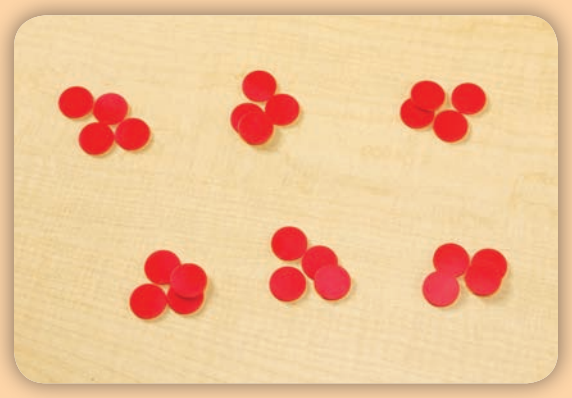
Here is a problem about exploring division.

There are 24 students in Mrs. Lopez's class. Mrs. Lopez divided the class into groups of 4 students. How many groups are there?

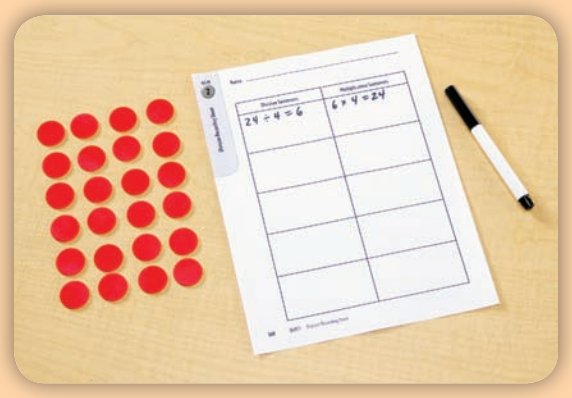
Introduce the problem. Then have students do the activity to solve the problem. Pass out Two-Color Counters and a Division Recording Sheet (BLM 2) to students.

Materials

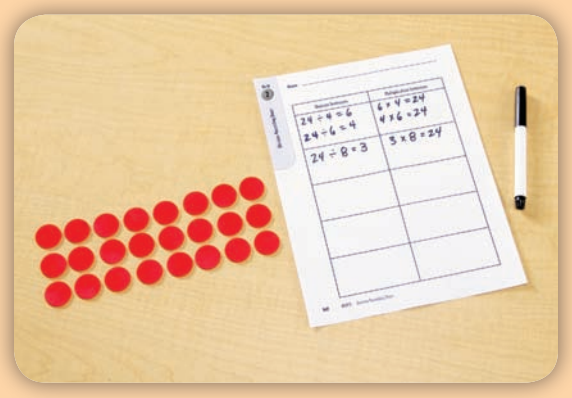
- Two-Color Counters (24 per pair)
- Division Recording Sheet (BLM 2; 1 per student)
- paper (1 sheet per student)
- pencils (1 per student)



1. Say: *We are going to divide our counters into equal groups of 4.* Tell students that this is one way to show 24 divided into equal groups of 4. **Ask:** *What division sentence are we modeling?*



2. Have students use their groups to construct an array to show the product of 6 and 4. **Ask:** *What multiplication sentence is displayed?* Have students fill out the Division Recording Sheet, using counters to assist them.



3. Have students use arrays of counters to model the other ways of dividing 24 into equal groups. They should write a division and multiplication sentence for the models they built on the recording sheet.

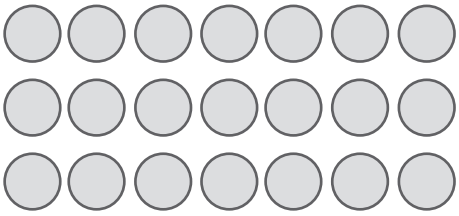
⚠ Look Out!

If students have difficulty using arrays to perform division, you may wish to show them a multiplication array. Point out that they need 4 columns, and they have 24 counters to use up. Have students put 1 counter in each column, adding rows until the counters have all been used. Also, watch for students who can divide using paper and pencil but cannot display the operation using manipulatives. This may indicate that the student lacks number sense and is relying on the memorization of facts.

Answer Key

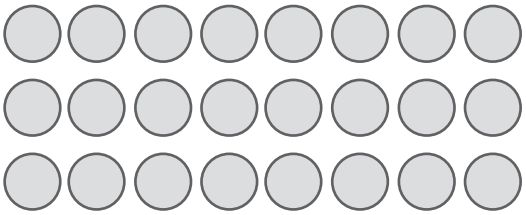
Use Two-Color Counters to build each array. Rearrange the counters into groups of the size shown. Complete each division sentence.

(Check students' work.)

1. 

put into groups of 

21 ÷ 3 = 7

2. 

put into groups of 

24 ÷ 4 = 6

Build each array using Two-Color Counters. Group the counters to be able to complete each division sentence. (Check students' work.)

3. 45 into 9 groups

45 ÷ 9 = 5

4. 32 into 4 groups

32 ÷ 4 = 8

5. 30 into 6 groups

30 ÷ 6 = 5

Write each division sentence. Write a related multiplication sentence.

6. 15 into 3 groups

15 ÷ 3 = 5

5 × 3 = 15

7. 28 into 7 groups

28 ÷ 7 = 4

4 × 7 = 28

8. 20 into 2 groups

20 ÷ 2 = 10

2 × 10 = 20

9. 35 into 5 groups

35 ÷ 5 = 7

5 × 7 = 35

10. 48 into 8 groups

48 ÷ 8 = 6

6 × 8 = 48

11. 81 into 9 groups

81 ÷ 9 = 9

9 × 9 = 81

Answer Key

Challenge! Problem 2 shows three rows of 8 Two-Color Counters for a total of 24 counters. Write a fact family for the model shown. Write a fact family for the model you create from the 24 counters. Explain how the number 24 can have two different fact families.

Challenge: (Sample) The fact family 3, 8, and 24 have the following: 3 × 8 = 24; 8 × 3 = 24; 24 ÷ 8 = 3; and 24 ÷ 3 = 8. The fact family 4, 6, and 24 have the following: 4 × 6 = 24; 6 × 4 = 24; 24 ÷ 4 = 6; and 24 ÷ 6 = 4. The number 24 has more than one factor pair.

LESSON 4

Objective

Explore the meaning of division.

Common Core State Standards

- **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Meaning of Division

Students explore division as sharing and repeated subtraction, and they should be learning to recognize the relationship between division and multiplication. Both concepts contribute to developing students' number sense, which enables them to understand when and how to use each of the two operations.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is the inverse of repeated subtraction?*
- **Ask:** *Why is distributing among groups called* sharing?
- **Ask:** *How does knowing that multiplication is the inverse of division help you when you are dividing?*

Solve It

Reread the problem with students. Have students compare the models in Steps 1 and 2 and write an equation for each situation.

More Ideas

For other ways to teach about the meaning of division—

- Have students work in groups of 3 or 4 using Color Tiles to build arrays of different sizes. Students can take turns building arrays and writing division and multiplication sentences to represent them.
- Distribute Base Ten Blocks (10 rods and 20 units per group) to groups of 4 students. Have students model various problems, such as 56 ÷ 4. Students should first model 56 using 5 rods and 6 units, and then divide the rods and units into 4 equal groups. They will need to regroup one rod as 10 units to find the quotient of 14. Present real-world problems to give students practice recognizing when to use division and identifying the meaning of the divisor.

Formative Assessment

Have students try the following problem.

Nikki is displaying 60 pictures in equal rows. If there are 5 rows, how many pictures are in each row?

A. 10 **B.** 12 **C.** 14 **D.** 20

Try It! 20 minutes | Groups of 4

Here is a problem about division of whole numbers.

Two schools are each sending 72 students to a spelling bee. School A is sending six students from each of their 3rd grade classes. How many 3rd grade classes does school A have? School B has six 3rd grade classes. How many students can be sent from each class?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Centimeter Cubes, paper, and pencils. Help students recognize this as a division situation.

Materials

- Centimeter Cubes (72 per group)
- paper (2 sheets per group)
- pencils (1 per group)

1. Say: *You can think about division as repeated subtraction.* Have students model the 72 students with Centimeter Cubes and subtract 6 cubes at a time until no cubes remain. **Ask:** *How many times did you subtract 6 cubes?*

2. Say: *You also can share the cubes among different groups to solve the problem.* Have students distribute the 72 cubes among 6 groups until no cubes remain. **Ask:** *How many cubes are in each group?*

3. Say: *Another way to model the problem is to build an array with 72 cubes and 6 cubes in each row.* **Say:** *The array also models multiplication. Write a division sentence and a multiplication sentence for the array.*

⚠ Look Out!

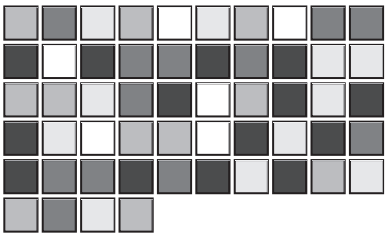
Some students may have difficulty forming groups or building arrays. Students can draw 6 circles on the sheet of paper for Step 2, and then place one cube at a time in each circle until all the cubes have been used. For Step 3, encourage students to line up the rows and columns as they build each row so they can easily see the number of rows and columns.

Answer Key

Use Centimeter Cubes to match the set shown. Divide the set into equal groups of the given size. Write a number sentence to show the quotient.

(Check students' work.)

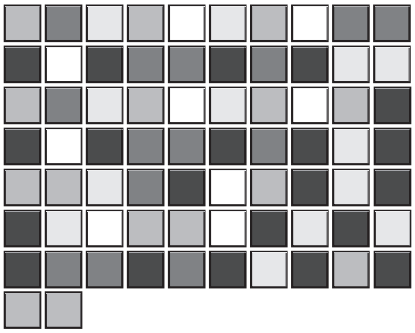
1.



Divide into groups of 9.

54 ÷ 9 = 6

2.



Divide into groups of 8.

72 ÷ 8 = 9

Using Centimeter Cubes, model the division problem. Sketch the model. Write a number sentence for the model.

(Check students' models.)

3. 39 cubes into 13 groups

39 ÷ 13 = 3

Write a number sentence for each quotient.

4. 81 cubes into 9 groups

81 ÷ 9 = 9

5. 77 cubes into 11 groups

77 ÷ 11 = 7

6. 70 cubes into 14 groups

70 ÷ 14 = 5

7. 63 cubes into 7 groups

63 ÷ 7 = 9

8. 96 cubes into 8 groups

96 ÷ 8 = 12

9. 75 cubes into 15 groups

75 ÷ 15 = 5

Answer Key

Challenge! For Problems 7–9, write a different division sentence using the same numbers and the same quotient. Choose one and draw a picture to support your sentence.

Challenge: (Sample) For Problem 7, 63 ÷ 9 = 7. For Problem 8, 96 ÷ 12 = 8. For Problem 9, 75 ÷ 5 = 15.

LESSON 5

Objective

Divide by a two-digit number.

Common Core State Standards

- **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Operations and Algebraic Thinking

More Division

Dividing larger numbers with a two-digit divisor is most often done using a tool such as an algorithm or a calculator. Before students can use these tools, they need to observe how a number is divided visually. Otherwise, the process is too abstract for students to understand due to the size of the numbers involved.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How many Centimeter Cubes and index cards would you need if you wanted to divide 78 by 15? How do you know?*
- Have groups compare their answers. **Ask:** *Did you end up with a remainder? How can you tell? Is it possible to have a remainder when you divide by a two-digit number?*
- Direct students to look at the division sentence they wrote. **Ask:** *Which number did you show using cubes? Which number did you show using index cards?*

Solve It

With students, reread the problem. Have students draw a picture showing how the cherries in the bowl were divided up, then write the corresponding division sentence under the picture.

More Ideas

For other ways to teach about dividing by a two-digit number—

- Have students use Two-Color Counters as units for the dividend in a division problem with a two-digit divisor. Students should begin with all counters flipped to the same color side. As they begin to divide the units into groups, they can flip the counters over to help keep track of which counters still need to be grouped.
- Have students practice writing word problems involving division. Then invite students to partner up and solve each other's division problems using Base Ten Blocks.

Formative Assessment

Have students try the following problem.

The Science Club earned 42 Fun Fair tickets for planting flowers around the school. There are 14 students in the club. How many tickets will each student get?

A. 3 **B.** 4 **C.** 5 **D.** 6

Try It! 25 minutes | Groups of 4

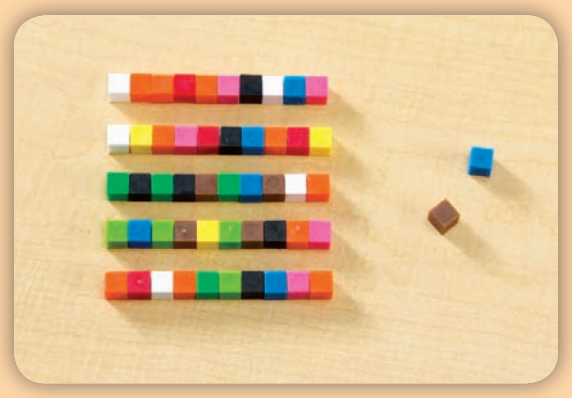
Here is a problem about dividing by a two-digit number.

There are 13 students in the dance club that meets after school. The end of the school year is tomorrow, so Juan brought in a large bowl of cherries to share with the other students in the club. The bowl has 52 cherries in it. How many cherries will each student get?

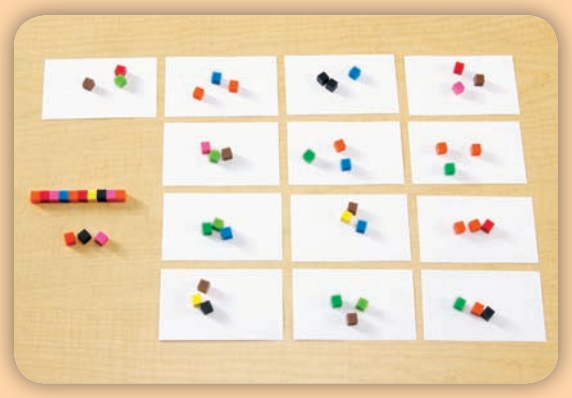
Introduce the problem. Then have students do the activity to solve the problem. Distribute Centimeter Cubes and index cards to groups of students.

Materials

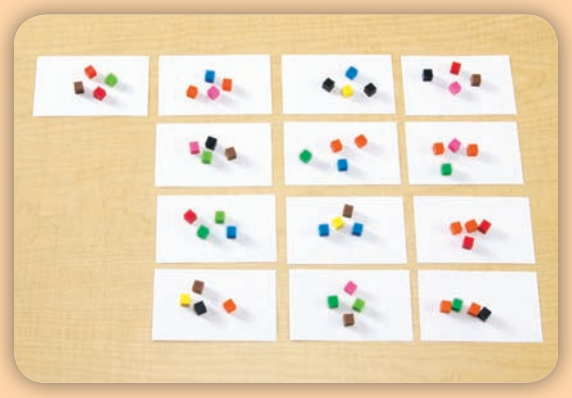
- Centimeter Cubes (60 per group)
- index cards (13 per group)



1. Say: *You need to divide 52 by 13.* Have students count out 52 cubes. Tell students that it may be easier to count out the cubes by placing them in groups of 10.



2. Direct students to place their 13 index cards on the desk or table so that all are visible and there is space between them. Have students divide the 52 cubes evenly, placing them on the 13 index cards until all the cubes are gone.



3. Tell students to count the cubes on each index card. **Ask:** *How can you show what you just did as a division sentence?* Help students write the division sentence 52 ÷ 13 = 4.

⚠ Look Out!

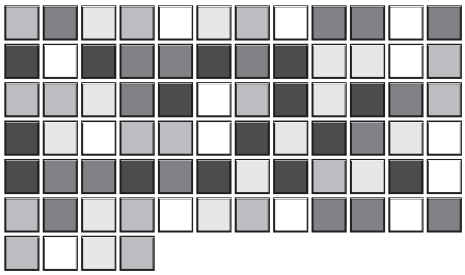
Watch for students who confuse two-digit dividends and divisors. Help students stress key words. For example, remind students that they want to divide 52 *by* 13, so they should separate 52 cubes into 13 groups.

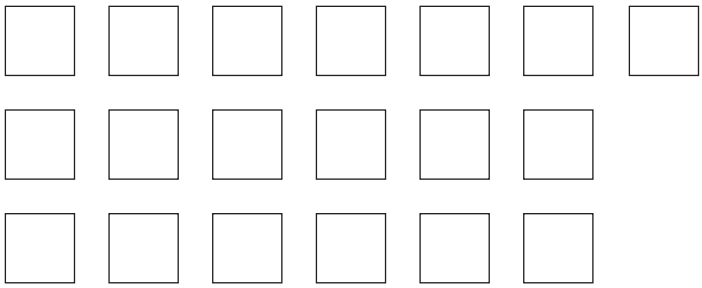
Answer Key

Use Centimeter Cubes and index cards to model the division problem. Divide the cubes equally among the index cards. Write the quotient.

(Check students' work.)

1.

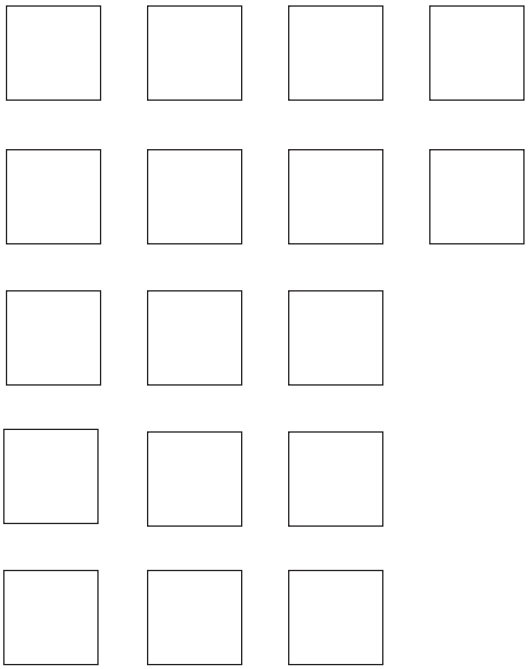




76 ÷ 19 = 4

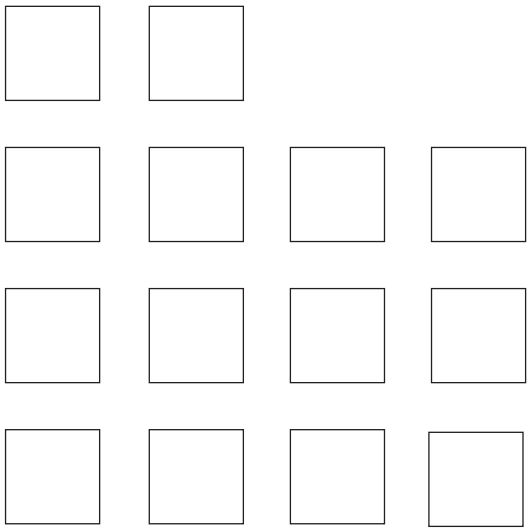
Using Centimeter Cubes and index cards, model each division. Sketch the cubes on the cards shown below. Write the division sentence.

2. 51 cubes



51 ÷ 17 = 3

3. 84 cubes



84 ÷ 14 = 6

Find each quotient.

4. 75 ÷ 15 = 5

5. 72 ÷ 12 = 6

6. 80 ÷ 16 = 5

7. 105 ÷ 15 = 7

8. 162 ÷ 18 = 9

9. 91 ÷ 13 = 7

Answer Key

Challenge! When you use Centimeters Cubes and index cards to model division, do the cubes or the index cards represent the dividend? What represents the quotient? Explain your answers.

Challenge: (Sample) Because the cubes represent the number of items being divided, it is the dividend. The number of cubes on each index card represents the quotient.

LESSON 6

Objective

Solve problems using either multiplication or division.

Common Core State Standards

- **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*
- **3.OA.6** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
- **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Operations and Algebraic Thinking

Multiplication and Division

Beginning with concrete models and moving on to paper and pencil experiences, students begin to develop computational fluency. The more they work with numbers, the stronger this foundation becomes. Using the inverse operation of a problem as a means of checking answers is another way to practice computations and build fluency. In this lesson, students identify whether multiplication or division should be used to solve problems and then create arrays to model and find the answers.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *Look at the multiplication problem you wrote.* **Ask:** *How did you turn it into a division problem to find the missing factor?*
- **Ask:** *What two multiplication problems describe the array you made with the Color Tiles?* Students should conclude that 4 × 7 and 7 × 4 describe the array.
- **Ask:** *How can you use multiplication to make sure you perform division problems correctly?*

Solve It

With students, reread the problem. Have students write instructions telling how to solve the problem using multiplication, and then how to solve it using division. Then have students illustrate the solution on paper.

More Ideas

For other ways to teach about solving problems using multiplication or division—

- Display the following multiplication problems on the board: 6 × _____ = 42, 7 × _____ = 35, 9 × _____ = 27, and 3 × _____ = 45. Have students model and solve each problem using Centimeter Cubes. Then find the corresponding division problems.
- Have one student write a multiplication or division problem and model it using Color Tiles. Then direct his or her partner to make an inverse model to find the same answer.

Formative Assessment

Have students try the following problem.

Which of the following is another way to write and solve 20 × _____ = 80?

A. 2 × 80 = 160

B. 80 ÷ 20 = 4

C. 80 – 20 = 60

D. 80 + 20 = 100

Try It! 25 Minutes | Pairs

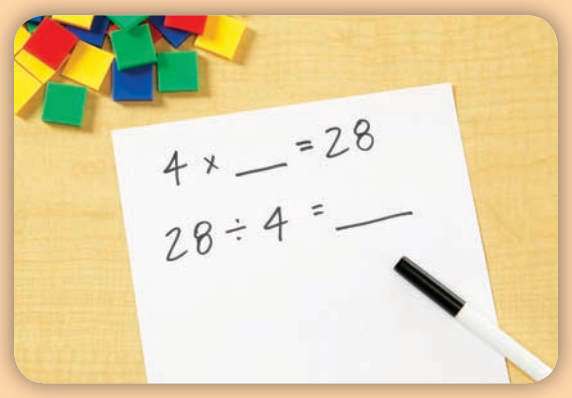
Here is a problem about using either multiplication or division.

There are 28 desks in the classroom. If we arrange the desks evenly in 4 rows, how many desks will there be in each row?

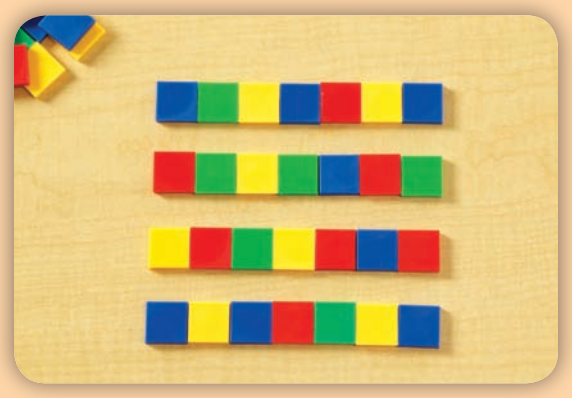
Introduce the problem. Then have students do the activity to solve the problem. Distribute Color Tiles, paper, and pencils to pairs of students.

Materials

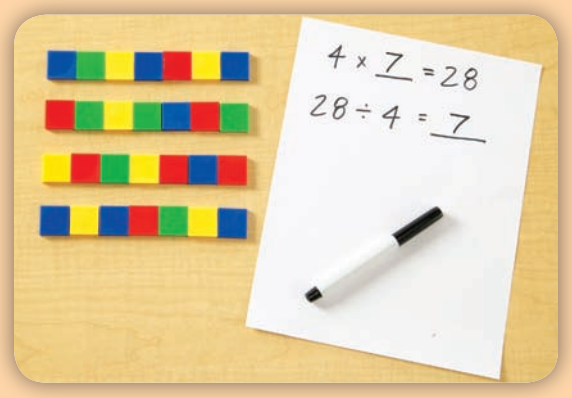
- Color Tiles (50 per pair)
- paper (1 sheet per pair)
- pencils (1 per pair)



1. Say: *We know that there are 28 desks arranged evenly in 4 rows.* Write *4* × _____ = *28* on the board, and have students write it on their papers. **Say:** *We want to find the missing number.* **Ask:** *What kind of problem can we write to find how many desks will be in each row?* Guide students to conclude that they should write a division problem. Write *28* ÷ *4* = _____ on the board, and have students write it on their papers.



2. Tell students that they can make an array using the tiles to solve a division problem. **Ask:** *How many tiles will you use in your array?* Students should see that they will use 28 tiles, since that is the number they want to divide. **Ask:** *How many rows of tiles will there be in the array?* Guide students to conclude that there will be 4 rows. Have students make the array.



3. Ask: *How can you use the array to find the answer to 28 ÷ 4 =* _____ . Have students count the tiles in a row to solve and then fill in the blanks in the multiplication and division problems on their paper.

⚠ Look Out!

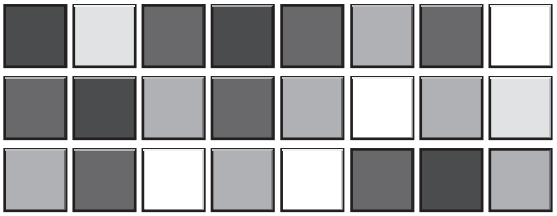
Watch for students who have difficulty formulating a division problem to solve a multiplication problem. Write *4* × *5* = *20* and *20* ÷ *4* = *5* on the board. Point out how the same three numbers are used in both problems, and how they are related. Invite students to write their own pairs of simple, corresponding division and multiplication problems.

Answer Key

Use Color Tiles to model each array. Complete the multiplication sentence. Write a related division sentence.

(Check students' work.)

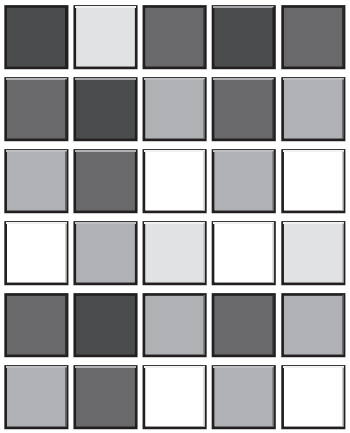
1.



$3 \times$ 8 $= 24$

24 ÷ 3 = 8

2.



$6 \times$ 5 $= 30$

30 ÷ 6 = 5

Using Color Tiles, model an array for each number sentence. Sketch the model. Complete the multiplication sentence. Write a related divison sentence.

(Check students' models.)

3. $8 \times$ 6 $= 48$

48 ÷ 8 = 6

4. $5 \times$ 4 $= 20$

20 ÷ 5 = 4

Complete each multiplication sentence. Write a related division sentence.

5. $7 \times$ 6 $= 42$

42 ÷ 7 = 6

6. $4 \times$ 9 $= 36$

36 ÷ 4 = 9

7. $8 \times$ 5 $= 40$

40 ÷ 8 = 5

8. $7 \times$ 8 $= 56$

56 ÷ 7 = 8

Answer Key

Challenge! Write another division sentence for Problem 1. Describe the model for this division sentence. Write the other two sentences in this fact family.

Challenge: (Sample) 24 ÷ 8 = 3, 8 rows of 3 Color Tiles for a total of 24 Color Tiles; 3 × 8 = 24; 8 × 3 = 24

LESSON 7

Objective

Model and identify the Commutative Property of Multiplication.

Common Core State Standards

- **3.OA.5** Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

Operations and Algebraic Thinking

Commutative Property of Multiplication

The Commutative Property of Multiplication states that you can multiply factors in any order and get the same product. For any two values, *a* and *b*, $a \times b = b \times a$. Students will apply the Commutative Property in their work in algebra with variables.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What answer did you get for 3 × 6? 6 × 3?*
- **Ask:** *What numbers can you change the order of using the Commutative Property of Multiplication? Can you switch a factor and the product? Why or why not?* Model for students what will happen if a product is changed for a factor. For example, write on the board *3 × 4 = 12*. **Ask:** *If you change the number sentence to 3 × 12 = 4, is it still true?*
- **Ask:** *Do you think the Commutative Property of Multiplication works for multiplication sentences with more than two factors?* Encourage students to test the property using number sentences with three factors.

Solve It

With students, reread the problem. Have students write two multiplication sentences that could be used to find the amount of fruit each class has. Then ask them to write a sentence telling why both multiplication sentences have the same answer.

More Ideas

For other ways to teach about the Commutative Property of Multiplication—

- Have one student model an array with Color Tiles. Then have his or her partner model an array that uses the same factors in a different order. Both students write multiplication sentences to represent the arrays.
- Prepare a paper bag that contains different multiplication sentences written on slips of paper. Have students pick a slip from the bag without looking and rewrite the multiplication sentence using the Commutative Property. Students should make arrays of Centimeter Cubes for both sentences.

Formative Assessment

Have students try the following problem.

What is another way to write 10 × 12 = 120?

A. 12 + 10 = 120 **B.** 1 × 10 = 10 **C.** 12 × 10 = 120 **D.** 2 × 20 = 120

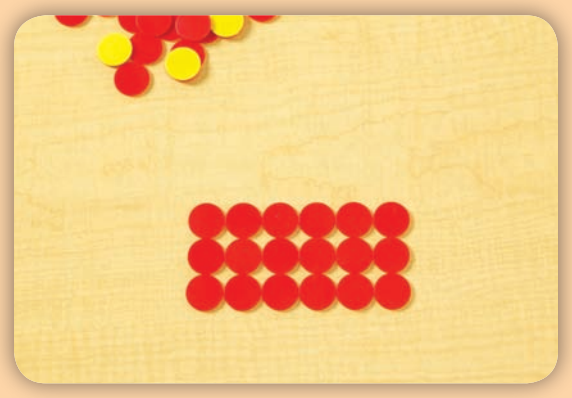
Try It! 30 minutes | Groups of 3

Here is a problem about the Commutative Property of Multiplication.

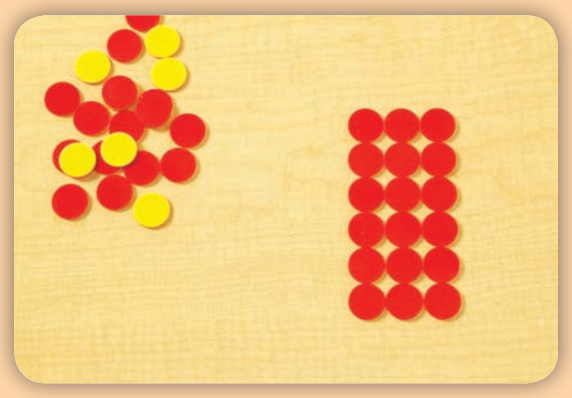
Third- and fourth-grade classes can choose different kinds of fruit for a snack. Mrs. Marshall's class has 3 types of fruit and 6 pieces of each type of fruit. Mr. Kim's class has 6 types of fruit and 3 pieces of each type of fruit. How many pieces of fruit does each class have?

Introduce the problem. Then have students do the activity to solve the problem. Write *3 × 6 =* _____ and *6 × 3 =* _____ on the board. Distribute Two-Color Counters to each group of students. Define the terms *factor* and *product.*

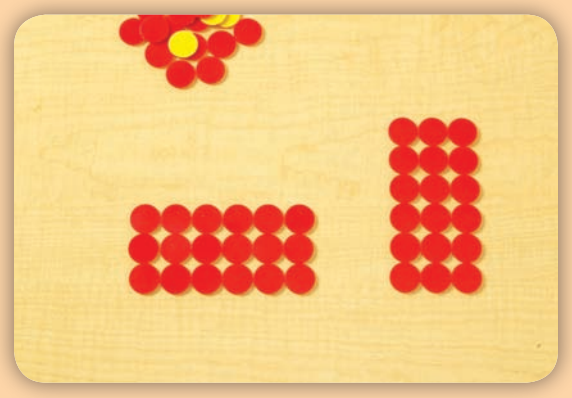
Materials
- Two-Color Counters (40 per group)



1. Have students model 3 groups of 6 using counters in an array. The array should have 3 rows with 6 counters in each row. Ask students to find the product. Fill in the answer for 3 × 6 on the board.



2. Have students model 6 groups of 3 using counters in an array. The array should have 6 rows of counters with 3 in each row. Ask students to find the product. Fill in the answer for 6 × 3 on the board.



3. Ask students to compare the arrays they have made to confirm that both have the same number of counters. Explain that the factors in the number sentence can be switched because of the Commutative Property of Multiplication. Have students compare the two arrays side-by-side to see that they reflect the same quantities.

⚠ Look Out!

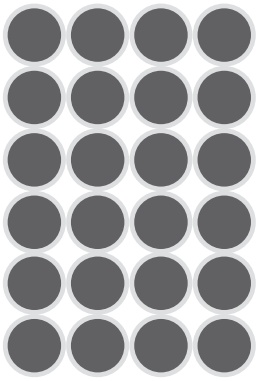
Students may believe that they can use the Commutative Property of Multiplication to exchange the product with one of the factors. Reinforce that the Commutative Property of Multiplication says that you can change the order of the factors with one another but not with the product. Use arrays of counters to show that when a factor and product are switched, the resulting number sentence will be incorrect. Also, be aware that some students may overgeneralize and try to use the Commutative Property to do division. Demonstrate with counters that when the order of numbers in a division sentence is changed, the answer is changed as well.

Answer Key

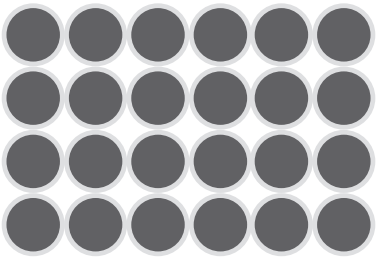
Use Two-Color Counters to build each model. Write number sentences that show the Commutative Property of Multiplication.

(Check students' work.)

1.



6 × 4 = 24



4 × 6 = 24

Using Two-Color Counters, model the Commutative Property of Multiplication for each pair of factors. Sketch the models. Write both number sentences. (Check students' models.)

2. 4, 8

4 × 8 = 32

8 × 4 = 32

3. 3, 7

3 × 7 = 21

7 × 3 = 21

Write two number sentences using each pair of factors that show the Commutative Property of Multiplication.

4. 2, 8

2 × 8 = 16

8 × 2 = 16

5. 5, 9

5 × 9 = 45

9 × 5 = 45

6. 6, 7

6 × 7 = 42

7 × 6 = 42

7. 3, 1

3 × 1 = 3

1 × 3 = 3

8. 8, 3

8 × 3 = 24

3 × 8 = 24

9. 6, 9

6 × 9 = 54

9 × 6 = 54

Answer Key

Challenge! Problems 1 and 8 both have products of 24. Use 24 Two-Color Counters to find another pair of factors for 24. Describe your model. Write two multiplication sentences for your model that show the Commutative Property of Multiplication.

Challenge: (Sample) 2 rows of 12 counters model a product of 24. 12 rows of 2 also model a product of 24; 2 × 12 = 24; 12 × 2 = 24.

LESSON 8

Objective

Model and identify the Associative Property of Multiplication.

Common Core State Standards

- **3.OA.5** Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

Operations and Algebraic Thinking

Associative Property of Multiplication

The Associative Property of Multiplication allows you to group and regroup the factors in a multiplication equation without changing the product. For any three values, *a, b,* and *c,* $(a \times b) \times c = a \times (b \times c)$. Students need to understand that regrouping numbers will allow them to multiply number pairs. The Associative Property is used in algebraic relations when working with equations.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *When you grouped the factors differently, did you get different products?* Emphasize that the Associative Property of Multiplication allows students to use different groups of factors to get the same product.
- **Ask:** *Which way was easier to multiply, 2 × (4 × 5) or (2 × 4) × 5? Why do you think the Associative Property is useful sometimes?* Guide students to understand that some number pairs are easier to multiply than others.
- **Ask:** *How is the Associative Property of Multiplication like the Associative Property of Addition? How are the properties different?*

Solve It

With students, reread the problem. Have students write to tell what products Jack and Meg calculated and explain why their answers were the same.

More Ideas

For other ways to teach about the Associative Property of Multiplication—

- Ask students to use Color Tiles to group factor arrays in different ways to get the same product. Have students draw large parentheses on paper to put around the tiles to show which factors are being multiplied first.
- Have students work in pairs with Color Tiles. Give students a product and have them work backward to find factors using the Associative Property. For example, if you gave students 50 as a product, they could come up with the number sentences (25 × 2) × 1 = 50 or (2 × 5) × 5 = 50.

Formative Assessment

Have students try the following problem.

What is another way to write (4 × 9) × 8 = 288 using the Associative Property of Multiplication?

A. 4 + (9 + 8) = 288
B. 8 × 9 × 4 = 288
C. 4 × (9 × 288) = 8
D. 4 × (9 × 8) = 288

Try It! 25 minutes | Groups of 4

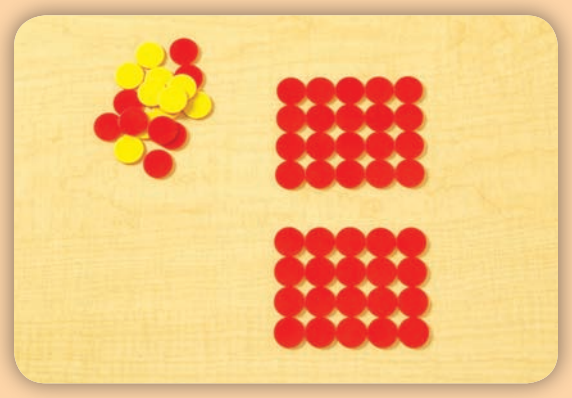
Here is a problem about the Associative Property of Multiplication.

Mrs. Larson's third-grade class is practicing multiplication. Mrs. Larson gives Jack the following problem to solve: 2 × (4 × 5) = _____ . She gives Meg another problem: (2 × 4) × 5 = _____ . What answers do the students get?

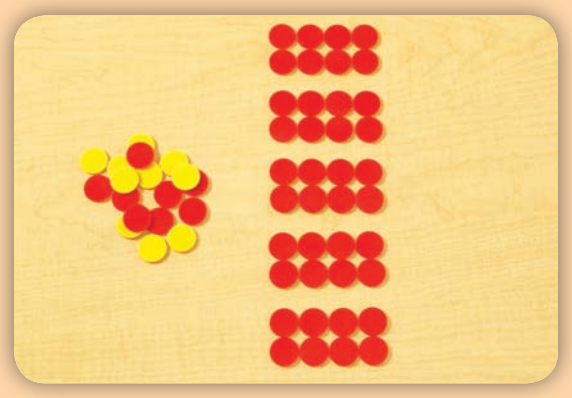
Introduce the problem. Then have students do the activity to solve the problem. Distribute Two-Color Counters to groups of students. Review the terms *factor* and *product* and the use of arrays if necessary. Write the problem *2 × 4 × 5* = _____ on the board.

Materials

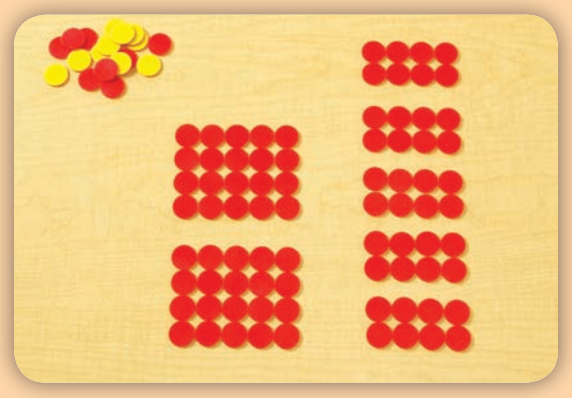
- Two-Color Counters (80 per group)
- paper (1 sheet per group)
- pencils (1 per group)



1. Have students model the number sentence 2 × (4 × 5) = _____ by showing 2 groups of 4 × 5 arrays. Then have them find the product.



2. Have students model the number sentence (2 × 4) × 5 = _____ by showing 5 groups of 2 × 4 arrays. Then have them find the product.



3. Have students compare the two sets of arrays they made out of counters. Ask them to write two number sentences to represent their models using parentheses to group the numbers that are multiplied first.

⚠ Look Out!

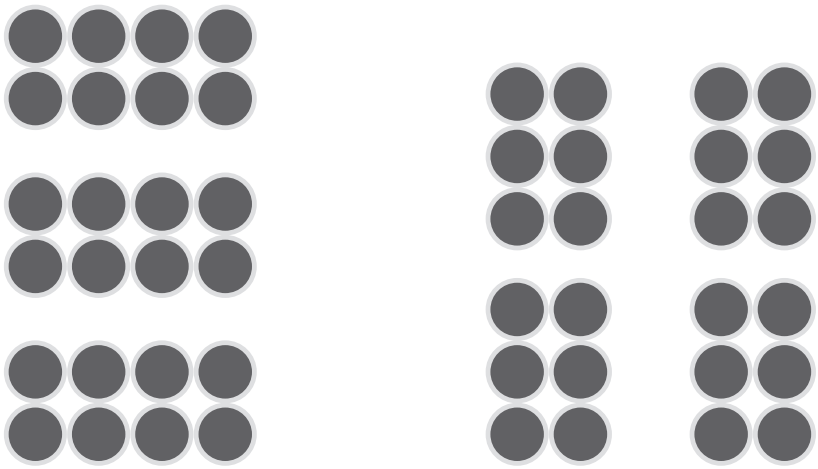
Students may confuse the Associative Property of Multiplication with the Commutative Property of Multiplication. Remind students that the Associative Property allows you to group numbers in different ways. The Commutative Property allows you to shift the factors' places in the number sentence. Also, watch for students who try to use the Associative Property when there is more than one operation present. For example, students may think that (6 + 4) × 3 is the same as 6 + (4 × 3). Model for students on the board that number sentences like this will have two different answers. It may be helpful to allow students to represent their products on grid paper.

Answer Key

Use Two-Color Counters to build each model. Write two number sentences that show the Associative Property of Multiplication.

(Check students' work.)

1.



3 × (2 × 4) = 24

(3 × 2) × 4 = 24

Using Two-Color Counters, model the Associative Property of Multiplication for each set of factors. Sketch the models. Write number sentences for both models. (Check students' models.)

2. 3, 5, 6

3 × (5 × 6) = 90

(3 × 5) × 6 = 90

Write two number sentences for each set of factors that show the Associative Property of Multiplication.

3. 2, 3, 5

2 × (3 × 5) = 30

(2 × 3) × 5 = 30

4. 2, 4, 5

2 × (4 × 5) = 40

(2 × 4) × 5 = 40

5. 3, 6, 8

3 × (6 × 8) = 144

(3 × 6) × 8 = 144

Answer Key

Challenge! In Problem 4, one way that you can associate the numbers makes the problem simpler because you get a multiple of 10 multiplied by a single-digit number. In what other problem on the previous page can you use the Associative Property to get a multiple of 10 times a single-digit number? Write the number sentence showing the association.

Challenge: (Sample) Problem 2 when you use the association 3 × (5 × 6) = 90, because you have 3 × 30 to find the answer of 90.

LESSON 9

Objective

Model and identify the Distributive Property.

Common Core State Standards

- **3.OA.5** Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

Operations and Algebraic Thinking

Distributive Property

The Distributive Property may be particularly challenging to students at this age because it involves more than one operation. The Distributive Property shows that, for the whole numbers *a, b,* and *c:* $(a \times b) + (a \times c) = a \times (b + c)$. While learning to apply the Distributive Property, students must address the order of operations and explore the consequences of ignoring it.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *First you made two rectangles.* **Ask:** *What part of the problem on the board does each model represent?*
- **Say:** *Both rectangles had one side that was two Color Tiles long. Then you pushed the two rectangles together to make one.* **Ask:** *What number sentence shows the multiplication problem modeled by the tiles?*
- **Say:** *The Distributive Property worked for (2 × 3) + (2 × 5) because the multiplication problems have a* common factor, *or one factor that is the same in both.* Have students identify the common factor (2).

Solve It

With students, reread the problem. Have students draw a picture of the tables and computers in the problem. **Say:** *Now draw a picture that shows the tables pushed together to make one long table.* Have students write a short paragraph telling how many computers there are in all.

More Ideas

For other ways to teach about the Distributive Property—

- Discuss the order of operations with students. Write *3 × 5 + 3 × 4* on the board. Have students use Centimeter Cubes to model each problem twice: once following the proper order of operations and once ignoring it. Have students use pencils or cut out parentheses to group the models. Then have students apply the Distributive Property and solve the problem.
- Make two columns on the board. On the left, write *(5 × 4) + (5 × 2)*, *(3 × 6) + (3 × 5)*, and *(2 × 8) + (2 × 3)*. On the right, write *2 × (8 + 3)*, *5 × (4 + 2)*, and *3 × (6 + 5)*. Have groups of students use Color Tiles to model each problem. Then have students identify the equivalent models and problems.

Formative Assessment

Have students try the following problem.

Circle the choice that shows (3 × 2) + (3 × 4) rewritten using the Distributive Property.

A. 3 × (2 + 4) **B.** 2 × (3 × 4) **C.** 4 × (3 + 2) **D.** 2 × (3 + 4)

Try It! 30 minutes | Pairs

Here is a problem about modeling and identifying the Distributive Property.

The computer lab at Lincoln Elementary School rearranged the computers. One table has 2 rows of 3 computers each, and the other table has 2 rows of 5 computers each. How many computers are there in all?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Color Tiles to students. Write *(2 × 3) + (2 × 5) =* _____ on the board. Remind students that the numbers within the parentheses should be multiplied before their products are added together.

Materials

- Color Tiles (30 of one color and 30 of another color per pair)

1. Tell students to use one color of tiles to model a 2 × 3 rectangle and another color to model a 2 × 5 rectangle. **Ask:** *How many counters are in each rectangle? How many in all?*

2. Point out that both rectangles have one side that is 2 tiles long, and explain that this means they can push the rectangles together to make one larger one. Have students push their 2 rectangles together to make one larger 2 × 8 rectangle.

3. Say: *Your model shows one side that is 2 tiles long and another that is 3 + 5, or 8, tiles long.* Write *2 × (3 + 5)* and *2 × 8* on the board. **Ask:** *How many tiles are there altogether?* Explain that their models show an example of the Distributive Property. **Say:** *Making a 2 × 3 array and adding it to a 2 × 5 array is the same as making a 2 × 8—or 2 × (3 + 5)—array.*

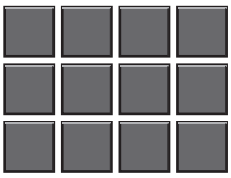
⚠ Look Out!

Watch for students who try to push their models together into a shape that is not a rectangle. For example, students might make a shape that has four rows instead of two. Have students start over, then ask them to count to find the side that is the same length in both rectangles. Tell students to make these two sides meet when they push the rectangles together.

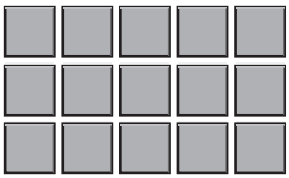
Answer Key

Use Color Tiles to model each array. Write the multiplication expression for each array. Then write the Distributive Property sentence modeled by the arrays. (Check students' work.)

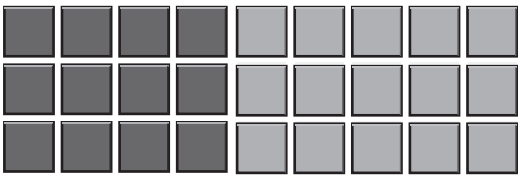
1.



3 × 4



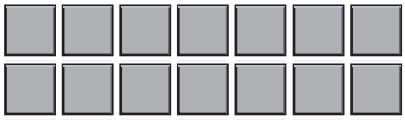
3 × 5



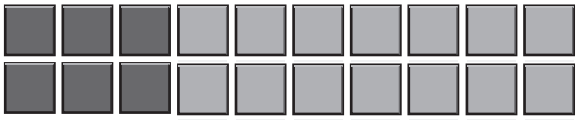
3 × (4 + 5) = 27

2.

2 × 3



2 × 7



2 × (3 + 7) = 20

Using Color Tiles, model arrays to show the Distributive Property. Sketch the models. Write the sentence modeled. (Check students' models.)

3. (3 × 3) + (3 × 4)

3 × (3 + 4) = 21

4. (5 × 2) + (5 × 7)

5 × (2 + 7) = 45

Write a number sentence for the total using the Distributive Property. Then find the total.

5. (2 × 4) + (2 × 6)

2 × (4 + 6) = 20

6. (3 × 8) + (3 × 1)

3 × (8 + 1) = 27

7. (4 × 3) + (4 × 7)

4 × (3 + 7) = 40

8. (6 × 2) + (6 × 3)

6 × (2 + 3) = 30

Answer Key

Challenge! Problems 1 and 6 both simplify to 27. Explain how this can be correct even though the problems have different numbers being multiplied. Draw pictures to help.

Challenge: (Sample) Problem 1 simplified to 3 × (4 + 5) and Problem 6 simplified to 3 × (8 + 1). Because they both have sums that are multiplied by 3, the sums have to be the same. The sum of 4 + 5 is 9 and the sum of 8 + 1 is 9.