The Number System

Number system learning at this level builds on students' previous learning of the base ten number system. Students continue developing their overall understanding of numbers, learning ways to represent them and discovering relationships among them. In particular, students extend the notion of number to include negative numbers. That is, they learn to work with the full set of **rational numbers**, which are the whole numbers, fractions, and decimals they have been learning to work with, plus their negatives.

The Grade 6 Common Core State Standards for The Number System specify that students should–

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Students often experience frustration when faced with problems that involve multiplication and division with fractions. This might be due to previous learning experiences that involved memorization of algorithms instead of engagement in activities that promote conceptual understanding.

The following hands-on activities enable teachers to help students learn the concepts of rational numbers in a rich and meaningful way. Teachers will want to encourage students to communicate about and explain why certain methods, for instance those for dividing fractions, make sense. When students do this, they will be on track to develop a deep and true understanding of the concepts.

The Number System

Contents

LESSON 1

Objective

Divide fractions by fractions.

Common Core State Standards

- **6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Fraction Division

When students develop their understanding of division, they make the connection that the divisor is either the number of groups to form or the number in each group, and that the dividend is the quantity that is divided into groups. The same model can be used when dividing a fraction by a fraction, which is useful as a tool in algebraic processes such as factoring and simplifying compound fractions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What operation can we use to solve this problem? What does it mean to divide by* $\frac{1}{8}$*?*
- **Ask:** *How do you choose your tower height?*
- **Ask:** *What do you do if the towers do not end up equal?*
- Introduce the idea that dividing by a fraction is the same as multiplying by its reciprocal, and give students the formula $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$.

Solve It

Read the problem with students. Have students perform the division. Ask students to show their work by creating a model with Fraction Tower® Equivalency Cubes. Have students write the steps involved in solving the problem.

More Ideas

For another way to teach about dividing a fraction by a fraction—

- Use Deluxe Rainbow Fraction® Squares to represent the divisor and the dividend. Cover the dividend with Fraction Squares the size of the divisor. Count the number of divisor pieces that were used to cover the dividend exactly. That is the quotient. Give students other fractions to divide. They should check their work by solving the problems with Fraction Tower Equivalency Cubes.

Formative Assessment

Have students try the following problem.

George wants to jog $\frac{2}{5}$ *mile. He increases his speed every* $\frac{1}{10}$ *mile. How many different speeds does George run?*

A. $\frac{1}{25}$ **B.** $\frac{1}{4}$

C. 2 **D.** 4

Try It! 15 minutes | Groups of 3

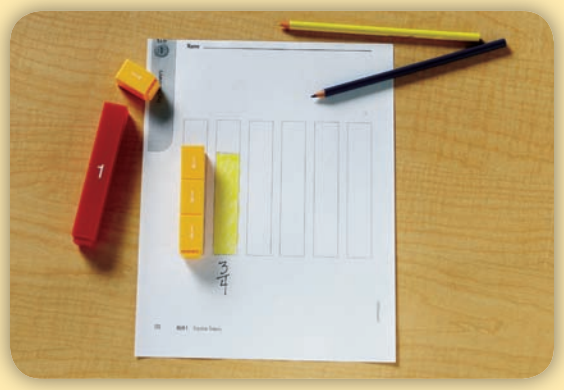
Here is a problem about dividing a fraction by a fraction.

Caryn is planning a party. She knows that a pitcher of juice holds $\frac{3}{4}$ gallon. Each serving is $\frac{1}{8}$ gallon. How many servings does the pitcher hold?

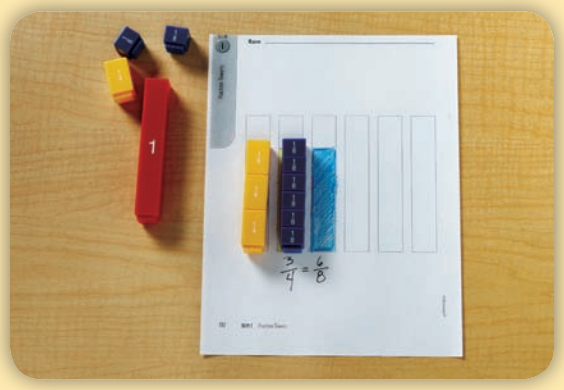
Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Equivalency Cubes, Fraction Tower templates, and pencils to students.

Materials

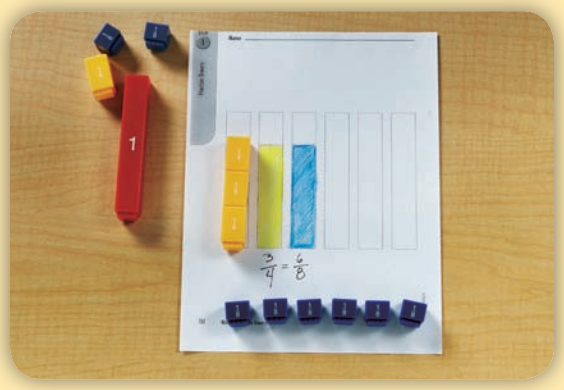
- Fraction Tower® Equivalency Cubes
- Fraction Towers (BLM 1; one per student)
- pencils (1 per group)



1. Say: *Use the towers to represent the amount the pitcher holds. Let the outline of a tower equal 1 whole gallon.* Have students build a tower that represents $\frac{3}{4}$ gallon on Fraction Towers BLM 1 and shade it in. Have students write the fraction the tower represents.



2. Using $\frac{1}{8}$ Fraction Tower Equivalency Cubes, have students build a tower next to the original tower that is the same height. Then, students should shade another tower on the BLM to represent this new equivalent tower. Have the students label the shaded tower with the equivalent fraction.



3. Ask: *How do you know how many $\frac{1}{8}$ gallon are in $\frac{3}{4}$ gallon?* Have students count the number of $\frac{1}{8}$ cubes in their tower as they take the tower apart. On their shaded tower, they should show the division lines for each $\frac{1}{8}$ cube in the tower. Then, they should number each shaded cube. **Ask:** *How many servings will the pitcher hold?* Have students write the division problem and quotient.

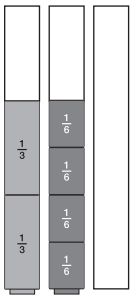
⚠ Look Out!

Students connect the process of division with breaking something into smaller parts. Remind them, however, that with fractions divided by fractions, the resulting quotient can be larger. Use Fraction Tower Equivalency Cubes to model fraction division. Then use the cubes to model the quotient. Compare the divisor, dividend, and quotient.

Answer Key

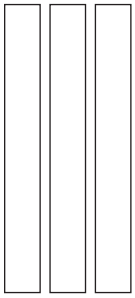
Use Fraction Towers to model the problem. Perform the division. (Check students' work.)

1. $\frac{2}{3} \div \frac{1}{6} =$ 4



Use Fraction Towers to model the problem. Sketch the model. Perform the division.

2. $\frac{5}{6} \div \frac{5}{12} =$ 2



Use Fraction Towers to model the problem. Solve the problem.

3. Aidan has $\frac{4}{5}$ of a gallon of juice. He wants to pour it into $\frac{1}{10}$-gallon jars. How many jars can he fill?

$\frac{4}{5} \div \frac{1}{10} =$ 8

Divide. Simplify, if possible.

4. $\frac{1}{2} \div \frac{1}{2} =$ 1

5. $\frac{3}{4} \div \frac{3}{8} =$ 2

6. $\frac{5}{8} \div \frac{5}{6} =$ $\frac{3}{4}$

7. $\frac{2}{5} \div \frac{3}{5} =$ $\frac{2}{3}$

8. $\frac{7}{10} \div \frac{4}{5} =$ $\frac{7}{8}$

9. $\frac{7}{10} \div \frac{7}{8} =$ $\frac{4}{5}$

Answer Key

Challenge! Create a story problem and draw a model to show $\frac{5}{6} \div \frac{1}{3}$. Divide to answer the problem. Simplify, if possible.

Challenge: (Sample) Jen has $\frac{5}{6}$ of a book left to read. If she reads $\frac{1}{3}$ of it each day, how many days will it take her to finish the book?

$\frac{5}{6} \div \frac{1}{3} = \frac{5}{2} = 2\frac{1}{2}$ days

LESSON 2

Objective

Represent, compare, and order integers.

Common Core State Standards

- **6.NS.5** Understanding that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- **6.NS.6a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite.
- **6.NS.6c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs or integers and other rational numbers on a coordinate plane.
- **6.NS.7a** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*
- **6.NS.7b** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*

Introduction to Integers

Students are typically introduced to negative numbers using temperatures below zero, losses in games, and money owed. The concepts of *magnitude, direction,* and *opposites* are important for understanding integers. Students can think about an integer as a distance (the magnitude, or absolute value) in one or the other direction on the number line. Numbers to the right of zero are positive, and numbers to the left of zero are negative. A number some distance from zero in one direction is the opposite of the number the same distance from zero in the other direction.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Discuss the correct way to say negative numbers—for example, *negative four* for –4. **Ask:** *What situations can you represent using negative integers?*
- **Ask:** *What is the opposite of 3? –4? 0?* Have students represent the integers and their opposites on a number line. Point out that 0 is neither positive nor negative and is its own opposite.
- **Ask:** *Is it possible for a positive integer to be less than a negative integer? Why or why not?* **Say:** *Explain why –4 is less than –1.*

Solve It

Reread the problem with students. Have them explain how they know that 3 is the first-place score and –4 is the last-place score for the round.

More Ideas

For other ways to teach integers—

- Have students use a vertical number line (BLM 2 in portrait orientation) to show temperature or altitude, with tick marks representing increments of 1, 2, 5, or 10. With the number line as a guide, students use color tiles to represent temperatures or altitudes greater than or less than a given value. Have students use < or > to demonstrate comparisons between the integers.
- Have students use an integer number line (BLM 2) with only certain numbers labeled, such as –4, 0, and 4. Have them use Two-Color Counters to locate various integers on the number line, such as –3 or the opposite of 2. Have students write the integers below the tick marks.

Formative Assessment

Have students try the following problem.

Which of the following is the lowest altitude?

A. –32 meters **B.** –8 meters **C.** –3 meters **D.** 2 meters

Try It! 20 Minutes | Pairs

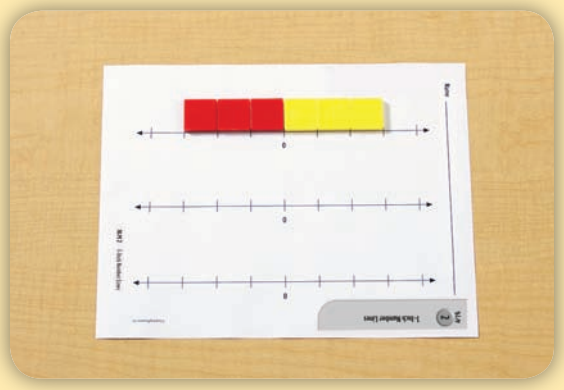
Here is a problem about representing and ordering integers.

Dustin, Kyle, and Emma are playing a word game. After one round of play, Dustin lost 4 points, Emma gained 3 points, and Kyle lost 1 point. How can you use integers to represent the scores for this round of play? Which scores represent first place and last place?

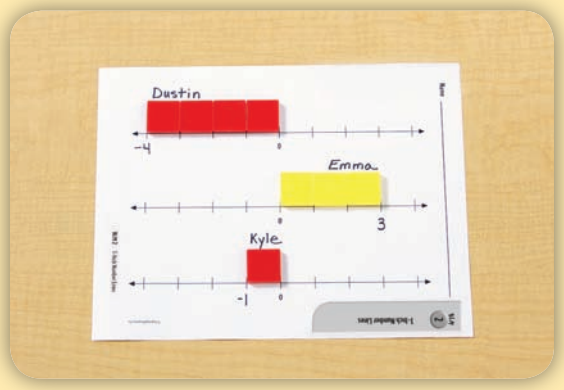
Introduce the problem. Then have students do the activity to solve the problem. Distribute Color Tiles, number lines sheets, paper, and pencils to students. Draw a number line from –4 to 4 on the board. Label the tick marks. Have students copy the numbering onto the number lines on the sheet.

Materials

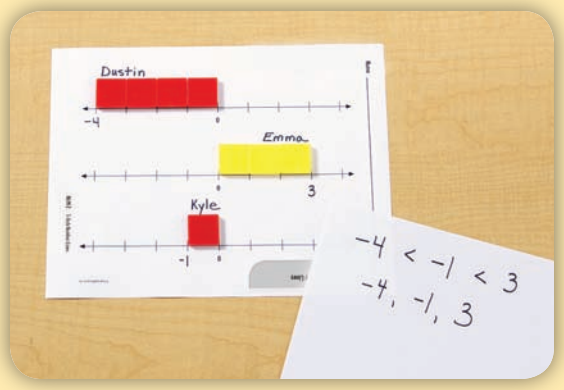
- Color Tiles (11 per pair)
- 1-Inch Number Lines (BLM 2; 1 per pair)
- paper (1 sheet per pair)
- pencils (1 per pair)



1. Say: *Integers are the counting numbers, their opposites, and 0.* Have students place 3 red tiles to the left of 0 and 3 yellow tiles to the right of 0 on the number line. **Say:** *–3 and 3 are opposites because they are the same distance from 0 on the number line.*



2. Say: *Negative integers can represent losses, below-zero temperatures, and below-sea-level altitudes.* **Ask:** *What do positive integers represent?* Have students represent the losses and gains from the game on the number lines.



3. Say: *You can use number lines to compare and order integers. The values of the numbers increase as you move left to right.* Have students use symbols to compare the three integers from the game. Then have them write the integers in order from least to greatest.

⚠ Look Out!

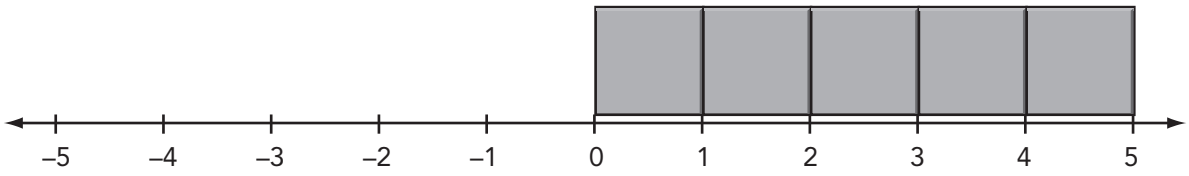
Some students may be confused by an integer such as –8 is less than –1. Use real-world contexts to help them see that the farther a negative integer is from 0, the less is its value. For example, the person who owes $8 has less money than the person who owes $1, or –8°F is below –1°F on the thermometer.

Answer Key

Use Color Tiles and a number line to model each integer. Write the integer.

(Check students' work.)

1.



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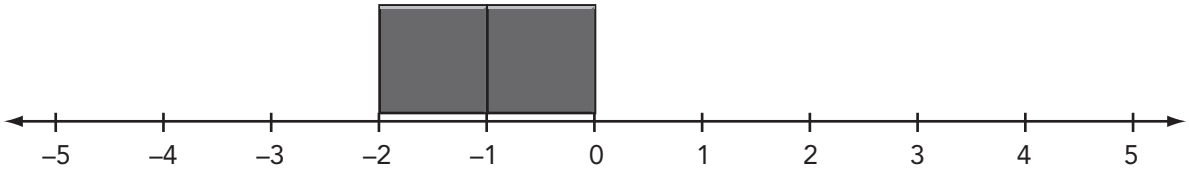
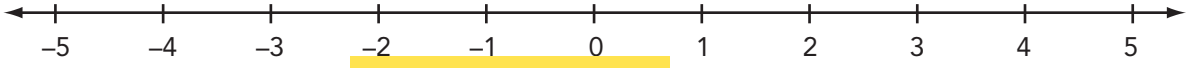
2.



–2

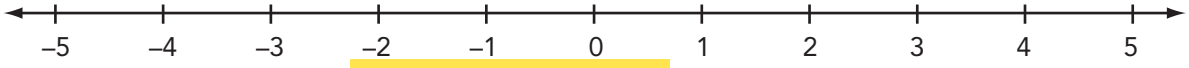
Using Color Tiles, model each integer. Sketch the model.

3. –4



Check students' model.

4. 1



Check students' model.

Use a number line to locate and compare each pair of integers. Write an inequality.

5. 5 > –2

6. –8 < –6

7. 9 > –9

8. 4 > 3

9. –10 < 11

10. –7 < –6

Use <, =, or > to complete each inequality.

11. 87 (>) –78

12. –31 (<) 28

13. –914 (=) –914

Answer Key

Challenge! When comparing integers, a number line is not always available or practical. Write guidelines you can use when comparing integers without a number line.

Challenge: (Sample) Negative numbers are always less than positive numbers. When comparing negative numbers, look at the number without the negative sign; the greater that number is, the lesser the negative number.

LESSON 3

The Number System

4-Quadrant Graphing

Students build upon their experiences with integers and graphing points in the first quadrant to graph points in the coordinate plane. They become familiar with all four quadrants by locating and plotting points and giving directions from one point to the next. These skills are useful in coordinate geometry and in graphing equations.

Objective

Graph points in the coordinate plane.

Common Core State Standards

- **6.NS.6b** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- **6.NS.6c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- **6.NS.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *When you use an ordered pair to describe a position, which coordinate is related to the east and west directions? To the north and south directions?*
- **Ask:** *Why does Quadrant II have a negative x-coordinate and a positive y-coordinate?*
- **Ask:** *If you make Alison's house the origin, what are the coordinates of the school?*

Solve It

Reread the problem with students. Have students write the directions from school to Alison's house, from Alison's house to the movie theater, and from the movie theater to Janet's house. Have students identify the coordinates and quadrant of each location.

More Ideas

For other ways to teach graphing points in the coordinate plane—

- Make a series of steps for students to follow, such as *place a peg at (–3, 0), move 4 units right and 5 units down; then move 2 units right and 8 units up.* For each step, have students place a peg and write the coordinates for the location that they have arrived at.
- Extend the lesson by having students graph various landmarks around the school, such as buildings, parks, and offices. Give coordinates and have students locate the landmark on a pegboard or graph grid. Then have students write instructions on how to move from one landmark to another.

Formative Assessment

Have students try the following problem.

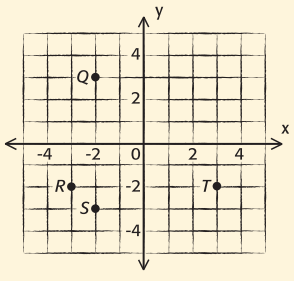
Which point is at (–3, –2)?

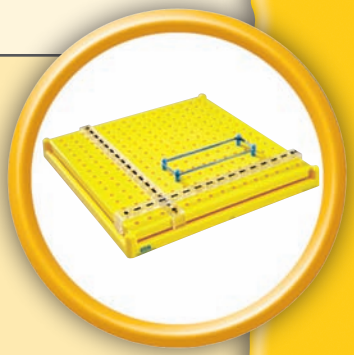
A. Point *Q*

B. Point *R*

C. Point *S*

D. Point *T*





Try It! 20 minutes | Pairs

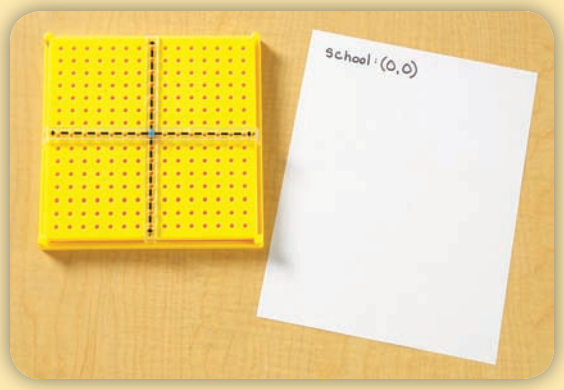
Here is a problem about graphing points in the coordinate plane.

Janet is going to Alison's house after school. Then they are going to the movie theater. Alison uses a grid to make a map for Janet. School is located at the origin. Each unit is one block. Directions east of the school are positive x-coordinates, north are positive y-coordinates, west are negative x-coordinates, and south are negative y-coordinates. Alison's house is 4 blocks east and 3 blocks south of school. Janet's house is 2 blocks west and 7 blocks south of school. The theater is 7 blocks west and 9 blocks north of Alison's house. What does the map look like?

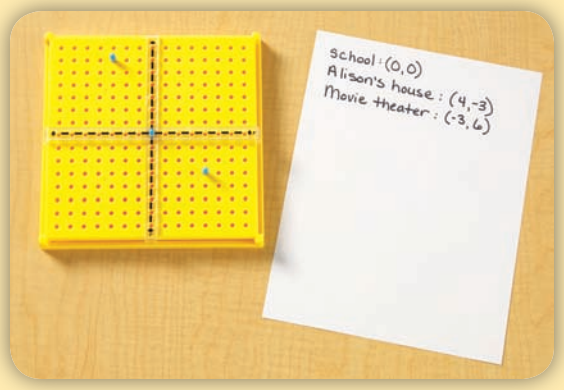
Introduce the problem. Then have students do the activity to solve the problem. Distribute pegboards, pegs, rubber bands, paper, and pencils. Draw a coordinate plane on the board, label the *x*- and *y*-axes, and label each quadrant with its number and sign pair, e.g., Quadrant II (–, +).

Materials

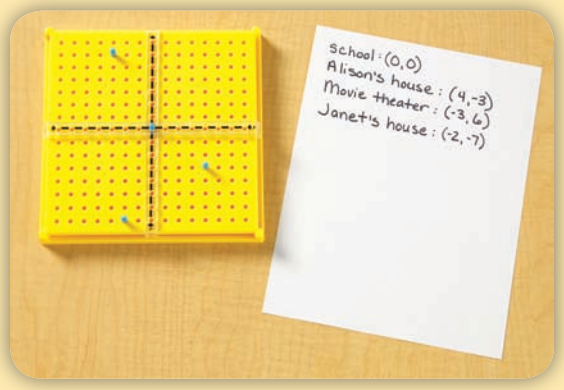
- XY Coordinate Pegboards (1 per pair)
- paper (1 sheet per pair)
- pencils (1 per pair)



1. Say: *Look at the coordinate plane on the board.* **Ask:** *What is the origin of the coordinate plane?* Guide students to mark the origin of the pegboard with a peg.



2. Have students locate and place a peg at Alison's house (4, –3). **Ask:** *What quadrant is Alison's house in?* **Say:** *Write the ordered pair describing the location of Alison's house.* Have students locate and place a peg at the movie theater (–3, 6). **Ask:** *What quadrant is the movie theater in?* **Say:** *Write the ordered pair describing the location of the movie theater.*



3. Have students locate and place a peg at Janet's house (–2, –7). **Ask:** *What quadrant is Janet's house in?* **Say:** *Write the ordered pair describing the location of Janet's house.*

⚠ Look Out!

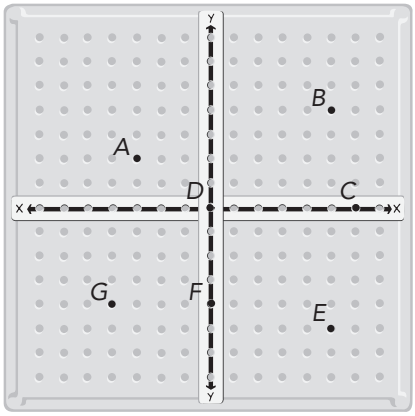
If students are confused about the directions, have them label the axes of the pegboard *North, South, East,* and *West* using masking tape. Explain that any location northeast of school is in Quadrant I, any location northwest is in Quadrant II, any location southwest is in Quadrant III, and any location southeast is in Quadrant IV.

Answer Key

Use an XY Coordinate Pegboard to plot each point. Write the ordered pair for each labeled point.

(Check students' work.)

1.



A (–3, 2) B (5, 4)

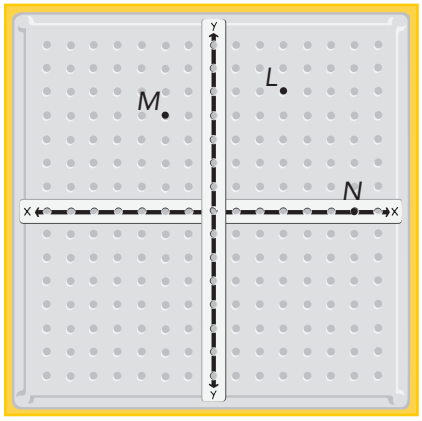
C (6, 0) D (0, 0)

E (5, –5) F (0, –4)

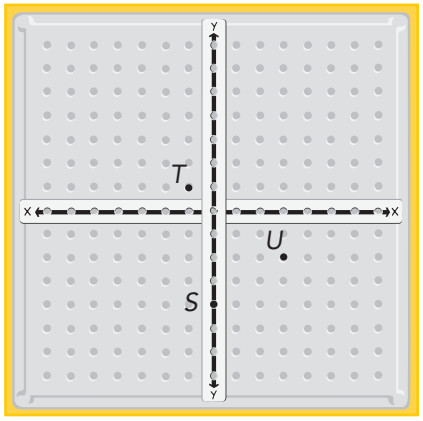
G (–4, –4)

Using an XY Coordinate Pegboard, plot the ordered pairs. Sketch the points on the graph below. Label the points.

2. *L* (3, 5) *M* (–2, 4) *N* (6, 0)



3. *S* (0, –4) *T* (–1, 1) *U* (3, –2)



Graph and label each ordered pair on the coordinate plane.

4.

A (–1, 2) *B* (3, 0)

C (4, 6) *D* (1, –5)

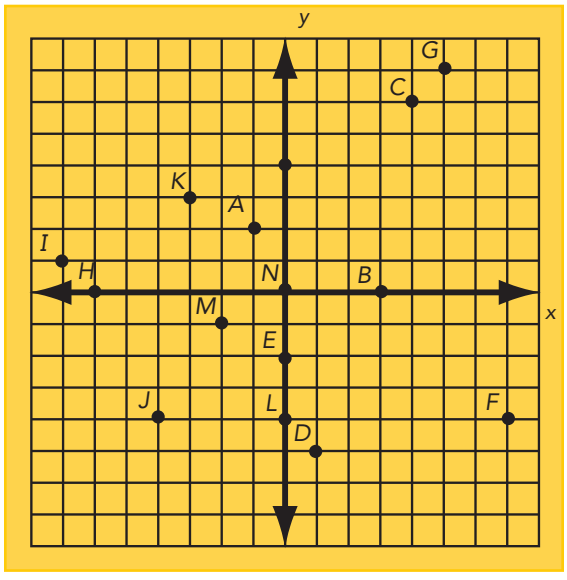
E (0, –2) *F* (7, –4)

G (5, 7) *H* (–6, 0)

I (–7, 1) *J* (–4, –4)

K (–3, 3) *L* (0, –4)

M (–2, –1) *N* (0, 0)



Answer Key

Challenge! Identify the signs of the ordered pairs in each quadrant of the coordinate plane. Draw a picture to help.

Challenge: (Sample) In the first quadrant the signs are (+, +). In the second quadrant the signs are (–, +). In the third quadrant the signs are (–, –). In the fourth quadrant the signs are (+, –).

LESSON 4

Objective

Compare and order fractions and decimals.

Common Core State Standards

- **6.NS.6c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

The Number System

Compare and Order Fractions and Decimals

The number line is a useful tool for comparing and ordering fractions and decimals. Students should be able to draw and mark a number line with fractions, with decimals, and with a combination of both types of numbers. They should be able to translate between forms and use whichever form is more convenient or more appropriate in the context of the problem.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *One pink Fraction Tower piece represents* $\frac{1}{2}$. *What decimal is equivalent to* $\frac{1}{2}$*?*
- **Say:** *Three green Fraction Tower pieces together represent* $\frac{3}{5}$. *What decimal is equivalent to* $\frac{3}{5}$*?* Note that each green tower piece is $\frac{1}{5}$, or 0.2. Three pieces equal $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$, or 0.2 + 0.2 + 0.2 = 0.6.

Solve It

Reread the problem with students. Students find the equivalent decimals for the fractions using the equivalency cubes. Students locate these decimals on a number line, reading from left to right: 0.375, 0.5, 0.6, 0.75, 0.8. Next they write the foods in order: salami, Swiss, Cheddar, turkey, ham.

More Ideas

For other ways to teach about comparing and ordering fractions and decimals—

- Have students use Base Ten Blocks to compare fractions and decimals. Provide students with decimals and fractions in simplest form. Have students write each number as a fraction with denominator 100 and model with Base Ten Blocks. Have students order the numbers.
- Have students use the Rainbow Fraction® Circle Rings to compare the sizes of the Fraction Circle pieces in the set. Ask students to place each piece inside the fraction ring and mark and label each piece starting from the 0. Then have students use the decimal ring to label each of the fraction pieces they marked.

Formative Assessment

Have students try the following problem.

Which of the following statements is true?

A. $0.5 < \frac{5}{8}$ **B.** $0.15 > \frac{3}{10}$ **C.** $\frac{4}{5} > 0.8$ **D.** $\frac{1}{3} < 0.25$

Try It! 20 minutes | Groups of 3

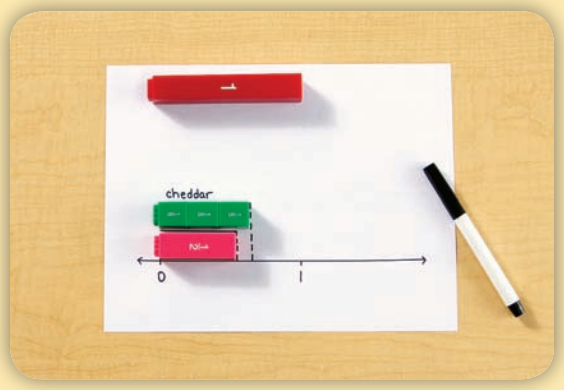
Here is a problem about ordering fractions and decimals.

Bob is making a snack tray for a party. He bought two packages of cheese, $\frac{1}{2}$ pound of Swiss and $\frac{3}{5}$ pound of Cheddar. The deli clerk sliced meats for Bob. Bob got 0.8 pound of ham, 0.75 pound of turkey, and 0.375 pound of salami. Write the foods in order from least to greatest weight.

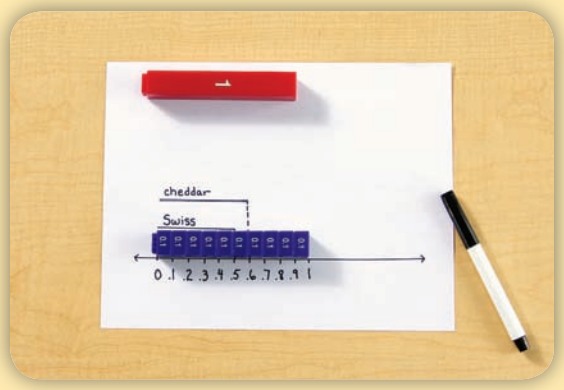
Introduce the problem. Then have students do the activity to solve the problem. Give each group of students Fraction Tower Equivalency Cubes, a straightedge, paper, and pencil.

Materials

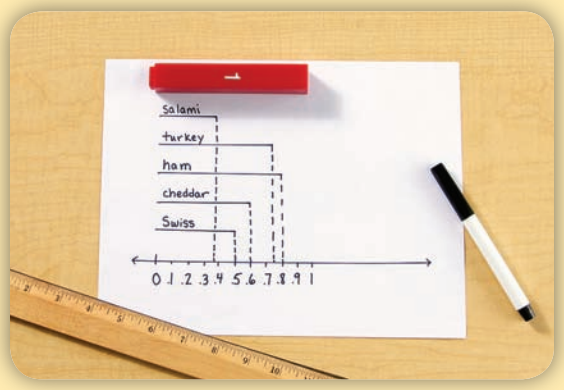
- Fraction Tower® Equivalency Cubes (1 set per group)
- straightedges (1 per group)
- paper (1 sheet per group)
- pencils (1 per group)



1. Say: *Select Fraction Tower pieces to represent the Swiss cheese and the Cheddar cheese. Draw a blank number line. Mark zero. Use a red tower to mark 1. Use the towers to draw a segment for each type of cheese.* Students trace along the tops of the pink and green towers and write *Swiss* and *Cheddar.*



2. Say: *Now, use the decimal side of the purple Fraction Tower to mark all the tenths on your number line between 0 and 1.* Students mark and label 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.



3. Say: *Draw and label a segment for each meat. Notice that 0.75 falls between two divisions and that 0.375 does too. Write the foods in order from least to greatest weight.*

⚠ Look Out!

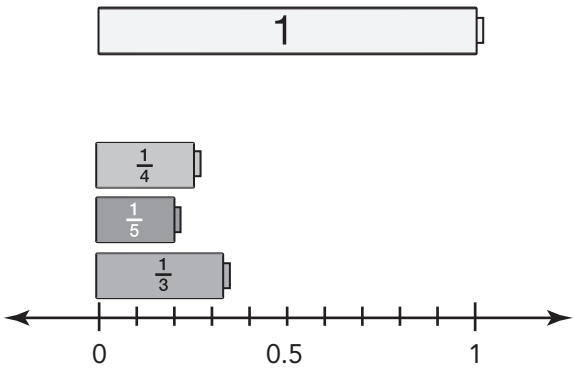
Make sure students think about place value when they compare decimals such as 0.8 (ham) and 0.375 (salami). Some students might think that 0.8 < 0.375 because 8 < 375. Point out that 0.8 = 0.800, so they should compare 0.800 with 0.375. Since 800 > 375, 0.800 > 0.375, and therefore 0.8 > 0.375. When comparing 0.75 (turkey) with 0.8 (ham), students should see that there's a little more ham than turkey. In this case, 0.80 > 0.75 because 80 > 75.

Answer Key

Use Fraction Towers to model each fraction on a number line. Write the fractions as decimals in order from least to greatest.

(Check students' work.)

1.



0.2 _____ 0.25 _____ $0.\overline{3}$

Using Fraction Towers, model each fraction. Sketch the models using the number line. Write the equivalent decimals in order from least to greatest.

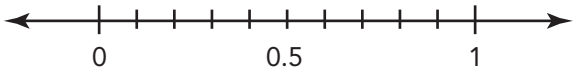
2. $\frac{3}{8}, \frac{1}{6}, \frac{3}{10}, \frac{2}{5}$

least $0.1\overline{6}$

0.3

0.375

greatest 0.4



Write the fractions as decimals in order from least to greatest.

3. $\frac{2}{3}, \frac{3}{4}, \frac{7}{10}$

$0.\overline{6}$; 0.7; 0.75

4. $\frac{3}{5}, \frac{3}{10}, \frac{7}{12}$

0.3; $0.58\overline{3}$; 0.6

5. $\frac{3}{8}, \frac{1}{3}, \frac{5}{12}$

$0.\overline{3}$; 0.375; $0.41\overline{6}$

6. $\frac{5}{6}, \frac{7}{8}, \frac{3}{4}$

0.75; $0.83\overline{3}$; 0.875

Answer Key

Challenge! Draw a diagram to show why $\frac{1}{5}$ is greater than $\frac{1}{6}$. Use this to compare the fractions $\frac{4}{5}$ and $\frac{4}{6}$. Explain.

Challenge: (Sample) When a circle is divided into 5 parts, one of the parts is larger than a circle divided into 6 parts. Four of the five parts of a circle are also larger than 4 out of 6 parts of a circle. $\frac{4}{5} > \frac{4}{6}$

LESSON 5

Objective

Estimate fractional numbers.

Common Core State Standards

- **6.NS.6c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

The Number System

Estimating Fractional Numbers

Students at this level are becoming increasingly accomplished in working with fractions, decimals, and percentages. Students should now begin thinking about fractions quantitatively. Practice with estimation, such as is provided in this lesson, helps students solidify their number sense, and it helps them build quantitative reasoning skills. This activity will also help students reinforce their understanding of the relative size of fractions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What color cubes did you use to represent the books on jewelry making? The books about pets? The other types of books?*
- **Ask:** *Did you use any "tricks" or strategies to help you estimate the fractions? If so, what were they?*
- **Ask:** *What percentage of shelf space is dedicated to each type of book? What are the decimal equivalents of these percentages?*

Solve It

Reread the problem with students. Ask students to explain how they made their estimates. They should write a brief note to the library aides with suggestions for making more accurate estimates.

More Ideas

For other ways to teach about estimating fractional numbers—

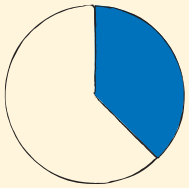
Note that the following activities may be done with decimals and percentages, as well as with fractional parts.

- Have students draw a 10-inch square on a large piece of unlined paper. Have them estimate a fractional part of it (e.g., $\frac{3}{5}$) and sketch in their best guess with a pencil. Students should then check their accuracy by filling in the section they indicated with Color Tiles. This activity may also be done with a square 10 centimeters on a side and Centimeter Cubes.
- Have students repeat the activity using Deluxe Rainbow Fraction® Circles and Rainbow Fraction Circle Rings. Discuss with students which is easier for them—estimating a fractional part of a line (linear estimation), estimating a fractional part of a square (area estimation), or estimating a fractional part of a circle (area estimation).

Formative Assessment

Have students try the following problem.

What portion of this circle is shaded?



A. $\frac{2}{7}$ B. $\frac{1}{3}$ C. $\frac{3}{8}$ D. $\frac{3}{5}$

Try It! 20 minutes | Groups of 3

Here is a problem about estimating fractional numbers.

Ms. Pérez, the school librarian, is moving books onto some new shelving units. She has asked her aides to reorganize the bookshelves to avoid overcrowding and to leave room for any new titles that might be added later. She has told the aides to fill each shelf this way (see chart). The aides do not have any measuring tools. Can you help them estimate the portion of each shelf to fill?

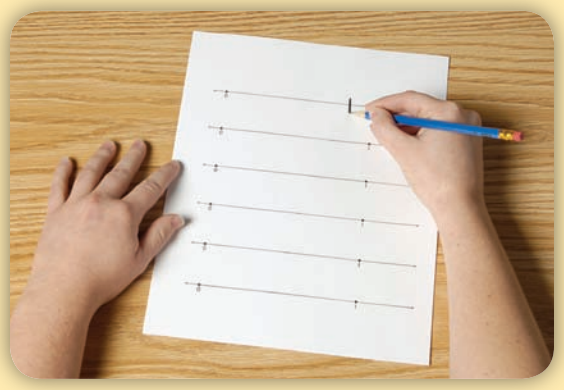
Type of Book	Portion of Shelf to Use
Fantasy	$\frac{5}{6}$
Jewelry Making	$\frac{7}{10}$
Pets	$\frac{3}{4}$
Sports Biographies	$\frac{2}{5}$
Woodworking	$\frac{2}{3}$

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Equivalency Cubes and number lines to students.

Materials

- Fraction Tower® Equivalency Cubes
- Fraction Tower Number Lines (BLM 3; 1 per group)

Caution students to ignore the tenon on the bottom of each Fraction Tower Cube, which allows the pieces to be snapped together. It should not be included in the length of the piece.



1. Say: *Look at the first line segment on the BLM. This will represent the length of the fantasy bookshelf. You can fill the shelf to only $\frac{5}{6}$ of its capacity. Use a pencil to mark where you estimate the books will end.*

2. Say: *Now check your work. Select the fraction cubes that represent sixths (teal) and stack five of them together. Align them with the zero on the number line.* **Ask:** *How close was your estimate to the actual amount?*

3. Say: *Repeat the procedure for the other types of books.*

⚠ Look Out!

Watch for students who confuse the rules for comparing and ordering whole numbers with those for comparing and ordering fractions. For example, 3 is greater than 2, but $\frac{1}{3}$ is less than $\frac{1}{2}$.

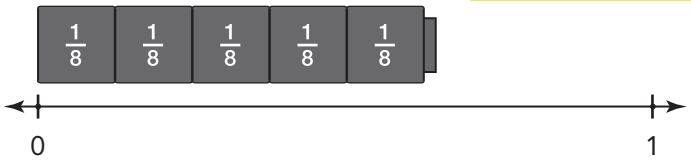
Be sure that students are estimating length on the basis of the line segment (between the two points) on the BLM, and not on one or both arrow ends.

Answer Key

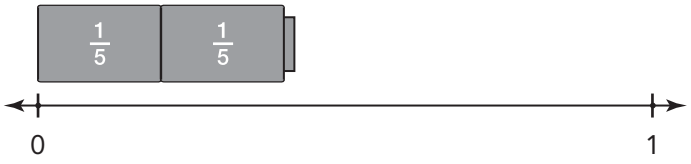
Use Fraction Towers to model each fraction on a number line. Tell whether the fraction is closer to 0 or 1.

(Check students' work.)

1.



$\frac{5}{8}$ 1



$\frac{2}{5}$ 0

$\frac{9}{10}$ 1

Using Fraction Towers, model each fraction. Sketch the model on a number line. Tell whether the fraction is closer to 0 or 1.

2. $\frac{5}{12}$ 0

3. $\frac{1}{3}$ 0

Estimate each fraction. Tell whether the fraction is closer to 0 or 1.

4. $\frac{7}{8}$ 1

5. $\frac{3}{10}$ 0

6. $\frac{3}{4}$ 1

7. $\frac{4}{5}$ 1

8. $\frac{9}{12}$ 1

9. $\frac{2}{6}$ 0

Answer Key

Challenge! How are the rounding rules for fractions similar to the rounding rules for whole numbers?

Challenge: (Sample) When rounding whole numbers, the test number is 5. If a test number is less than 5, you keep the number in the place you are rounding as is. If a number is 5 or greater, you round that number up. For fractions less than 1, the test number is $\frac{1}{2}$. If a fraction is less than $\frac{1}{2}$, you round down to 0. If a fraction is from $\frac{1}{2}$ to 1, you round up.

LESSON 6

The Number System

Comparing Rational Numbers

Students at this level are becoming increasingly accomplished in working with fractions, decimals, and percentages. They are also becoming more flexible in their mathematical thinking. This lesson brings comparing and ordering rational numbers to the concrete level by having students line up Fraction Tower Cubes on a number line. Allowing students to see when a negative is closer to being positive helps them to conceptualize the relative magnitude of rational numbers.

Objective

Compare rational numbers using a number line.

Common Core State Standards

- **6.NS.6c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
- **6.NS.7b** Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 °C > –7 °C to express the fact that –3 °C is warmer than –7 °C.*

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is a rational number?*
- **Ask:** *What number is represented by each of the bars on the graph? How do you know whether the number is positive or negative?*
- **Ask:** *Which Fraction Tower Cubes will you use to represent each bar? How many Fraction Tower Cubes of each color will you need to accurately represent the information on the graph?*
- **Say:** *Explain how a large tower can represent a smaller number than a small tower when they are placed on the negative side of the number line.*

Solve It

Reread the problem with students. Have students write a brief report to Mr. Grimley presenting their results and how they arrived at them.

More Ideas

For another way to teach about comparing rational numbers—

- Have students solve this and similar problems by using the other faces of the Fraction Tower Cubes to express the given quantities as decimals and percentages.

Formative Assessment

Have students try the following problem.

Order the following set of numbers from largest to smallest: $-\frac{3}{10}, \frac{5}{12}, -\frac{3}{8}, \frac{1}{4}$.

A. $\frac{5}{12}, \frac{1}{4}, -\frac{3}{8}, -\frac{3}{10}$

B. $\frac{5}{12}, \frac{1}{4}, -\frac{3}{10}, -\frac{3}{8}$

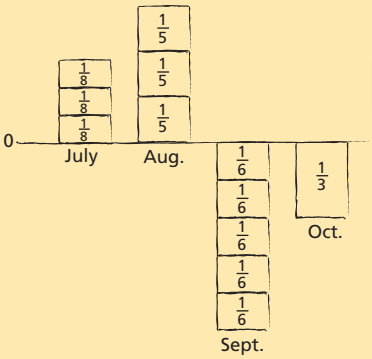
C. $\frac{1}{4}, \frac{5}{12}, -\frac{3}{8}, -\frac{3}{10}$

D. $\frac{1}{4}, \frac{5}{12}, -\frac{3}{10}, -\frac{3}{8}$

Try It! 30 minutes | Groups of 3

Here is a problem about comparing rational numbers.

The following chart shows changes in profits for Acme Motors for the months of July, August, September, and October of last year. Mr. Grimley, your boss, wants you to write the increases and decreases as rational numbers. Then you are to order the numbers from largest to smallest.

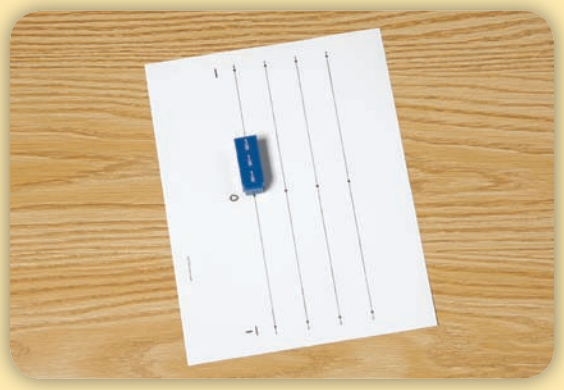


Introduce the problem. Then have students do the activity to solve the problem. Sketch the graphic from the story problem on the board. Distribute Fraction Tower Equivalency Cubes and number lines to students.

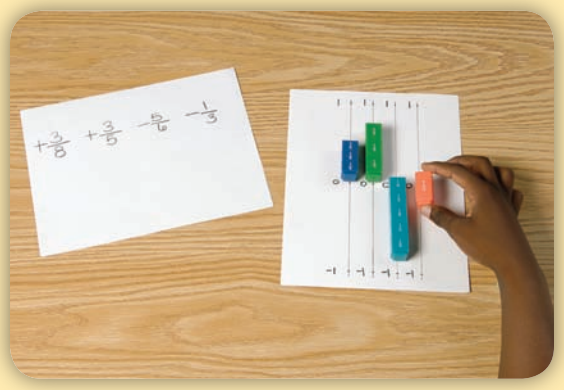
Materials

- Fraction Tower® Equivalency Cubes
- Double Fraction Tower Number Line (BLM 4; 1 per group)

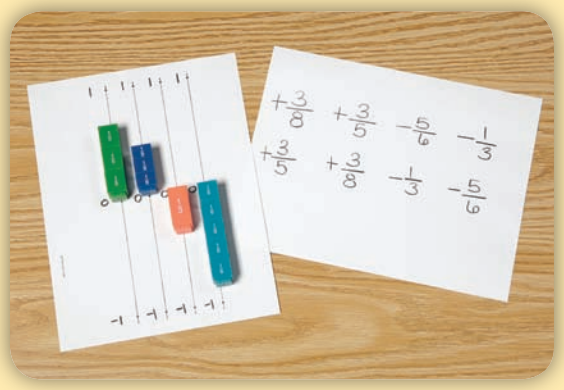
Caution students to ignore the tenon on the bottom of each Fraction Tower Cube, which allows the pieces to be snapped together. It should not be included in the length of the piece.



1. Ask: *Look at the first bar on the graph. What number does it represent?* ($\frac{3}{8}$) **Say:** *Write the number it represents on a piece of paper.* **Say:** *Now find the appropriate Fraction Tower Cubes to represent this amount. Place them on the first number line on the BLM.*



2. Say: *Follow the same procedure for the other bars on the graph.*



3. Say: *Now order the change in profits from largest to smallest. Move the towers into the correct positions.*

⚠ Look Out!

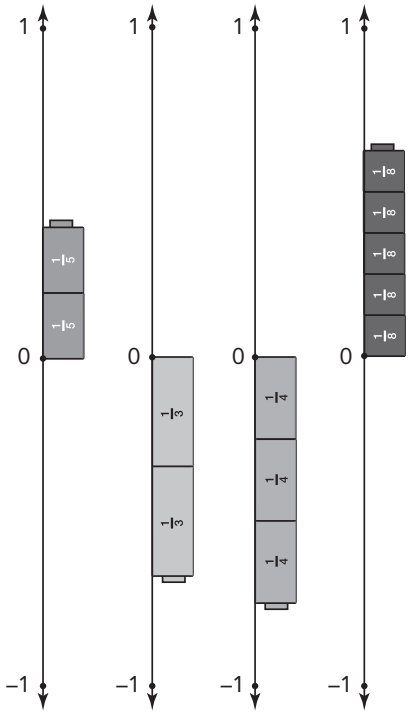
Some students may be confused to see that a large tower in the negative portion of the number line represents a smaller number than a shorter tower. (Compare the teal $\frac{5}{6}$ tower with the smaller orange $\frac{1}{3}$ tower in the photos above and at left.) Remind students that the teal tower is "more negative," while the orange tower is "closer to positive." It also may be helpful for students to mark the lower end of each tower on the BLM with a pencil. They can then remove the towers and read the relative values on the number line.

Answer Key

Use Fraction Towers to model each rational number on a number line. Write each number. Then write the numbers in order from least to greatest.

(Check students' work.)

1.



Numbers:

$\frac{2}{5}, -\frac{2}{3}, -\frac{3}{4}, \frac{5}{8}$

Ordered from least to greatest:

$-\frac{3}{4}, -\frac{2}{3}, \frac{2}{5}, \frac{5}{8}$

Using Fraction Towers, model each rational number. Sketch the models on number lines. Write the numbers in order from least to greatest.

2. $\frac{3}{8}, -\frac{1}{4}, \frac{7}{12}, -\frac{2}{5}$

Ordered from least to greatest: $-\frac{2}{5}, -\frac{1}{4}, \frac{3}{8}, \frac{7}{12}$

Use < or > to compare the numbers.

3. $\frac{7}{8}$ > $\frac{3}{4}$

4. $\frac{7}{10}$ < $\frac{9}{12}$

5. $\frac{1}{3}$ > $\frac{1}{4}$

6. $\frac{2}{5}$ < $\frac{1}{2}$

7. $\frac{1}{6}$ < $\frac{1}{4}$

8. $\frac{3}{12}$ < $\frac{2}{6}$

Answer Key

Challenge! Explain how comparing negative rational numbers is different than comparing positive rational numbers.

Challenge: (Sample) Negative numbers are left of 0 on a number line and in the reverse order of the positive numbers that are to the right of 0 on a number line.

LESSON 7

Objective

Distinguish absolute value from order.

Common Core State Standards

- **6.NS.7c** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*
- **6.NS.7d** Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

Absolute Value

Students must understand absolute value before learning how to add and subtract integers. Using concrete models will help students learn that opposite integers on a number line have the same absolute value and that comparisons of absolute value differ from statements about order. This learning will help students apply integers to real-world situations.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Write –15 and +6 on the board. **Ask:** *Where are these values on a number line? Which value is greater?*
- Write |–15| and |+6| on the board. **Ask:** *Which absolute value is greater?*
- Discuss the difference between absolute value (distance from 0) and order (position on a number line).
- Write –60 on the board. **Say:** *Think of a scenario that demonstrates the meaning of –60.* Discuss scenarios (e.g., 60 feet *below* sea level). Distinguish between magnitude (absolute value) and position.

Solve It

Reread the problem with students. Have students write |–10| = 10, |+5| = 5, and the inequality sentence that compares the amounts (10 > 5). Have students write why –10 is a greater change than +5 even though –10 is less than +5 on the number line.

More Ideas

For other ways to teach about distinguishing absolute value from order—

- Use Cuisenaire® Rods. Have students work in pairs and each choose a rod. Let one rod be a negative value and the other be a positive value. Have students give scenarios that demonstrate their integers in action. Then have them compare the lengths of their rods.
- Repeat the above activity, but make both values negative or both values positive.

Formative Assessment

Have students try the following problem.

Which of the following represents a debt greater than $50?

A. –$25 **B.** $75 **C.** $15 **D.** –$75

Try It! 15 minutes | Pairs

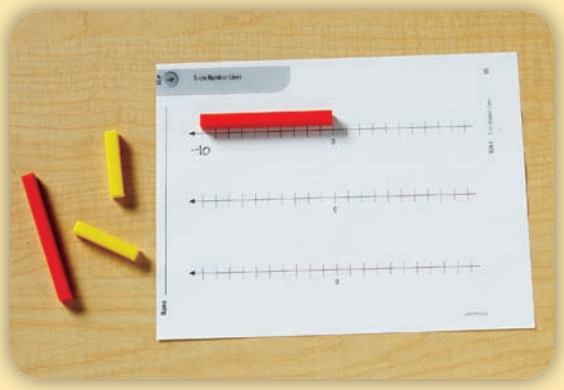
Here is a problem about distinguishing absolute value from order.

Jamie and Devon both have savings accounts. Within one week's time, Jamie's account balance had a change of –$10 and Devon's had a change of +$5. Who experienced the greater change in account balance?

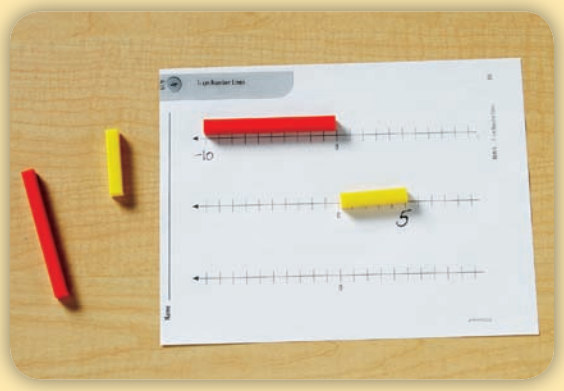
Introduce the problem. Then have students do the activity to solve the problem. Distribute Cuisenaire Rods and 1-cm Number Lines (BLM 5) to students.

Materials

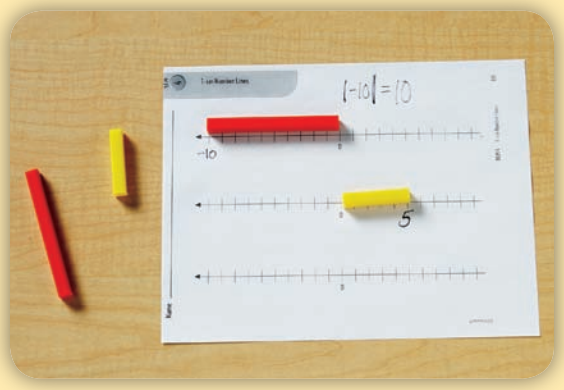
- Cuisenaire® Rods
- 1-cm Number Lines (BLM 5; 1 per pair)
- pencils (1 per pair)



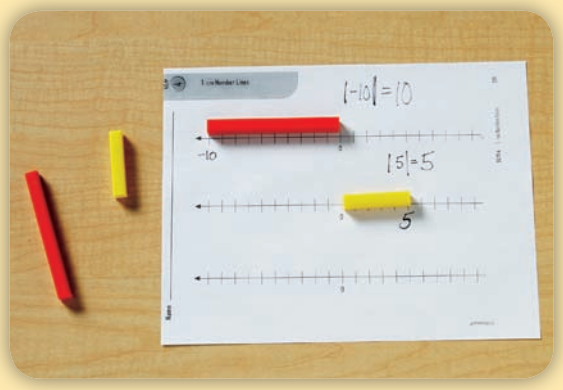
1. Say: *You are going to model the amount of change in both account balances. Assuming that the white Cuisenaire Rod represents 1, find a rod that represents 10.* If necessary, help students determine that the orange rods represent 10. **Say:** *Find –10 on the number line. Label it. Using the rod, cover the interval that stretches from –10 to 0.*



2. Say: *Find a rod that represents 5.* If necessary, help students determine that the yellow rods represent 5. **Say:** *Find +5 on the number line. Label it. Using the rod, cover the interval that stretches from 0 to +5.*



3. Ask: *What is the length of the interval from –10 to 0?* Elicit that the length, as indicated by the orange rod, is 10. Explain that the rod demonstrates how far –10 is from 0. Tell students that the distance a number is from 0 is called the absolute value of the number. Note that the symbol for absolute value is | |. Have students write |–10| = 10 above the orange rod.



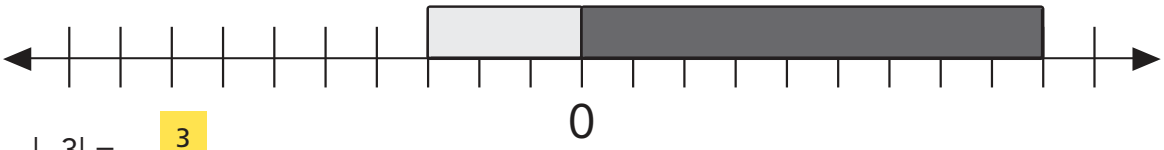
4. Ask: *What is the length of the interval from 0 to +5? What is the absolute value of +5?* Write |+5| = 5 above the yellow rod. **Ask:** *Which absolute value is greater? Which account balance experienced the greater change?* Elicit that the decrease in Jamie's account is greater than the increase in Devon's account.

Answer Key

Use Cuisenaire Rods and a number line. Model the numbers. Write the absolute values. Find the greater absolute value.

(Check students' work.)

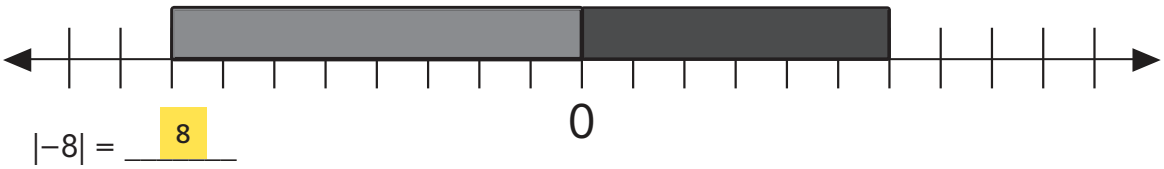
1.



$|-3|$ = 3

$|9|$ = 9 Greater absolute value: 9

2.



$|-8|$ = 8

$|6|$ = 6 Greater absolute value: 8

Write a situation that each integer could represent.

3. +17 Possible answer: climb 17 feet up a ladder

4. −61 Possible answer: owe $61

5. −9 Possible answer: temperature 9 degrees below zero

6. +12 Possible answer: earn $12 babysitting

Write the absolute value.

7. $|-40|$ = 40

8. $|33|$ = 33

9. $|16|$ = 16

10. $|-11|$ = 11

11. $|-90|$ = 90

12. $|4|$ = 4

Answer Key

Challenge! Consider point *A* on a number line. *A* represents a negative number. How would the absolute value of *A* change if it is moved 4 units to the left? Explain.

Challenge: The absolute value of *A* would increase by 4. A negative number that moves 4 units to the left moves farther from 0; therefore, it would increase by 4.