LESSON

7

Theoretical and Experimental Probability with Dice

Objective

Find the theoretical and experimental probabilities of an event involving dice.

Common Core State Standards

- **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- **7.SP.8a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.8b** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Students' experiences with determining theoretical probability continue with this activity in which they play a game involving dice. The students then gather experimental data by playing the game and comparing the results with the mathematically determined theoretical probabilities.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Based on your experiment, what is* P*(multiple of 3) and* P*(multiple of 4)? How do these values compare with the theoretical probabilities?* Have students share their results.

Solve It

Reread the problem with students. Discuss the differences between the experimental and theoretical probabilities. Ask students what they would do to obtain experimental probabilities closer to the theoretical probabilities. Elicit that they could increase the number of rolls.

More Ideas

For another way to teach about theoretical and experimental probabilities—

- Have students complete a similar activity using the 4-sided die and the 8-sided die. This time have them determine *P*(sum of 4) and *P*(sum of 5). Tell students to find the experimental probability by rolling the dice 32 times and the theoretical probability using a table of values.

Formative Assessment

Have students try the following problem.

A green 6-sided number cube and a blue 6-sided number cube are rolled, and the results are added together. All of the possible sums are listed in the table.

Which of the following probabilities is correct?

A. *P*(multiple of 10) = $\frac{1}{4}$

B. *P*(odd number) = $\frac{1}{2}$

C. *P*(even number) = $\frac{1}{36}$

D. *P*(greater than 5) = $\frac{1}{4}$

Green

Blue

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Try It!

25 minutes | Pairs

Here is a problem about theoretical probability vs. experimental probability.

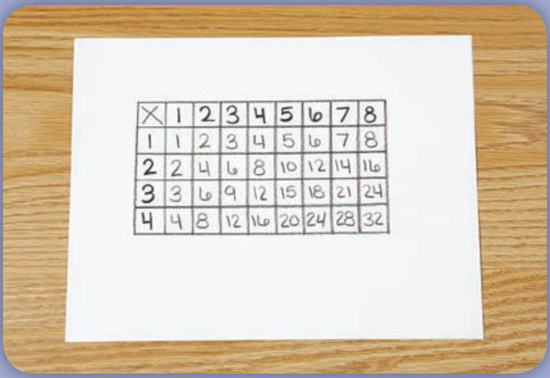
Matt and Jana are playing a game. Matt rolls a 4-sided die, and Jana rolls an 8-sided die. If the product of the rolls is a multiple of 3, Matt gets a point. If the product of the rolls is a multiple of 4, Jana gets a point. A game consists of 32 rolls of the dice. Who has a better chance of winning?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

- Polyhedral Dice Set (4-sided die and 8-sided die)

1. Have students take the roles of Matt and Jana and play a round (32 rolls of the dice) of the game. Students should record the product for each of the rolls. **Ask:** *Who won the game—Matt or Jana?*

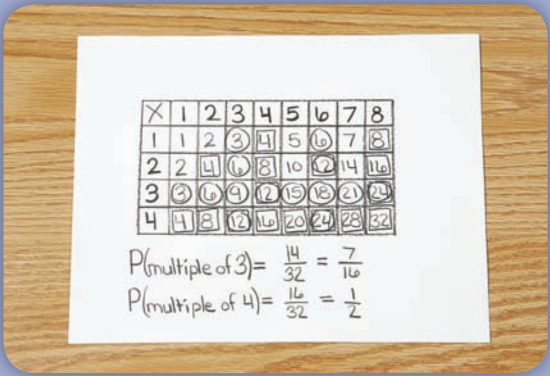


×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32

2. Say: *Now determine the theoretical probability.* Have students set up a table of values and fill in the products.

×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32

3. Have students mark the multiples of 3 and the multiples of 4. **Ask:** *How many values in the table are multiples of 3? How many are multiples of 4?* Elicit from students that there are fourteen multiples of 3 and sixteen multiples of 4.



×	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32

$P(\text{multiple of } 3) = \frac{14}{32} = \frac{7}{16}$

$P(\text{multiple of } 4) = \frac{16}{32} = \frac{1}{2}$

4. Have students then determine *P*(multiple of 4) and *P*(multiple of 3) using their table of values. **Ask:** *Who has a better chance of winning a round of this game?*

Use a 4-sided die and a 6-sided die to make a table of products when the dice are rolled. Use the table to find each probability.

(Check students' work.)

1.

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24

P(multiple of 6) $\frac{1}{3}$

P(multiple of 4) $\frac{11}{24}$

P(even product) $\frac{3}{4}$

P(multiple of 10) $\frac{1}{12}$

Using Polyhedral Dice, make a table to find each probability.

2. two 4-sided dice

Check students' tables.

P(multiple of 3) $\frac{7}{16}$

P(product < 15) $\frac{15}{16}$

P(product that is a prime number) $\frac{1}{4}$

P(multiple of 8) $\frac{3}{16}$

3. 6-sided die and 10-sided die

Check students' tables.

P(odd product) $\frac{1}{4}$

P(product < 10) $\frac{1}{3}$

P(product > 40) $\frac{1}{10}$

P(multiple of 5) $\frac{1}{3}$

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Answer Key

Challenge! An experiment has you roll an 8-sided die and a 12-sided die and multiply the face values of the dice. What is the number of outcomes for this experiment? What is the smallest product in the table? What is the largest product in the table? How many products are less than 10?

Challenge: 96; 1; 96; 25

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Name ______________________________

Use a 4-sided die and a 6-sided die to make a table of products when the dice are rolled. Use the table to find each probability.

1.

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24

P(multiple of 6) ____________________

P(multiple of 4) ____________________

P(even product) ____________________

P(multiple of 10) ____________________

Using Polyhedral Dice, make a table to find each probability.

2. two 4-sided dice

P(multiple of 3) ____________________

P(product that is a prime number) ______

P(product < 15) ____________________

P(multiple of 8) ____________________

3. 6-sided die and 10-sided die

P(odd product) ____________________

P(product > 40) ____________________

P(product < 10) ____________________

P(multiple of 5) ____________________

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Name ______________________________

Challenge! An experiment has you roll an 8-sided die and a 12-sided die and multiply the face values of the dice. What is the number of outcomes for this experiment? What is the smallest product in the table? What is the largest product in the table? How many products are less than 10?

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