# Measurement and Data

Students have learned that **measurement** is the process of comparing a quantity to a standard unit. In fifth grade, students convert units within a given measurement system and use these conversions in solving multi-step problems. They also make connections between the base ten number system and metric units while making conversions.

Students in fifth grade represent and interpret **data**. They make line plots to display data sets of measurements in fractions of a unit. They also use the operations of addition and subtraction with fractions to solve problems involving length, mass, and volume presented in line plots.

Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of unit cubes required to fill a space without gaps or overlaps and they recognize that volumes are therefore expressed using cubic units. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume and measure necessary attributes of shapes in order to solve real-world and mathematical problems.

**The Grade 5 Common Core State Standards for Measurement and Data specify that students should–**

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

The following hands-on activities enable students to develop their understanding of measurement and data concepts in a meaningful way. In particular, students will benefit from working with concrete representations of volume. A good concrete understanding of volume will help students engage in the abstract reasoning required to work flexibly with volume in a variety of contexts.

# Measurement and Data

## Contents

# LESSON 1

## Objective

Make and interpret a line plot for measurements involving fractions of units.

## Common Core State Standards

- **5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

### Measurement and Data

# Make and Use a Line Plot

Given a data set of measurements in fractions of units, students will make a line plot and use operations on fractions to solve problems involving the information given. Students will use what they have learned about making equivalent fractions and adding, subtracting, multiplying, and dividing fractions. Using concrete models will help students visualize the information so they can manipulate it.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- Draw the line plot on the board. **Say:** *The second part of the question wants us to find the total amount of solution and redistribute it equally to the 10 groups.*
- **Say:** *Let's use the line plot to convert the fractions to eighths and add them.* Have students write the equation and add: $\frac{2}{8} + \frac{6}{8} + \frac{16}{8} + \frac{10}{8} + \frac{6}{8} = \frac{40}{8}$. Have students change the improper fraction to a whole number and write $\frac{40}{8} = 5$.
- **Ask:** *How much solution is left over in all? If the 5 cups of solution were redistributed equally to the 10 groups, how much would each group get?*

## Solve It

With students, reread the problem. Have students draw a line plot to represent the problem and write the equation to add the data. Then have them divide the total amount by the number of groups.

## More Ideas

For other ways to teach about using line plots with measurements in fractions of units—

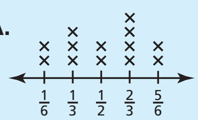
- Have students use Centimeter Cubes in place of the X's on a line plot to represent the data in the following problem: The 15 students in the Running Club each ran a fraction of a mile before school. The following are the distances they ran: $\frac{3}{10}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{7}{10}$, $\frac{2}{5}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{7}{10}$, and $\frac{3}{5}$. What was the average distance they ran?
- Have students use Fraction Tower® Equivalency Cubes or Cuisenaire® Rods to make a line plot of how far the fifth graders walk to school: 8 students walk $\frac{1}{8}$ mile, 6 walk $\frac{3}{8}$ mile, 7 walk $\frac{1}{4}$ mile, 7 walk $\frac{1}{2}$ mile, and 3 walk $\frac{3}{4}$ mile. How many students walk $\frac{1}{2}$ mile or less?

## Formative Assessment

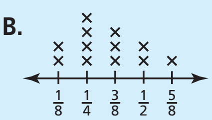
Have students try the following problem.

*Which line plot represents the following data: $\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{6}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{5}{6}$?*

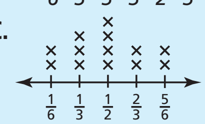
**A.**



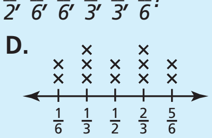

**B.**




**C.**




**D.**





## Try It! 20 minutes | Groups of 4

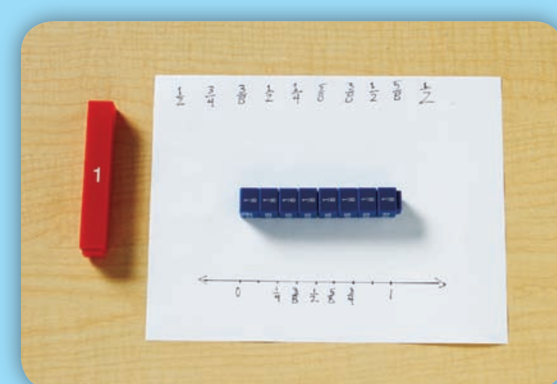
Here is a problem about using line plots with measurements in fractions of units.

*The fifth grade students used a saline solution in a science experiment. Afterward, each of the 10 groups had some solution left. The amounts of solution left over (in cups) were:* $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, *and* $\frac{1}{2}$. *Make a line plot of these amounts. Then find the amount of solution each group would have if the total amount was redistributed equally.*

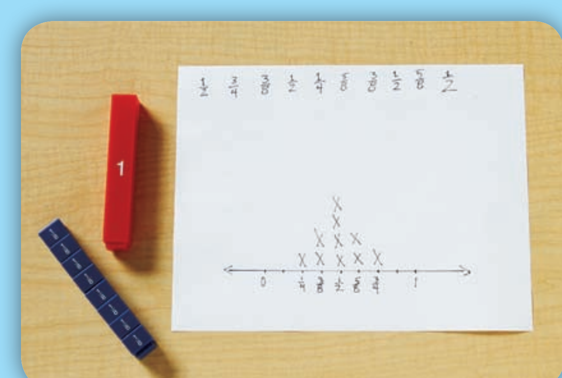
Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Equivalency Cubes, paper, and pencils to students.

### Materials
- Fraction Tower® Equivalency Cubes (1 set per group)
- paper (1 per group)
- pencils (1 per group)

**1. Say:** *Write the fractional amounts of solution left over on your paper. Find the common denominator of these fractions.* **Ask:** *Which Fraction Tower shows the common denominator of these fractions?*



**2. Say:** *Lay the eighths tower horizontally and use it to draw a number line. Mark the number line with the fractional amounts of solution left over.* Guide students as needed to mark $\frac{1}{4}$ at $\frac{2}{8}$, $\frac{1}{2}$ at $\frac{4}{8}$, and $\frac{3}{4}$ at $\frac{6}{8}$.



**3. Say:** *Now plot the data. Use an X to represent each amount of solution. We have 10 fractions, so you should have 10 X's on your line plot.* Guide students as necessary in marking 1 X for $\frac{1}{4}$, 2 X's for $\frac{3}{8}$, 4 X's for $\frac{1}{2}$, 2 X's for $\frac{5}{8}$, and 1 X for $\frac{3}{4}$.

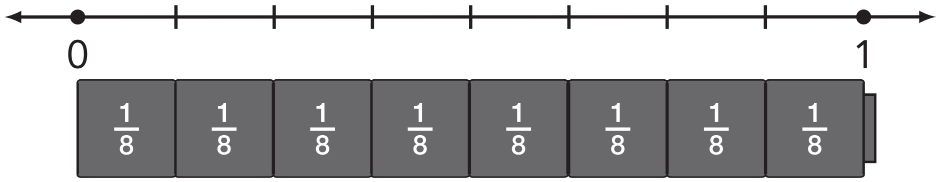
### ⚠ Look Out!

Make sure students convert their fractions properly. Have them put the $\frac{1}{2}$ Fraction Tower and $\frac{3}{4}$ Fraction Tower next to a tower of eighths to be sure they use an equivalent fraction in eighths.

**Use Fraction Towers to draw a line plot of the data. Answer the question.**

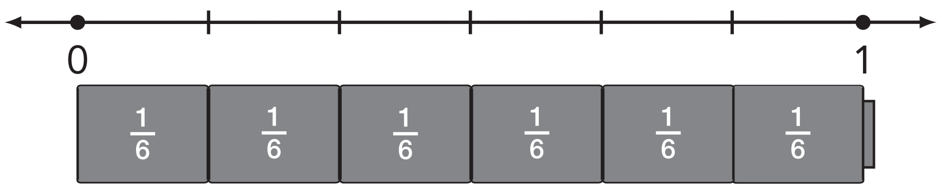
(Check students' work.)

**1.** Lisa is measuring pieces of ribbon in yards. They are $\frac{1}{4}$, $\frac{7}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$. How many pieces are $\frac{1}{2}$ yard or longer?




11 pieces; Number line should be marked in eighths. Line plot should have 1 X at $\frac{1}{4}$, 4 X's at $\frac{1}{2}$, 3 X's at $\frac{5}{8}$, 3 X's at $\frac{3}{4}$, and 1 X at $\frac{7}{8}$.

**2.** After students finished planting their seedlings, they turned in the leftover soil. In cups, the amounts left over are $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{3}$. How much soil is left over?




$6\frac{1}{3}$ cups; Number line should be marked in sixths. Line plot should have 3 X's at $\frac{1}{6}$, 5 X's at $\frac{1}{3}$, 4 X's at $\frac{1}{2}$, 2 X's at $\frac{2}{3}$, and 1 X at $\frac{5}{6}$.

**Draw a line plot to represent the data. Answer the question.**

**3.** Miles walked: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{5}{8}$.

(Check students' models.)

7 miles; Number line should be marked in eighths. Line plot should have 1 X at $\frac{3}{8}$, 3 X's at $\frac{1}{2}$, 3 X's at $\frac{5}{8}$, 2 X's at $\frac{3}{4}$, and 2 X's at $\frac{7}{8}$.

How many miles were walked in all? ________

**4.** Data: $\frac{2}{5}$, $\frac{7}{10}$, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{1}{2}$, $\frac{2}{5}$, $\frac{1}{2}$, and $\frac{3}{5}$.

$\frac{1}{2}$; Number line should be marked in tenths. Line plot should have 3 X's at $\frac{2}{5}$, 4 X's at $\frac{1}{2}$, 3 X's at $\frac{3}{5}$, 2 X's at $\frac{7}{10}$, and 1 X at $\frac{4}{5}$.

Which value occurs the most? _______

## *Answer Key*

**Challenge!** Create a story context for Problem 4. How many of the data values are greater than $\frac{1}{2}$?

Challenge: Stories will vary. Six are greater than $\frac{1}{2}$.

LESSON 2

## Objective

Find the volume of a rectangular solid.

## Common Core State Standards

- **5.MD.3a** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- **5.MD.3b** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
- **5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- **5.MD.5a** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- **5.MD.5b** Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

Measurement and Data

# Volume of a Rectangular Solid

Students explore the volume of a rectangular solid by constructing a model out of cubic units. By counting units, students build an understanding of volume and visualize the connection between volume and dimensions. With this understanding, students can develop formulas for volume and are prepared to consider the volumes of other solids.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *How is a cubic unit different from a square unit?* Have students use a Snap Cube® to demonstrate the difference.
- **Ask:** *How can you determine the number of cubes in the rectangular solid by looking at the completed model?*
- **Say:** *You can calculate the volume of a rectangular solid by finding the area of the solid's base and multiplying this by the solid's height.* **Ask:** *How do you write the formula represented by this method?* Guide students to write $V = B \times h$. **Ask:** *How do you find the area of the base?* Students should know to multiply length by width.

## Solve It

Reread the problem with students. Have them talk about the two ways of writing the formula for the volume of the freezer. Guide them to see how the formulas represent different ways of visualizing the same thing, and ask them to write a paragraph about what they have learned.

## More Ideas

For other ways to teach about volume of rectangular solids—

- Give pairs of students 48 Snap Cubes or Centimeter Cubes and have them build five different rectangular solids with a volume of 48 cubic units. Have them record and compare the dimensions of the solids.
- Give pairs of students the 2 cubes and the 2 rectangular prisms from the Relational GeoSolids® set and have them estimate the volumes of the solids and investigate the relationship between volume and side lengths.

## Formative Assessment

Have students try the following problem.

*Construction workers dug a hole that measures 5 meters long by 4 meters wide by 3 meters deep. What is the volume of the hole?*

**A.** 12 cubic meters **B.** 27 cubic meters **C.** 60 cubic meters **D.** 94 cubic meters

## Try It! 30 Minutes | Groups of 3

Here is a problem about the volume of a rectangular solid.

*Mr. Adams bought a freezer. The freezer is rectangular and the space inside it measures 3 feet long by 2 feet wide by 5 feet high. What is the volume of the space inside the freezer? Write a general formula for this volume that could be used for any length, width, and height.*

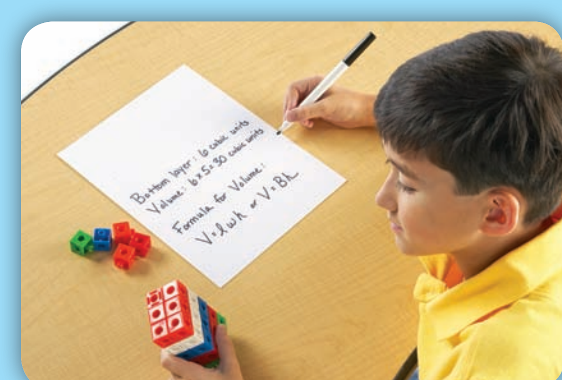
Introduce the problem. Then have students do the activity to solve the problem. Distribute Snap Cubes, paper, and pencils to students. **Say:** *Volume is a three-dimensional measure and is therefore expressed in cubic units.*

### Materials

- Snap Cubes® (30 per group)
- paper (1 sheet per group)
- pencils (1 per group)

**1. Say:** *Using cubes, build a 3-by-2 rectangle to represent the bottom layer of space inside the freezer.* **Ask:** *How many cubic units are in this layer of cubes?* Have students count cubes or multiply to find the number of cubic units.


**2. Say:** *Stack additional 3-by-2 layers on top of the first layer to make a solid that is five layers tall. The solid model represents the space inside the freezer.* **Ask:** *How many cubic units are in the model?* Guide students to multiply the number of cubic units in each layer by the number of layers. Have them record their results.




**3. Say:** *You can multiply length by width by height to find the volume.* Have students confirm that multiplying length by width by height produces the same answer. **Say:** *Write a formula for volume.* Help students write the formula.

### ⚠ Look Out!

Some students might not see the connection between the different forms of the volume formula: $V = lwh$ and $V = Bh$. Help students see that multiplying length by width is the same as calculating the area of the base. Reinforce this idea by presenting a table like the following and having students extend the table for additional layers.

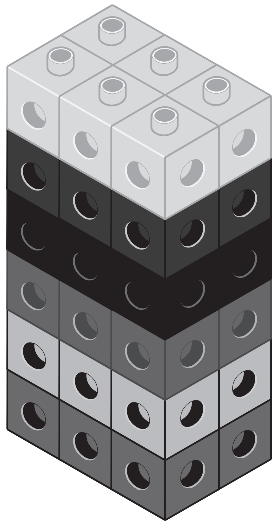
| Number of layers | Cubic units | $V = lwh$ | $V = Bh$ |
|---|---|---|---|
| one | 6 | $3 \times 2 \times 1 = 6$ | $6 \times 1 = 6$ |
| two | 12 | $3 \times 2 \times 2 = 12$ | $6 \times 2 = 12$ |

**Answer Key**

**Use Snap Cubes to model the rectangular solid. Find the volume.**

(Check students' work.)

**1.**



What is the length? 3 units

What is the width? 2 units

What is the height? 6 units

**What is the volume?** 36 cubic units

**Using Snap Cubes, model the solid with the given dimensions. Sketch the model. Find the volume.**

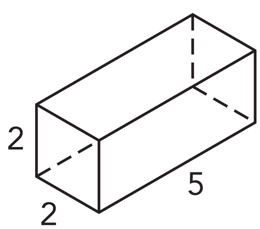
(Check students' models.)

**2.** length: 7 units, width: 5 units, height: 2 units

70 cubic units

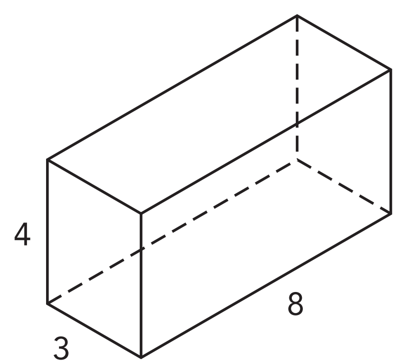
**Find the volume of each rectangular solid.**

**3.**




20 cubic units

**4.**




96 cubic units

**5.** length: 8 units
width: 3 units
height: 5 units

120 cubic units

## Answer Key

**Challenge!** Explain why the volume formulas $V = l \times w \times h$ and $V = B \times h$ give the same results. ($B$ represents the area of the base.)

Challenge: (Sample) The formula for area of the base is $l \times w$. Each layer or unit of the height that makes up the solid gets multiplied by that area.

LESSON 3



# Volume of a Composite Solid

Students will build upon their understanding of the volume of rectangular solids by using cubic units to construct a composite solid and explore its volume. They will work with a composite solid made of two right rectangular prisms. Knowing that volume is additive, they will count and/or use the formula for volume to find the volume of each prism and add the volumes to find the volume of the composite.

## Objective

Find the volume of a composite solid.

## Common Core State Standards

- **5.MD.5c** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *How does knowing that volume is additive help you find the volume of the box? What is the volume of the back part of the box? What is the volume of the front part? What is the total volume?*
- **Ask:** *What is another way you could have divided the box into two rectangular solids? What are the dimensions of the bottom part of the box? What are the dimensions of the top part? Is the total volume the same?*

## Solve It

Reread the problem with students. Have them sketch the box and label its dimensions, and then draw a dotted line between the two parts. Have students find the volume of each part and add to find the total.

## More Ideas

For other ways to teach finding the volume of a composite solid—

- Give groups of students 72 Snap Cubes® or Centimeter Cubes. Have them build four different composite solids made up of two rectangular prisms using any number of the cubes. Have them record and compare the dimensions of the composite solids, and calculate the volume of each.
- Give groups of students 72 Snap Cubes or Centimeter Cubes, but now have them use all of the cubes each time to build four different composite solids made up of two rectangular prisms. Have them record the dimensions and volume, and explain why the volume is the same each time.

## Formative Assessment

Have students try the following problem.

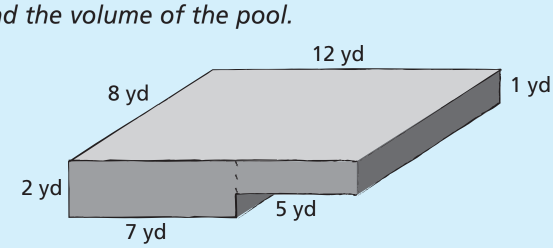
*The swimming pool has a deep end and a shallow end. The dimensions of the pool are shown at right. Find the volume of the pool.*

**A.** 208 cubic yards

**B.** 172 cubic yards

**C.** 152 cubic yards

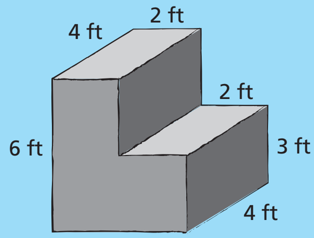
**D.** 115 cubic yards



## Try It!
15 minutes | Groups of 4

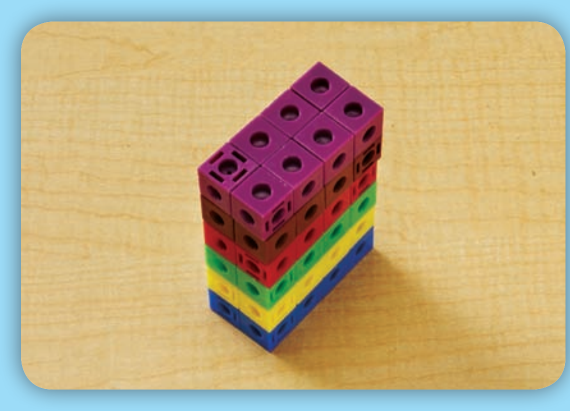
Here is a problem about finding the volume of a composite solid.

*Angelo is building a shipping box in which to pack an antique writing desk with a glass cabinet top. The box he is building is shown at right. Find the volume of the box.*

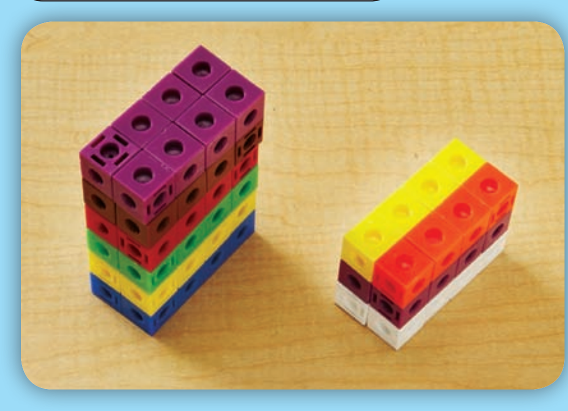



Introduce the problem. Then have students do the activity to solve the problem. Distribute Snap Cubes, paper, and pencils to students.

### Materials
- Snap Cubes® (75 per group)
- paper
- pencils



**1. Say:** *Using cubes, build the back part of the box.* **Ask:** *What are the dimensions of the back part of the box? How many cubic units are in the back part of the box?* Have students count cubes or multiply length by width by height to find the number of cubic units.



**2. Say:** *Now build the front part of the box.* **Ask:** *What are the dimensions of the front part of the box? How many cubic units are in the front part of the box?* Have students count cubes or multiply to find the number of cubic units.

**3. Ask:** *How can we find the volume of the entire box?* Explain that to find the volume of the box, they need to add the volume of the back part to the volume of the front part. Have students add the two volumes to find the total number of cubic units.

### Look Out!

Students may want to use the total width for each part instead of dividing the total width in two for the two parts. Have them draw a sketch of the box and draw a dotted line separating the two rectangular solids that make up the box, so they can clearly see the division.

**Answer Key**

**Use Snap Cubes to model the composite solid. Find the volume of each part. Then find the total volume.** (Check students' work.)

**1.** Part 1: What is the height? 6

What is the width? 2

What is the depth? 3

What is the volume of Part 1? 36 cubic units

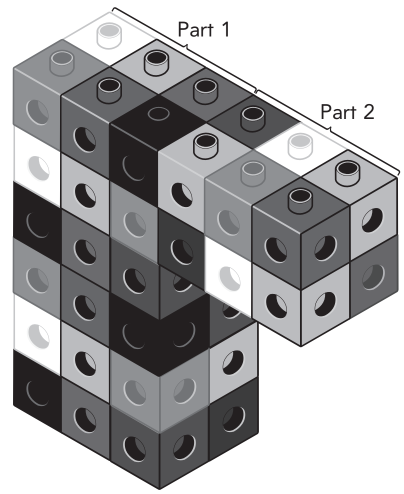
Part 2: What is the height? 2

What is the width? 2

What is the depth? 3

What is the volume of Part 2? 12 cubic units

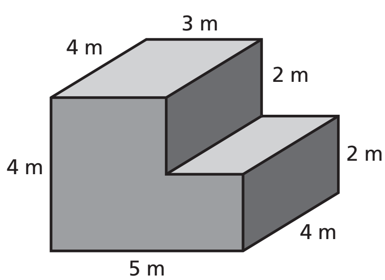
What is the total volume of the two parts? 48 cubic units




**Using Snap Cubes, model the composite solid. Sketch the model. Find the volume of each part. Then find the total volume.**

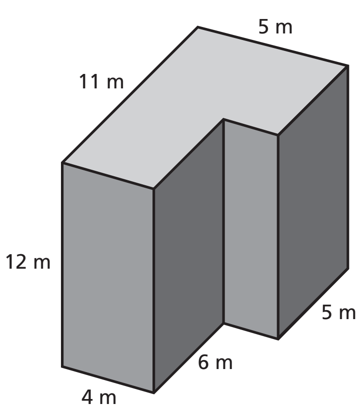
**2.** 40 + 24 = 64 cubic meters; or 48 + 16 = 64 cubic meters

(Check students' models.)




**Find the volume of the composite solid.**

**3.** 288 + 300 = 588 cubic meters; or 528 + 60 = 588 cubic meters.



## *Answer Key*

**Challenge!** Explain why each of the composite solids in the previous problems can be divided in two different ways and how that affects the total volume.

Challenge: (Sample) Each composite solid is made of two non-overlapping right rectangular prisms that can be viewed two ways—for example, two prisms with dimensions 4 m × 6 m × 12 m and 5 m × 5 m × 12 m, or two prisms with dimensions 4 m × 11 m × 12 m and 1 m × 5 m × 12 m. The total volume is the same either way.