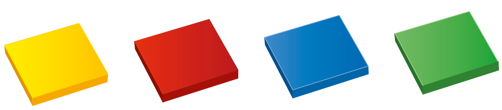
The S-Shaped Figure

OBJECTIVE

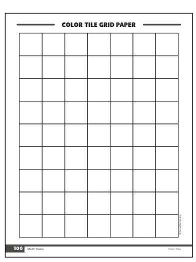
Students will explore and use ratios and proportions to solve problems using scale drawings.

WHAT YOU WILL NEED

Color Tiles, 150 per pair



Color Tile Grid Paper, page 100

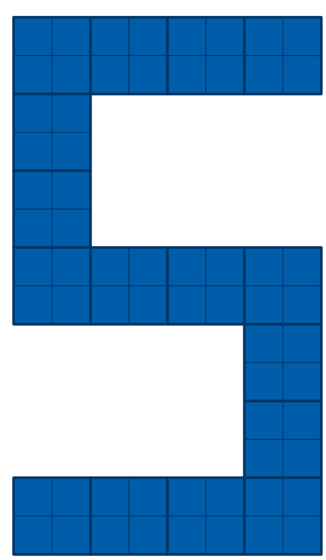


OVERVIEW

In this activity, given a scale model of a shape made from Color Tiles and the area of the larger shape it represents, students figure out the scale used and the perimeter of the larger polygon.

THE BIG IDEA

As students consider the problem of finding the perimeter of the larger S, they may begin by doubling each length of the small S. To do this, they might replace each Color Tile with a square made from 4 Color Tiles as shown below.



However, students may realize that the area of this S is only 64 square inches, not nearly as large as the 400 square inches of the large S shape in the problem. Alternatively, some students might use numbers, reasoning that if each tile edge of the scale model were equal to 2 inches in the larger S, then the area of one tile would be 2 × 2, or 4. Since 16 tiles × 4 is only 64, it follows that a side of a tile must be worth more than 2 inches.

Another strategy students may use is to break down the problem into steps. They may realize that in order to find the perimeter of the larger S, they need to know how the side of a square in the small S relates to the side of a square in the larger S. They might begin to write down what they know about each S and look for ways to compare the information:

area of larger S = 400 square inches

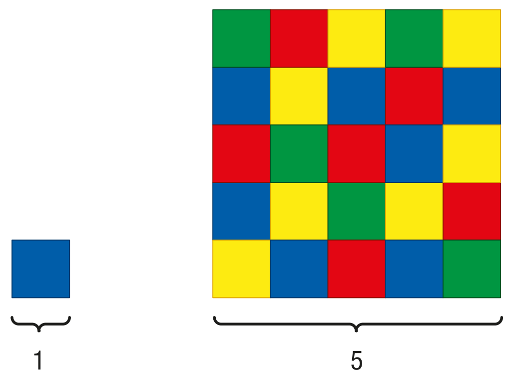
area of small S = 16 square inches

ratio of areas = 400 to 16 = 400/16 = 25/1

The area of the larger S is 25 times the area of the small S.

Students who reach this point might jump to the conclusion that the perimeter and area relationships are the same—the perimeter of the larger S is 25 times that of the small S, or 25 × 34, or 850 inches. That is not the case. Students who do not reach this false conclusion might continue to reason along these lines:

The area of the larger S is 25 times the area of the small S. So each tile in the small S represents 25 square inches in the larger S. So the side of each tile in the small S stands for 5 inches of perimeter in the larger S.



1 represents 5.

The perimeter of the small S is 34 inches.

The perimeter of the larger S is 34 times 5, or 170 inches.

Students may realize that they can use ratios to summarize and describe the relationships in side lengths, areas, and perimeters between the two figures:

ratio of areas = 400 to 16 = 25 to 1

ratio of side lengths = 5 to 1

ratio of perimeters = 170 to 34 = 5 to 1

After students have had the opportunity to compare and discuss their methods and results, they may be able to come up with a generalization: Side lengths and perimeter increase in the same ratio. In this problem, the scale is 5 to 1. The area increases with the square of the side length, so the ratio of the areas is 5^2 to 1^2, or 25 to 1.

1 INTRODUCTION

- Show students a rectangle made with 3 Color Tiles. Ask how to find the area and the perimeter of the rectangle.
- Now ask students to think of the longer side as 6 feet. Explain that this makes the rectangle a scale model of a much larger rectangle.
- Ask students what length is represented by the edge of each Color Tile in the scale model. Confirm that if the length of the rectangle is 6 feet, each edge represents 2 feet.
- To check students' understanding, have them find the area and perimeter of the larger rectangle that the scale model represents.

2 ON THEIR OWN

Students will complete the On Their Own. During this time, the teacher's role is to:

- ask probing questions to guide and extend
- record student thinking
- record student conversation that promotes collaboration

Use the information gathered to inform the Math Talk.

3 MATH TALK

Use prompts such as these to promote class discussion:

- What is the perimeter of the larger S?
- How did you figure its perimeter?
- What length does the side of 1 tile in the model represent?
- How are the perimeters of the small S and the larger S related?
- How are the areas of the small S and the larger S related?
- Are there other ways this problem could be solved? Explain.

4 EXTENSION

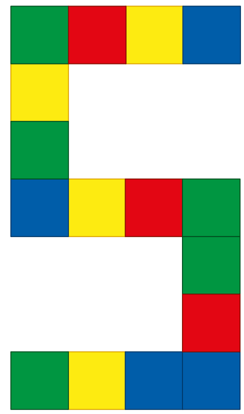
- Have students use Color Tiles to create S-shaped figures with side-length ratios of 1:2, 1:3, and 1:4. Have them find the area and perimeter of each S, make a chart of their findings, and discuss their results.
- Have students use Color Tiles to create a figure shaped like a letter other than S. Have them explore enlargements of their letter, recording data for side lengths, areas, and perimeters.

The S-Shaped Figure

ON THEIR OWN

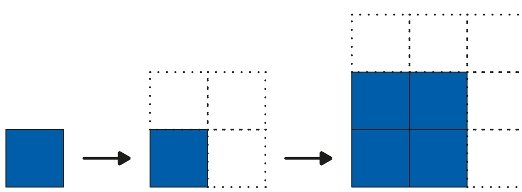
Can you use Color-Tile models of a figure to help you find the perimeter of a larger version of the same figure?

1. Work with a partner. Use Color Tiles to make this S-shaped figure. Find its perimeter (in inches) and its area (in square inches).

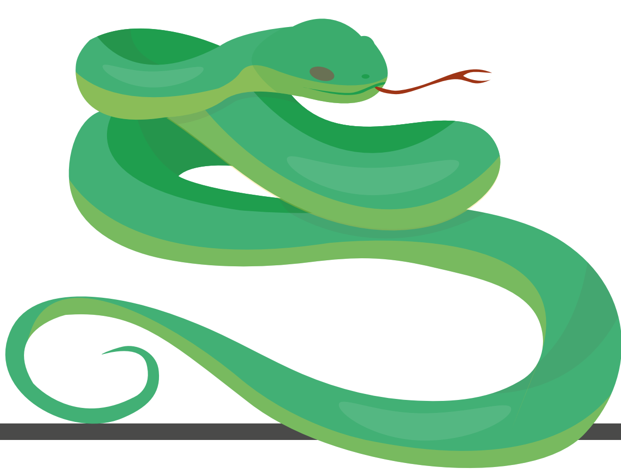


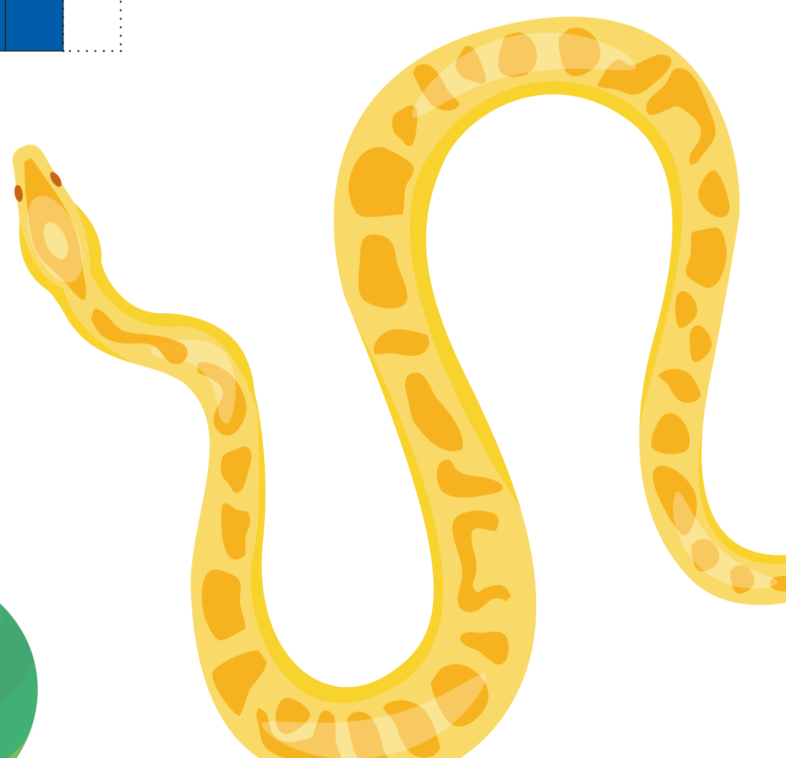
2. Now, consider this to be a a scale model of a bigger S-shaped figure that has an area of 400 square inches. Find the perimeter of the larger figure:

   - Build larger models and look for patterns that will help you answer the question.
   - Remember that as your figures become larger, each Color Tile must grow into a larger square.



3. Use both words and diagrams to tell how you arrived at your solution.





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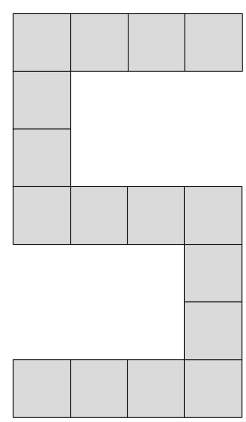

❶ Work with a partner. Use Color Tiles to make this S-shaped figure. Find its perimeter (in inches) and its area (in square inches).

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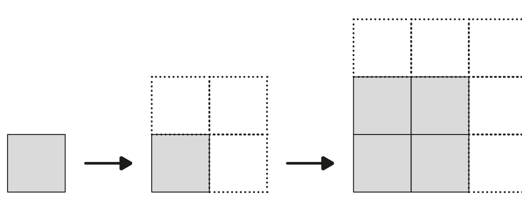

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