Number and Operations–Fractions

Students in third grade develop an understanding of fractions, viewing fractions as being built out of unit fractions. They understand that the size of a fractional part is viewed relative to the size of the whole. Students express fractions as fair sharing, parts of a whole, and parts of a set. They use fractions to represent numbers equal to, less than, and greater than 1. Additionally, they solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators and denominators. They understand a fraction as a number on a number line and represent fractions on a number line diagram. This is the first time students work with a number line for numbers between whole numbers.

Students also explain equivalence of fractions and compare fractions by reasoning about their sizes. At this level, students explore equivalent fractions primarily through the use of visual area models and number lines. They understand two fractions as equivalent (equal) if they are the same size or lie at the same point on a number line.

The Grade 3 Common Core State Standards for Number and Operations–Fractions specify that students should–

- Develop understanding of fractions as numbers.

The following hands-on activities provide students with opportunities to use a variety of models and contexts to develop foundational understanding of fractions. Experiences involving area models and number lines are important to developing number sense. It is also important that students have opportunities to represent problem-solving situations involving fractions in multiple ways, such as building and drawing, and using numbers and objects. Opportunities to model and explain connections among representations will lead to greater mathematical proficiency.

Number and Operations—Fractions

Contents

LESSON 1

Objective

Identify and write fractions.

Common Core State Standards

- **3.NF.1** Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

Number and Operations–Fractions

Identify and Write Fractions

Students have explored fractions in earlier grades and by this time should understand that a fraction shows a part of a whole. Here, students begin to divide that whole into smaller pieces. Identifying and writing fractions in this lesson will lay the foundation for students' work identifying fractions as parts of sets and finding equivalent fractions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What does the bottom number, or denominator, mean in our fraction? What about the top number?*
- **Say:** *We use fractions to show a part or parts of a whole.* **Ask:** *Do you think it is important that all the parts of a fraction—like all 6 parts in the circle—are equal? Why or why not?*
- **Ask:** *What other fractions could you show using the sixths circle? How would you show one whole using sixths fraction pieces?*

Solve It

With students, reread the problem. Have students trace the 6 parts of the circle onto white paper. Then have them label 2 pieces of the circle to show the slices of pizza William ate. Below the picture, they should write $\frac{2}{6}$.

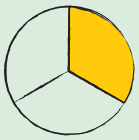
More Ideas

For other ways to teach about identifying and writing fractions—

- Use Pattern Blocks to help build fraction sense. For example, have students select a hexagon and several triangles. Ask students to cover the hexagon with triangles. Then ask students to show $\frac{1}{6}$ of the hexagon using the triangles, then $\frac{2}{6}$, and so on. Repeat with the trapezoid.
- Have students use Deluxe Rainbow Fraction® Circles to create their own fraction models. Have students trace a whole circle, then use smaller pieces to trace within the circle, dividing it into equal parts. Students should then color some of the pieces to show a fraction of their choice.
- Have students use Geoboards to show fractions. Students should create a rectangle or square, then divide it evenly using other rubber bands.

Formative Assessment

Have students try the following problem.



What fraction of the circle is shaded?

A. $\frac{3}{1}$ **B.** $\frac{2}{3}$ **C.** $\frac{1}{2}$ **D.** $\frac{1}{3}$

Try It! 25 minutes | Groups of 4

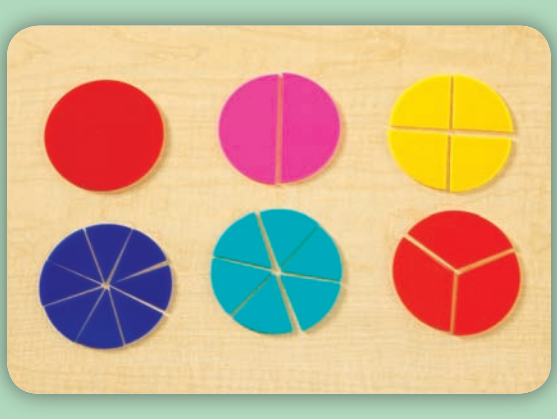
Here is a problem about identifying and writing fractions.

William's class had a pizza party on the last day of school. One of the pizzas was divided into 6 pieces. William ate 2 of the pieces. What fraction of the pizza did William eat?

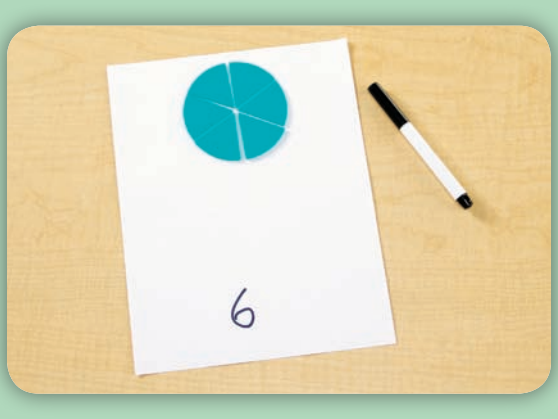
Introduce the problem. Then have students do the activity to solve the problem. Distribute Deluxe Rainbow Fraction Circles, paper, and pencils to groups of students.

Materials

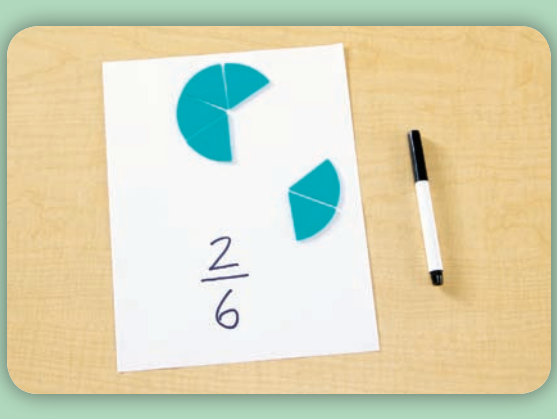
- Deluxe Rainbow Fraction® Circles (1 set per group)
- paper (1 sheet per group)
- pencils (1 per group)



1. Say: *In the problem above, the pizza is divided into 6 pieces. Find the circle that is divided into 6 sections, or "sixths."* Have students look at the various circles and select the one that is divided into 6 sections.



2. Have students count the number of pieces in the fraction and write it on their paper. **Say:** *This number will become the* denominator, *or the bottom number of our fraction.*



3. Ask students to find 2 slices of the sixths circle to show the 2 slices of pizza that William ate. **Say:** *This will become the* numerator, *or the top number of our fraction.* Instruct students to draw a line over the 6 on their paper and write a 2 on top of it.

⚠ Look Out!

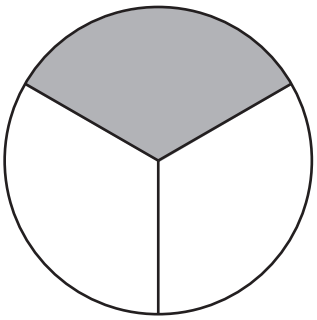
Watch for students who mistakenly think they are "taking away" pieces of the circles. Have students trace the circles onto paper and color 4 of the pieces blue and 2 of the pieces yellow to represent $\frac{2}{6}$.

Answer Key

Use Fraction Circles to model each fraction. Write each fraction.

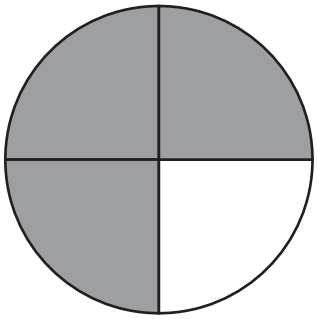
(Check students' work.)

1.



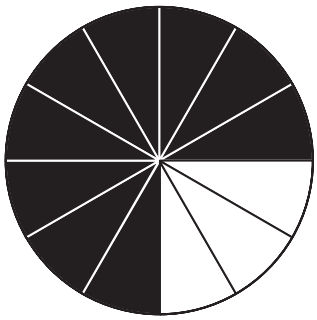
$\frac{1}{3}$

2.



$\frac{3}{4}$

3.

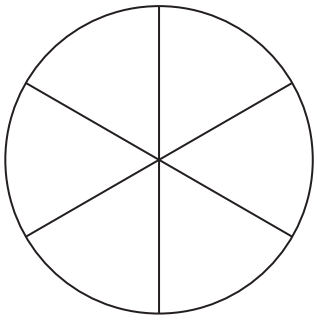


$\frac{9}{12}$

Using Fraction Circles, model each fraction. Shade each circle to represent the fraction. Write the fraction.

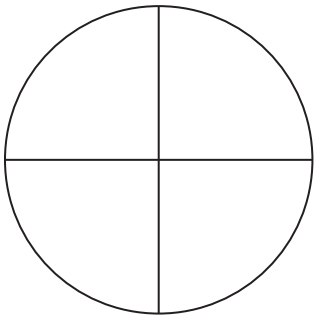
(Check students' models.)

4. 5 sections of $\frac{1}{6}$



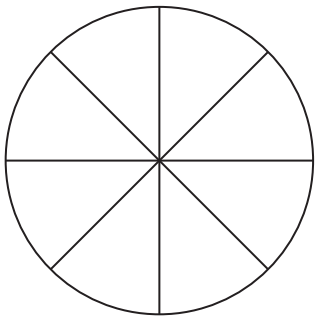
$\frac{5}{6}$

5. 2 sections of $\frac{1}{4}$



$\frac{2}{4}$

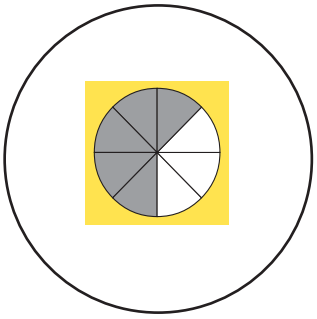
6. 3 sections of $\frac{1}{8}$



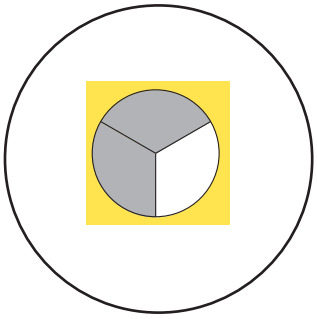
$\frac{3}{8}$

Draw a model for each fraction.

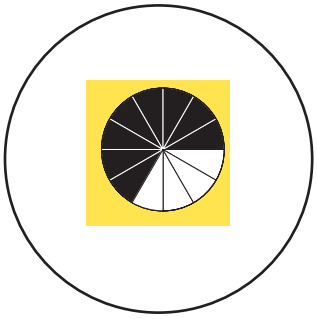
7. $\frac{5}{8}$



8. $\frac{2}{3}$



9. $\frac{8}{12}$



Answer Key

Challenge! Describe how you choose which set of Fraction Circles to use to model $\frac{5}{6}$.

Challenge: (Sample) The number in the denominator needs to match the number of equal sections that make a whole circle.

LESSON 2

Objective

Represent a unit fraction on a number line.

Common Core State Standards

- **3.NF.2a** Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

Number and Operations–Fractions

Fractions on a Number Line

In this lesson, students build upon their previous understanding of a fraction as a part of a whole. They learn that the size of a fractional part $\frac{1}{b}$ is viewed relative to the size of the whole by partitioning the interval from 0 to 1 on a number line into *b* equal parts. Students use Fraction Tower® Cubes with number lines to visualize how unit fractions relate to the size of the whole.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How many fourths are equal to one whole?* Have students use Fraction Towers and the BLM to show that four fourths equal a whole. **Ask:** *What do we call one of the 4 pieces that make a whole?* Write $\frac{1}{4}$ on the board.
- **Say:** *Let's say Melinda and her mom were making 6 bows from one piece of ribbon.* **Ask:** *How many pieces would they divide the ribbon into? What would you call one of the pieces then?* Write $\frac{1}{6}$ and have students use the turquoise Fraction Tower to divide the second number line on the BLM into sixths. Have them identify the first sixth and label the first tick $\frac{1}{6}$.
- **Ask:** *What happens to the size of the parts when we break the whole into more pieces?*

Solve It

With students, reread the problem. Have them use the yellow Fraction Tower to draw a number line from 0 to 1 divided into fourths and label the first tick $\frac{1}{4}$. Have them write a sentence to answer the problem.

More Ideas

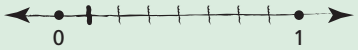
For other ways to teach about unit fractions on a number line—

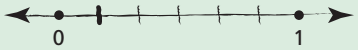
- Have pairs use Fraction Tower Cubes with Fraction Tower Number Lines (BLM 9) to explore other unit fractions. Have students divide and label each number line into thirds, sixths, and eighths. Have them identify one piece of the whole, and what it is called.
- Have students cut Fraction Tower Number Lines (BLM 9) into pieces that show 0 to 1. Have them fold one number line into halves, one into fourths, and one into eighths. Have them identify one piece of the whole, and what it is called.

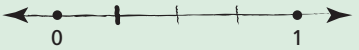
Formative Assessment

Have students try the following problem.

Which shows $\frac{1}{6}$?

A. 

B. 

C. 

D. 

Try It! 30 minutes | Groups of 4

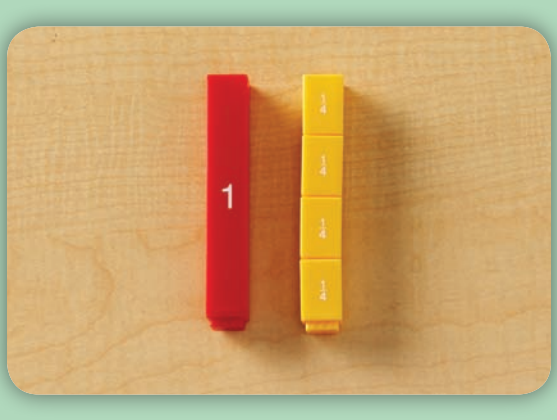
Here is a problem about unit fractions on a number line.

Melinda and her mom want to make 4 bows. They have one piece of ribbon that they want to divide equally to make the 4 bows. How much of the ribbon will each bow get?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Cubes, Fraction Tower Number Lines (BLM 9), and pencils to students.

Materials

- Fraction Tower® Cubes (1 set per group)
- Fraction Tower Number Lines (BLM 9; 1 per group)
- pencils (1 per group)



1. Ask: *How many pieces do Melinda and her mom need to divide the ribbon into? Which tower is divided into 4 pieces?* Have students find the yellow Fraction Tower and count the 4 pieces. Then have them compare the whole yellow tower to the whole red tower to show they are equal.

2. Say: *Lay the yellow tower on the first number line on your paper so that the ends fit between 0 and 1.* Explain that the 4 pieces of the tower make up the whole. Have students draw tick marks to partition the number line into fourths, using the yellow tower as a guide.

3. Have students break the yellow tower into 4 pieces and explain how 1 piece is $\frac{1}{4}$ of the whole. Have them use a piece to show that each interval they drew on the number line is $\frac{1}{4}$. Then have them identify the first fourth and label the first tick mark $\frac{1}{4}$.

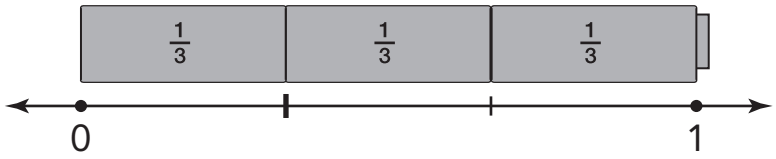
⚠ Look Out!

Help students recognize that each part has the size of $\frac{1}{4}$ piece, and that the endpoint of the first part based at 0 locates the first fourth, or $\frac{1}{4}$, on the number line.

Answer Key

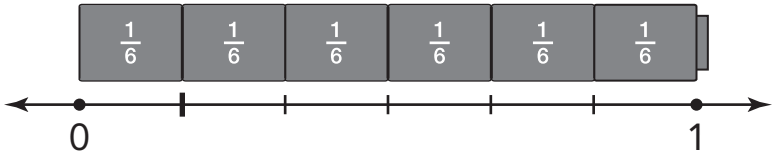
Use Fraction Tower Cubes and a number line to build each model. Circle the first part of the whole. Write the fraction. (Check students' work.)

1. Jason breaks a stick into 3 equal pieces.



first third should be circled, $\frac{1}{3}$

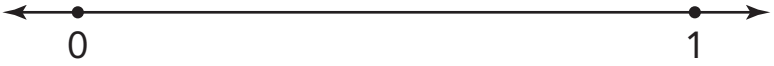
2. Bailie divides a bar of clay into 6 equal pieces.



first sixth should be circled, $\frac{1}{6}$

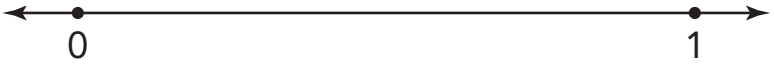
Use Fraction Tower Cubes and a number line to model each fraction. Draw the model. Color the first part of the whole. Mark the fraction on the number line. Write the fraction.

3. A string is cut into 4 equal pieces.



drawing should show fourths, first fourth shaded, first tick marked, $\frac{1}{4}$

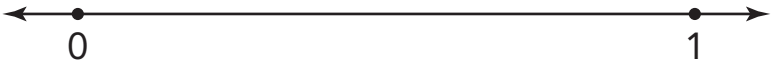
4. A banana is divided into 2 equal pieces.



drawing should show halves, first half shaded, half tick marked, $\frac{1}{2}$

Mark and label the fraction on the number line.

5. $\frac{1}{8}$ $\frac{1}{8}$ should be marked and labeled.



Answer Key

Challenge! Using Fraction Tower Cubes, draw a number line and show a whole divided into 10 equal parts. Color one piece of the whole. Write the fraction.

Challenge: Number line should be divided into 10 reasonably equal pieces with 1 piece colored; $\frac{1}{10}$.

LESSON 3

Objective

Represent a proper fraction on a number line.

Common Core State Standards

- **3.NF.2b** Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths *1*/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

Number and Operations–Fractions

Proper Fractions on a Number Line

Once students develop their understanding of unit fractions, they can progress to other proper fractions. In this lesson, students will begin to count more than one fractional part on a number line. Using manipulatives will make the counting more concrete.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How many thirds are in one whole?* Have students use the Fraction Tower® Cubes and the number line to show that three thirds equal a whole, and have them write $\frac{3}{3}$ under the 1 on the number line. **Ask:** *What do we call 2 of the 3 pieces that make a whole?* Write $\frac{2}{3}$ on the board.
- **Ask:** *What if Margot decided she wanted a shelf longer than $\frac{2}{3}$? What is the next division in thirds?* ($\frac{3}{3}$ or the whole) **Say:** *Look at the other Fraction Towers. Find one that you can use to make a tower longer than $\frac{2}{3}$ but shorter than a whole. Make the tower.* Guide students to find $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, or $\frac{6}{8}$.

Solve It

With students, reread the problem. Have students draw a number line to represent the whole piece of wood and divide it into thirds. Have them count and label $\frac{2}{3}$.

More Ideas

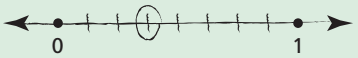
For other ways to teach about proper fractions on a number line—

- Have pairs use Fraction Tower Cubes with Fraction Tower Number Lines (BLM 9) to show fractions they create. Have one student choose a tower other than the one whole and use it to draw ticks on two of the number lines. Have the other student break the chosen tower into two parts. Have students mark and label each of the number lines with one of the fractional parts.
- Have students reach into a bag filled with Fraction Tower Cubes and grab 2 handfuls of cubes. Have them match up all of the like pieces and draw each resulting fraction on a number line. Have them label the fractions.

Formative Assessment

Have students try the following problem.

Which fraction is circled?



A. $\frac{3}{8}$ B. $\frac{5}{8}$ C. $\frac{4}{6}$ D. $\frac{3}{4}$

Try It! 30 minutes | Groups of 4

Here is a problem about proper fractions on a number line.

Margot is making a shelf from a board. She wants the shelf to be $\frac{2}{3}$ the length of the wood. Where should she cut the wood?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Cubes, Fraction Tower Number Lines (BLM 9), and pencils to students.

Materials

- Fraction Tower® Cubes (1 set per group)
- Fraction Tower Number Lines (BLM 9; 1 per group)
- pencils (1 per group)

1. Have students find the red whole Fraction Tower. **Say:** *Let's say this is Margot's piece of wood. She only wants to use $\frac{2}{3}$ of it.* **Ask:** *Which Fraction Tower can you use to show thirds?* Guide students to find the orange Fraction Tower. Have students put the two towers side by side to see that they are equal.

2. Say: *Lay the orange tower on the first number line on your paper so that the ends fit between 0 and 1.* Explain that the 3 pieces of the tower make up the whole and that 0 to 1 on the number line is the whole. Have students draw tick marks to partition the number line into thirds, using the orange tower as a guide.

3. Have students break the orange tower into 3 pieces, and discuss that 1 piece is $\frac{1}{3}$ of the whole. Have them lay one piece on the first section of the number line and label the first tick $\frac{1}{3}$. Then have them add a second piece to the tower. **Ask:** *How many thirds are there now?* Have students lay the tower on the number line and label the second tick $\frac{2}{3}$.

⚠ Look Out!

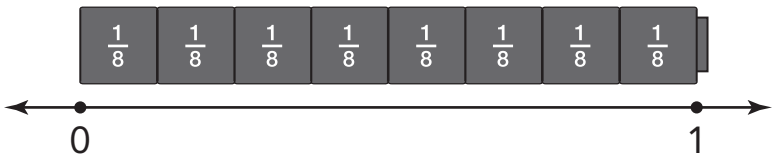
Make sure students count and label ticks from left to right so they can see the progression of the fractions from 0 to 1.

Answer Key

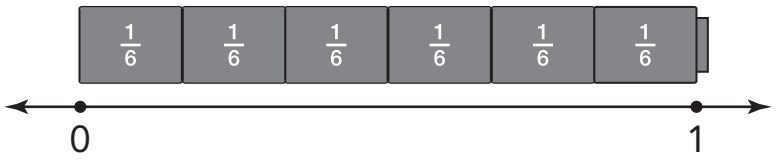
Use Fraction Towers and a number line to build each model. Mark and label the number line. Circle the fraction on the number line.

(Check students' work.)

1. $\frac{5}{8}$ number line divided into eighths with the $\frac{5}{8}$ label circled



2. $\frac{2}{6}$ number line divided into sixths with the $\frac{2}{6}$ label circled



Use Fraction Tower Cubes and a number line to model each fraction. Draw the model. Mark and label the number line. Circle the fraction on the number line.

3. $\frac{4}{5}$ model of fifths, number line divided into fifths with the $\frac{4}{5}$ label circled

4. $\frac{3}{4}$ model of fourths, number line divided into fourths with the $\frac{3}{4}$ label circled

Mark and label the number line. Circle the fraction.

5. $\frac{7}{10}$ number line divided into tenths with the $\frac{7}{10}$ label circled

Answer Key

Challenge! Using Fraction Tower Cubes, draw a number line and show a whole divided into 8 equal parts. Label the number line. Color $\frac{7}{8}$ of the whole. Write the fraction.

Challenge: Number line should be divided into 8 reasonably equal pieces with the first 7 pieces colored; $\frac{7}{8}$.

LESSON 4

Number and Operations–Fractions

Model Equivalent Fractions

When students look at equivalent fractions in written form, such as $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, it is hard for them to understand that they are looking at the same fraction. When students divide an object several ways, they can see that the overall size of the object does not change—it just has a different number of equal-sized units.

Objective

Identify equivalent fractions using models.

Common Core State Standards

- **3.NF.3a** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- **3.NF.3b** Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What fraction was left when you took away 3 pieces from the Deluxe Rainbow Fraction® Circle made of 6 equal parts?*
- Explain to students that the fractions they made that cover the same part of the circle are called *equivalent fractions.* **Ask:** *What were some equivalent fractions for* $\frac{3}{6}$*?* Make sure students are able to list $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$. **Say:** Equivalent *means same or equal.* **Ask:** *How do you know that the fractions you found are equivalent?*
- **Ask:** *What fraction did you have when you removed 1 piece of the circle made of 3 equal parts? What equivalent fractions did you find for* $\frac{2}{3}$*?*
- **Ask:** *Can you think of a situation in which you might want to know fractions that are equivalent to one another?*

Solve It

With students, reread the problem. Have students trace the circle pieces to show the different equivalents for $\frac{3}{6}$. Ask them to label each drawing with the fraction shown, drawing an equal sign between each equivalent fraction.

More Ideas

For other ways to teach about equivalent fractions—

- Have students use Fraction Tower® Cubes to make equivalent fractions. Give students a problem such as: *Sue had a granola bar. She divided it into 5 equal parts and ate 2 of them. Use cubes to show the fraction of granola bar that was left. Then find 1 equivalent fraction.*
- Have students work in groups using Geoboards to find other ways to make equivalent fractions. Ask students to show a fraction on the Geoboard, and then ask them to find an equivalent fraction.

Formative Assessment

Have students try the following problem.

Mrs. Daniel cut a pizza into 8 slices. The students ate 4 slices. The fractional part of the remaining pizza is $\frac{4}{8}$*. Which fraction below means the same as* $\frac{4}{8}$*?*

A. $\frac{2}{8}$ **B.** $\frac{1}{2}$ **C.** $\frac{4}{4}$ **D.** $\frac{8}{4}$

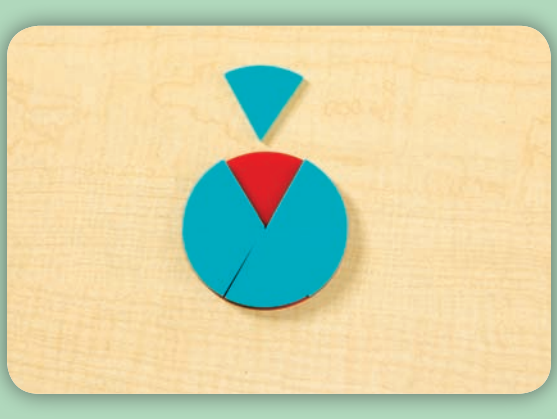
Try It! 25 minutes | Groups of 4

Here is a problem about equivalent fractions.

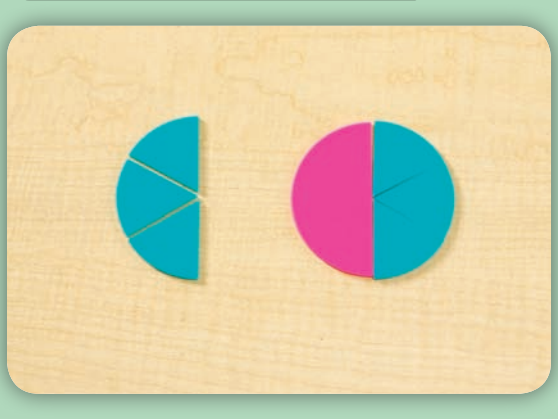
It is Darnell's birthday, so his mother brought a birthday cake to his after-school class for him to share with his friends. The cake was cut into 6 equal slices. If Darnell and his friends ate 3 of the 6 slices, what fraction of the cake was left over?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Deluxe Rainbow Fraction Circles to students.

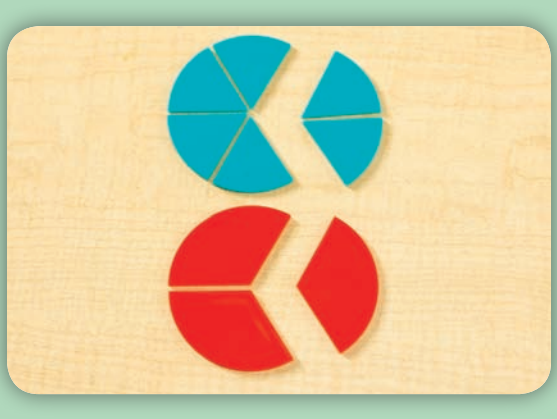
Materials
- Deluxe Rainbow Fraction® Circles (1 set per group)



1. Have students assemble all of the circles in the set and explore how each circle is divided into different numbers and sizes of pieces. **Say:** *Find the fraction pieces that make a circle out of 6 equal parts.* Make sure students use sixths to make a circle. Explain that the 6 pieces match the 6 equal pieces of the cake in the problem, and that combined, the 6 equal pieces make up 1 whole.



2. Say: *Let's take away the 3 pieces eaten by the kids.* **Ask:** *What is the fraction of the circle that is left? Can we use any other fractional parts to cover the $\frac{3}{6}$ that is left to make another fraction that means the same thing?* Students should find the equivalent fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$.



3. Have students create other equivalent fractions using circles. **Say:** *A pie is divided into 3 slices. One slice is removed.* **Ask:** *What fraction of the pie is left?* Then have students build models to show $\frac{2}{3}$ and then $\frac{4}{6}$.

⚠ Look Out!

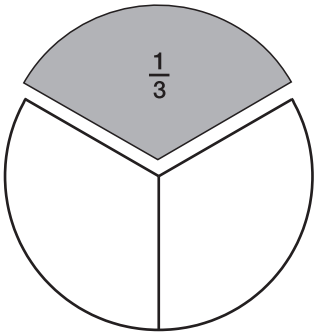
Stress that when finding equivalent fractions, students need to use the same size of the fractional parts. Watch for students who try to put together $\frac{1}{3}$ and $\frac{1}{6}$ to show $\frac{1}{2}$. Although these two fractions added together equal $\frac{1}{2}$, they are not creating an equivalent fraction for $\frac{1}{2}$. Stress the one-to-one correspondence of equivalent fractions: $\frac{1}{2} = \frac{3}{6}$. Although $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ is true, it is an addition sentence, not a set of equivalent fractions.

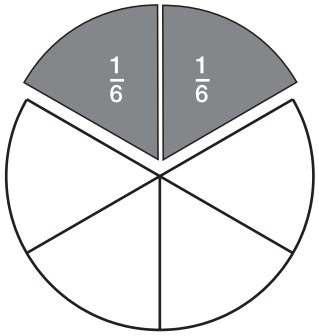
Answer Key

Use Fraction Circles to model each fraction. Write equivalent fractions for the shaded parts. Write equivalent fractions for the unshaded parts.

(Check students' work.)

1.

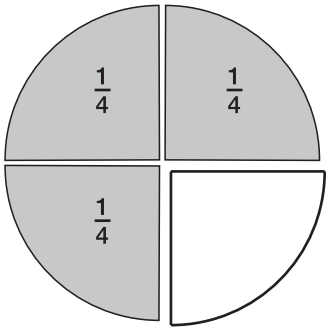


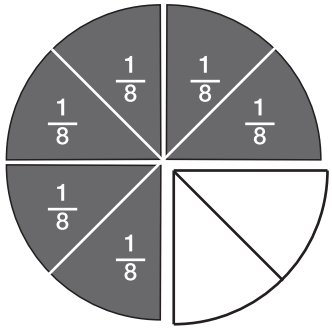


$\frac{1}{3} = \frac{2}{6}$

$\frac{2}{3} = \frac{4}{6}$

2.





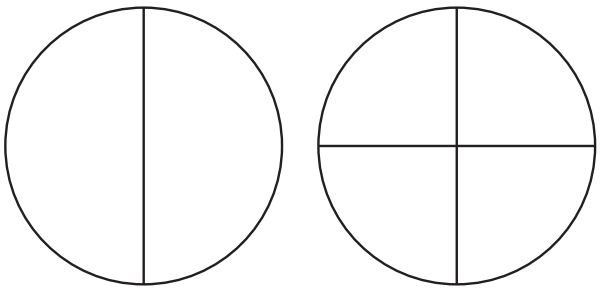
$\frac{3}{4} = \frac{6}{8}$

$\frac{1}{4} = \frac{2}{8}$

Using Fraction Circles, model the fraction. Then sketch a model or an equivalent fraction. Write the equivalent fraction.

3. $\frac{1}{2} = \frac{2}{4}$

(Check students' models.)



Write an equivalent fraction for each fraction.

4. $\frac{2}{3} = \frac{4}{6}$

5. $\frac{3}{4} = \frac{6}{8}$

6. $\frac{2}{8} = \frac{1}{4}$

7. $\frac{2}{4} = \frac{1}{2}$

8. $\frac{2}{6} = \frac{4}{12}$

9. $\frac{6}{8} = \frac{9}{12}$

Answer Key

Challenge! Name another fraction equivalent to the fractions in Problem 9. Explain how you know that it is equivalent.

Challenge: (Sample) $\frac{3}{4}$ is equivalent to both the fractions in Problem 9. Three fourths parts are the same size as six eighths parts and nine twelfths parts.

LESSON 5

Objective

Identify equivalent fractions using a number line model.

Common Core State Standards

- **3.NF.3a** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

Number and Operations–Fractions

Equivalent Fractions on a Number Line

As students develop a deeper understanding of fractions, they will see that there are different fractions that are the same point on a number line. Equivalent fractions need to be explored visually at first, so students can see the relationship. Exploring equivalent fractions on a number line prepares students to use operations to find equivalencies.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Draw a number line from 0 to 1 on the board. **Ask:** *How many parts should we divide the space between 0 and 1 into to show Adam's sandwich?* Have a student show halves on the number line. **Ask:** *How many parts should we divide the space between 0 and 1 into to show Maya's sandwich?* Have a student show fourths on the number line.
- **Say:** *Look at the space between 0 and* $\frac{1}{2}$. **Ask:** *How many fourths are in that space?* Write $\frac{1}{2} = \frac{2}{4}$ under the number line.
- **Ask:** *How can we see on the Fraction Number Line, and with your Fraction Towers, that* $\frac{1}{2}$ *and* $\frac{2}{4}$ *are equal?* Guide discussion to the fact that both fractions take up all the space between 0 and $\frac{1}{2}$.

Solve It

With students, reread the problem. Have students draw a number line and show that $\frac{2}{4}$ is the same point as $\frac{1}{2}$.

More Ideas

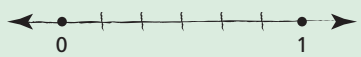
For other ways to teach about equivalent fractions—

- Have students use Fraction Tower® Cubes and Fraction Tower Number Lines (BLM 9) to find various equivalent fractions. Have students mark the equivalent fractions on a number line.
- Have students cut individual number lines from Fraction Tower Number Lines (BLM 9) and use them to find equivalent fractions. First have students fold one in half and mark the $\frac{1}{2}$ point. Then students can fold in half again and again, and unfold to see how many smaller parts are equal to one half. Have students write the fractions they discover.

Formative Assessment

Have students try the following problem.

Which statement is true?



A. $\frac{1}{3} = \frac{1}{6}$ B. $\frac{1}{3} = \frac{2}{6}$ C. $\frac{1}{3} = \frac{3}{6}$ D. $\frac{1}{3} = \frac{4}{6}$

Try It! 20 minutes | Groups of 4

Here is a problem about equivalent fractions.

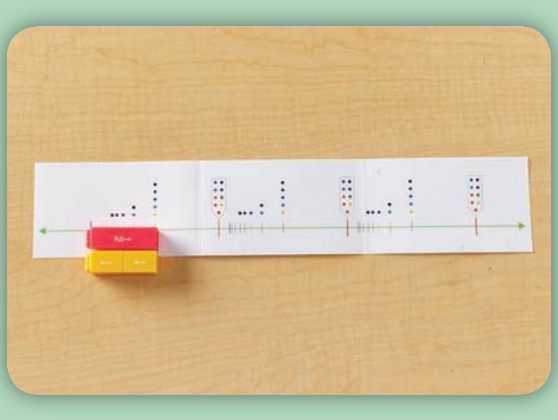
Adam and Maya are having lunch. Adam's sandwich is cut into 2 pieces, and Maya's sandwich is cut into 4 pieces. They want to trade half of their sandwiches. How many pieces does Adam give Maya? How many pieces does Maya give Adam?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Cubes and Fraction Number Lines to students.

Materials
- Fraction Tower® Cubes (1 set per group)
- Fraction Number Line (2 per group)

1. Have one pair of students model Adam's sandwich and another pair model Maya's using Fraction Towers. **Ask:** *Which tower shows halves? Which tower shows fourths?* Make sure students use the correct cubes.

2. Say: *Separate your towers into halves. Once you have two halves, trade one with your partners. Now compare the size of the tower you have left with the one you received.* **Ask:** *Are the towers the same size?* Discuss that one of the $\frac{1}{2}$ pieces is the same as two of the $\frac{1}{4}$ pieces.



3. Have students lay their towers below a Fraction Number Line with the colored dots, making sure to align the left ends of the towers with zero on the number line. **Ask:** *Are $\frac{1}{2}$ and $\frac{2}{4}$ equal? What do you think the colored dots above the one-half tick indicate? What other cubes could you use to build $\frac{1}{2}$?*

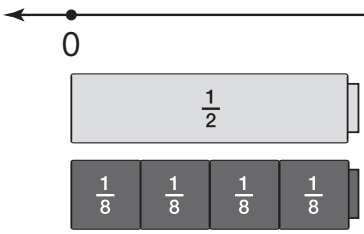
⚠ Look Out!

If students find the colored dots and lines on the Fraction Number Line confusing, use Fraction Tower Number Lines (BLM 9). Have them mark $\frac{1}{2}$ on a number line using a pink tower cube and then mark $\frac{2}{4}$ on the same number line using two yellow cubes.

Answer Key

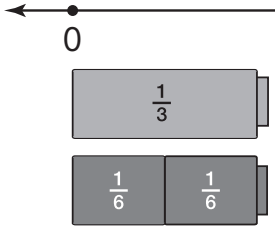
Use Fraction Tower Cubes and the Fraction Number Line to build the model. Write the equivalent fraction. (Check students' work.)

1. How many eighths are in $\frac{1}{2}$?



$\frac{1}{2} = \frac{4}{8}$

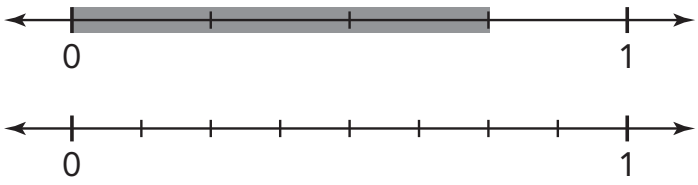
2. How many sixths are in $\frac{1}{3}$?



$\frac{1}{3} = \frac{2}{6}$

Look at each number line. Color and mark an equivalent fraction. Write the fractions. (Check students' work.)

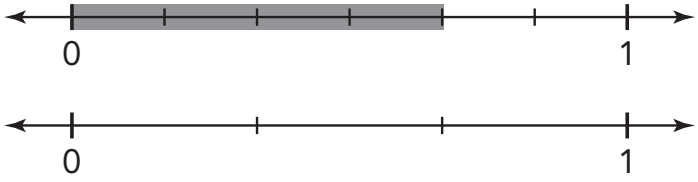
3.



$\frac{6}{8}$ colored

$\frac{3}{4} = \frac{6}{8}$

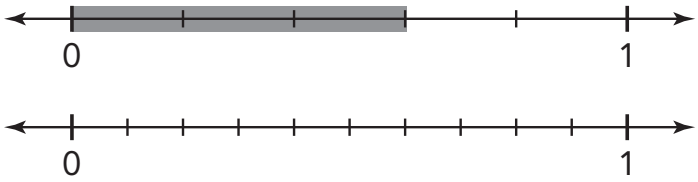
4.



$\frac{2}{3}$ colored

$\frac{4}{6} = \frac{2}{3}$

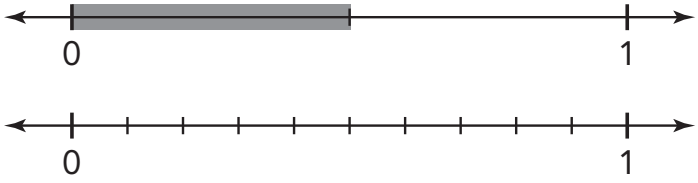
5.



$\frac{6}{10}$ colored

$\frac{3}{5} = \frac{6}{10}$

6.



$\frac{5}{10}$ colored

$\frac{1}{2} = \frac{5}{10}$

Answer Key

Challenge! Use Fraction Tower Cubes to draw and label two number lines. Then color and write two fractions equal to $\frac{1}{2}$.

$\frac{1}{2}$ = _____ = _____

Challenge: Number lines could show $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, or $\frac{6}{12}$.

LESSON 6

Objective

Express whole numbers as fractions.

Common Core State Standards

- **3.NF.3c** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

Number and Operations–Fractions

Whole Numbers as Fractions

In previous lessons, students have been developing their understanding of unit and proper fractions between 0 and 1. Now they will expand on that knowledge to understand whole numbers represented as fractions. These skills will build the foundation for students to progress to creating fractions that represent parts of a group.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Write $\frac{4}{4}$ on the board. **Say:** *Let's read the fraction and understand what it means. This fraction shows Aeron used 4 out of the 4 parts of the apple, which is the same as saying he used the whole apple.*
- **Say:** *When a fraction has the same numerator and denominator, it means the same as 1. Even if the fraction is $\frac{100}{100}$, it still means 1.*
- **Say:** *Any whole number can be written as a fraction by putting the whole number in the numerator and 1 in the denominator. The 1 in the denominator means the whole is divided into 1 part. The numerator tells the number of parts being used.*

Solve It

With students, reread the problem. Have students draw the number line model of the 3 apples and write the fraction. Have them draw a number line from 0 to 1 divided into fourths and write the fraction for the whole apple cut into fourths. Have students write a few sentences explaining $3 = \frac{3}{1}$ and $\frac{4}{4} = 1$.

More Ideas

For other ways to teach about expressing whole numbers as fractions—

- Have students use Fraction Tower® Cubes and the Fraction Number Line to express a whole using different fractions. Have students place each Fraction Tower on the number line, mark the parts from 0 to 1, and write the fraction that expresses the whole.
- Have pairs work with Deluxe Rainbow Fraction® Circles. Have one student create a story about several whole items or one whole being divided into parts. Have the other student show the story using Fraction Circles and write the fraction that represents the story.

Formative Assessment

Have students try the following problem.

Crosby cut a cake into 12 pieces. He had 11 friends at his party. Everyone had a piece of cake. What fraction shows how much cake was eaten?

A. $\frac{1}{1}$ **B.** $\frac{1}{12}$ **C.** $\frac{11}{12}$ **D.** $\frac{12}{12}$

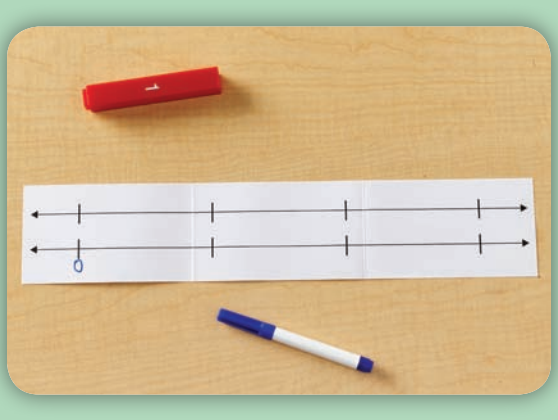
Try It! 30 minutes | Groups of 4

Here is a problem about expressing whole numbers as fractions.

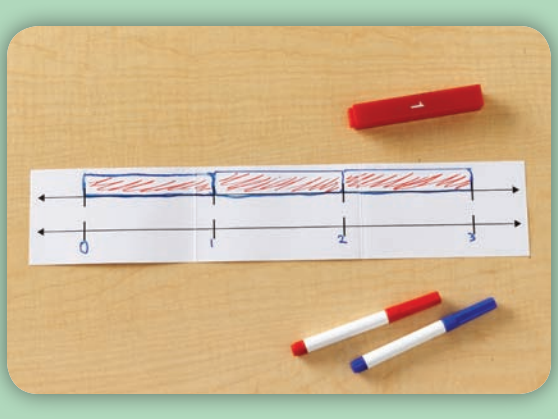
Aeron buys a bag of 3 apples. He wants to use one apple to feed his 4 lizards. What fraction can we use to show the bag of apples? What fraction shows he cut one apple into 4 pieces to feed his lizards?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Cubes, Fraction Number Lines, and markers to students.

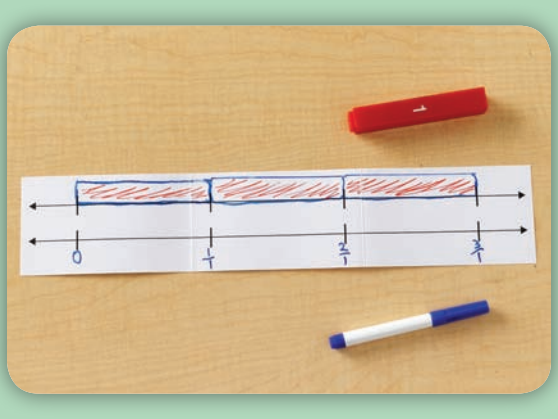
Materials
- Fraction Tower® Cubes (1 set per group)
- Fraction Number Line (1 per group)
- dry erase markers (1 set per group)



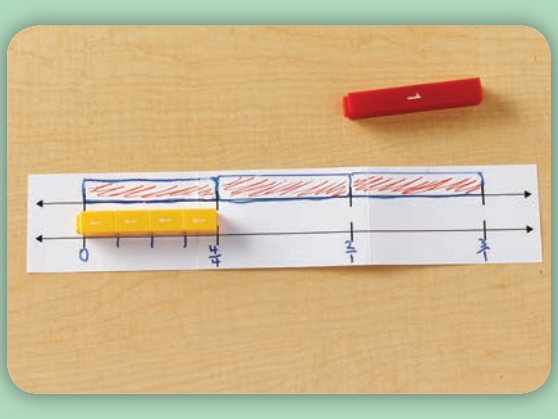
1. Say: *Let's represent the 3 apples.* Have students find the red Fraction Tower. Ask them to look at the number line cards and find the double number line. Have them label the leftmost tick 0 using a dry erase marker. **Ask:** *What does the red tower represent?* Elicit that it represents one whole apple.



2. Have students lay the red tower on the first interval of the top number line. **Say:** *The right end of the tower marks 1 apple. Label the tick mark and trace the tower.* Have students trace the tower onto the second interval, label the tick for the second apple, and do the same for the third apple.



3. Say: *The denominator of a fraction tells how many parts the whole is divided into.* **Ask:** *How many parts is the red tower divided into?* **Say:** *Just one. And the numerator tells the number of parts we are using.* **Ask:** *How many of the one whole part do we use for 1 apple? 2 apples? 3 apples?* Guide students to change the tick labels to $\frac{1}{1}$, $\frac{2}{1}$, and $\frac{3}{1}$. Discuss.

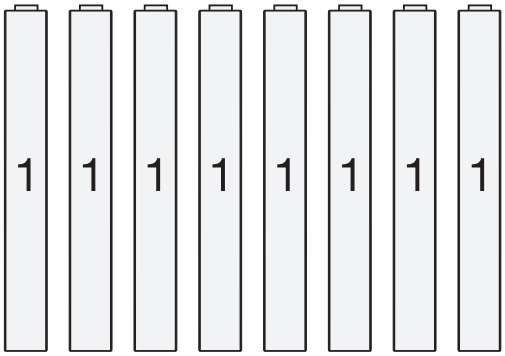


4. Say: *Aeron cut one apple into four parts for his lizards.* Have students lay a yellow fraction tower on the first interval of the bottom number line and mark the fourths. **Ask:** *How many parts does the tower represent?* **Say:** *Four parts for the four pieces of the apple.* **Ask:** *How many of the parts is Aeron using?* **Say:** *All four. Write $\frac{4}{4}$ under $\frac{1}{1}$.*

Answer Key

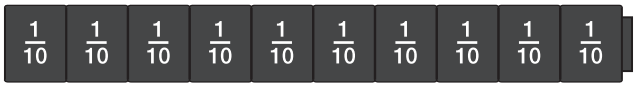
Use Fraction Tower Cubes and sketch paper to model each fraction. Then write the fraction. (Check students' work.)

1. Victor has 8 logs for the fireplace.



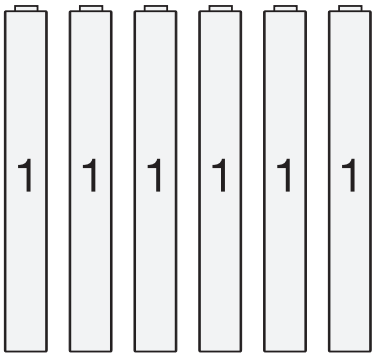
Fraction: $\frac{8}{1}$

2. Monica cut a loaf of bread into 10 pieces.



Fraction: $\frac{10}{10}$

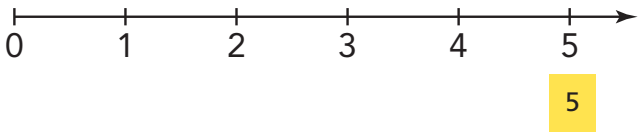
3. Jabar has 6 pencils for school.



Fraction: $\frac{6}{1}$

Show where the fraction belongs on the number line.

4. Amber had $\frac{5}{1}$ bananas in a basket.



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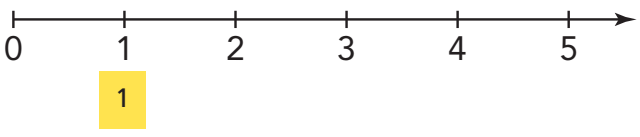
5. Kahlil used $\frac{2}{2}$ of the ribbon to wrap gifts.



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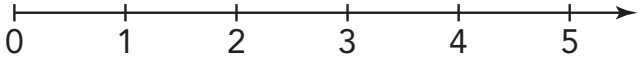
6. Ethan used $\frac{3}{3}$ of the string.



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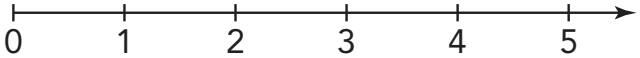
7. Cali used $\frac{3}{1}$ boxes to pack gifts.



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Answer Key

Challenge! Mark had a box of 4 pizzas for his party. Each pizza was cut into 8 pieces. After the party, all of the pizza was gone. His mom said $\frac{32}{32}$ of the pizza was eaten. Is she right? Use drawings to show if she is right or wrong. Explain.

Challenge: (Sample) Yes, she is correct. The students can show 4 pizzas each divided into 8 pieces, which gives 32 pieces of pizza. So if all of the pizzas together are 32 pieces, and all 32 pieces were eaten, the whole box of pizzas was eaten.

LESSON 7

Objective

Compare fractions with the same numerator or denominator.

Common Core State Standards

- **3.NF.3d** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Number and Operations–Fractions

Comparing Fractions

In previous lessons, students learned that the size of a fractional part is viewed relative to the size of the whole. Students can now compare fractions that have either the same numerator or same denominator to determine their relative sizes. In this lesson, students create fractional parts with manipulatives and compare them using <, =, or > symbols.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What if Trey had decided he would cut his orange into fourths and eat 3 pieces? Then who would eat more?* Have students show a Deluxe Rainbow Fraction® Circle that is divided into fourths and identify 3 fourths.
- **Say:** *Trey will eat $\frac{3}{4}$ and Alauna will eat $\frac{3}{8}$.* Write $\frac{3}{4}$ and $\frac{3}{8}$ on the board. **Ask:** *How can we compare these, when the numerators are the same, but the denominators are different?* Guide students to compare the size of the parts by placing one group on another to see which is greater.
- **Say:** *When something is broken into more parts, each part will be smaller.* **Ask:** *If one pie is cut into 6 pieces and another pie is cut into 8 pieces, which pie will have bigger pieces?*

Solve It

With students, reread the problem. Have students draw and write $\frac{3}{8}$ and $\frac{5}{8}$. Then have students write < or > to show which is greater.

More Ideas

For other ways to teach about comparing fractions—

- Have pairs use Fraction Tower® Cubes to model various fractions with the same numerators or denominators and determine whether they are greater than, less than, or equal to one another. Encourage students to discuss and justify their comparisons.
- Have students use Cuisenaire® Rods to model the following fractions: $\frac{2}{5}$ and $\frac{4}{5}$; $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{4}{6}$ and $\frac{2}{3}$. Have students write the fractions, compare the models, and use <, >, or = to compare the fractions.

Formative Assessment

Have students try the following problem.

Which is true?

A. $\frac{1}{6} > \frac{1}{10}$ **B.** $\frac{2}{9} > \frac{2}{3}$ **C.** $\frac{4}{8} > \frac{7}{8}$ **D.** $\frac{1}{4} > \frac{3}{4}$

Try It! 20 minutes | Groups of 4

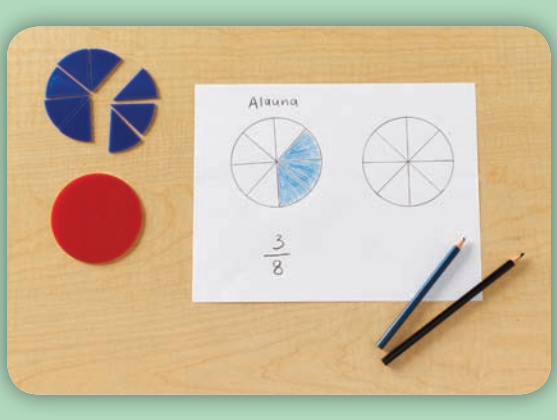
Here is a problem about comparing fractions.

Alauna and Trey each cut their orange into 8 equal parts. Alauna says she will eat $\frac{3}{8}$ of her orange. Trey says he will eat $\frac{5}{8}$ of his orange. Who will eat more of their orange?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Deluxe Rainbow Fraction Circles, paper, and pencils to students.

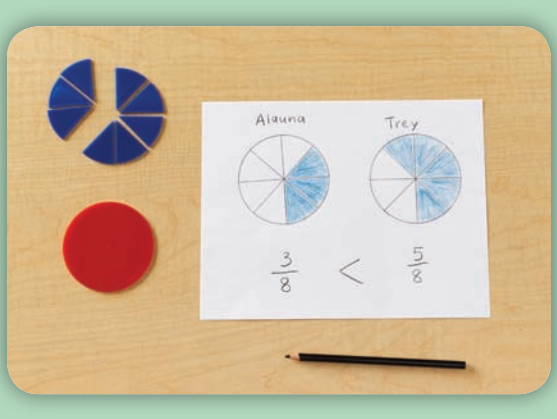
Materials

- Deluxe Rainbow Fraction® Circles (1 set per group)
- paper (1 sheet per group)
- pencils (1 per group)
- colored pencils (1 set per group)



1. Have students find the eighths Fraction Circle and the whole circle. Ask them to trace 2 whole circles side by side onto their paper, write "Alauna" above the first circle, and divide the first circle into eighths. **Say:** *Let's show how much Alauna will eat.* Have students color 3 sections of the first circle and write $\frac{3}{8}$ under the circle.

2. Have students write "Trey" above the second circle and divide it into eighths. **Say:** *Let's show how much Trey will eat.* Have students color 5 sections of the second circle and write $\frac{5}{8}$ under the circle. Have students separate pieces from the eighths Fraction Circle to help them confirm that they have shaded their circles correctly.



3. Ask: *Who will eat more of their orange?* Elicit from students that both Alauna and Trey are going to eat some number of eighths and that the numerators tell how many eighths each will eat. Have students write < or > between the fractions to show which is greater.

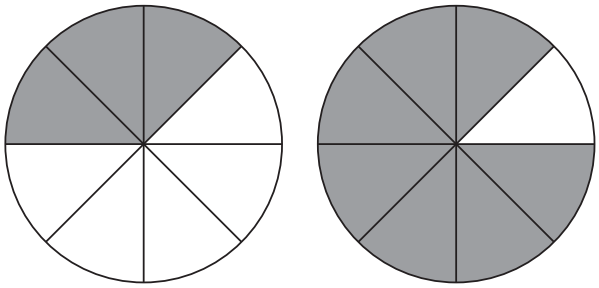
⚠ Look Out!

Watch for students who just compare numerators without making note that the denominators are the same. Have students make the models every time to be able to *see* the fractions.

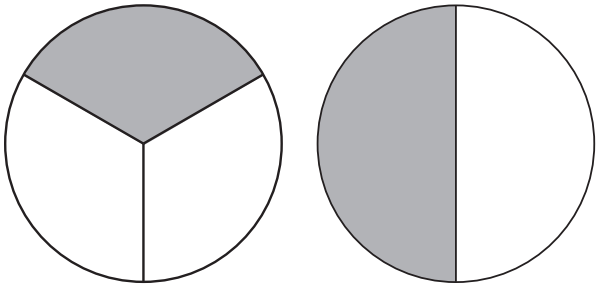
Answer Key

Use Fraction Circles to model each fraction. Compare the fractions. Write <, >, or = to compare. (Check students' work.)

1. $\frac{3}{8}$ < $\frac{7}{8}$

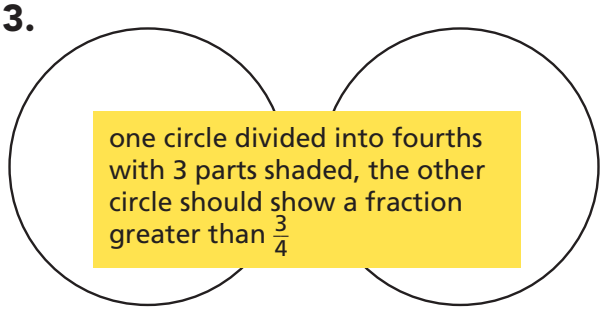


2. $\frac{1}{3}$ < $\frac{1}{2}$



Using Fraction Circles, model the fraction. Draw the model. Build and draw a second fraction that makes the number sentence true. Complete the number sentence.

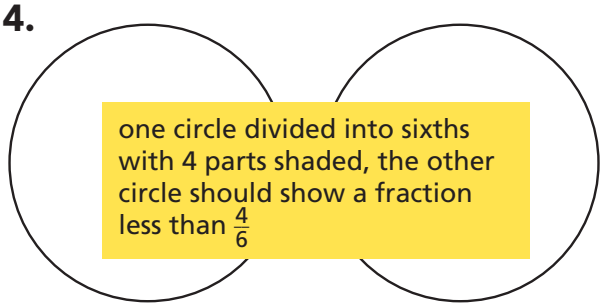
3.



one circle divided into fourths with 3 parts shaded, the other circle should show a fraction greater than $\frac{3}{4}$

$\frac{3}{4}$ < _____

4.



one circle divided into sixths with 4 parts shaded, the other circle should show a fraction less than $\frac{4}{6}$

$\frac{4}{6}$ > _____

Write <, >, or = in each circle to compare.

5. $\frac{1}{2}$ > $\frac{1}{4}$

6. $\frac{2}{4}$ > $\frac{2}{6}$

7. $\frac{3}{5}$ < $\frac{3}{4}$

8. $\frac{5}{8}$ < $\frac{6}{8}$

9. $\frac{2}{3}$ > $\frac{2}{6}$

10. $\frac{5}{10}$ < $\frac{5}{6}$

Answer Key

Challenge! When comparing fractions, why is it important that you compare fractions of the same whole? Is $\frac{3}{4}$ of an orange greater than $\frac{1}{2}$ of a watermelon?

Challenge: (Sample) The wholes could be different sizes, such as an orange and a watermelon. Since a watermelon is much bigger than an orange, $\frac{1}{2}$ of it would be greater than $\frac{3}{4}$ of an orange.