

# Math Mammoth End-of-the-Year Test, Grade 7, Answer Key

If you are using this test to evaluate a student's readiness for Algebra 1, I recommend that the student gain a score of 80% on the first four sections (Integers through Ratios, Proportions, and Percent). The subtotal for those is 118 points. A score of 94 points is 80%. I also recommend that the teacher or parent review with the student any content areas in which the student may be weak. Students scoring between 70% and 80% in the first four sections may also continue to Algebra 1, depending on the types of errors (careless errors or not remembering something, versus a lack of understanding). Use your judgment.

You can use the last four sections to evaluate the student's mastery of topics in Math Mammoth Grade 7 Curriculum. However, mastery of those sections is not essential for a student's success in an Algebra 1 course.

**A calculator is *not* allowed for the first three sections of the test: Integers, Rational Numbers, and Algebra.**

**A basic calculator *is* allowed for the last five sections of the test: Ratios, Proportions, and Percent; Geometry, The Pythagorean Theorem, Probability, and Statistics.**

My suggestion for points per item is as follows.

Question #	Max. points	Student score
<b>Integers</b>		
1	2 points	
2	2 points	
3	3 points	
4	6 points	
5	2 points	
6	3 points	
<i>subtotal</i>		/ 18
<b>Rational Numbers</b>		
7	8 points	
8	3 points	
9	3 points	
10	2 points	
11	4 points	
<i>subtotal</i>		/ 20
<b>Algebra</b>		
12	6 points	
13	3 points	
14	12 points	
15	2 points	
16a	1 point	
16b	2 points	
17	3 points	
18	4 points	

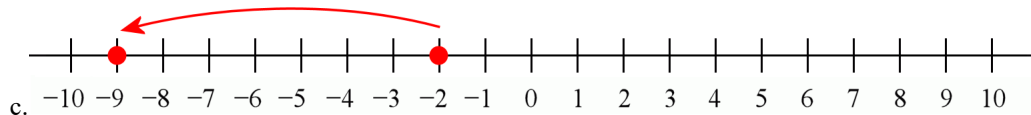
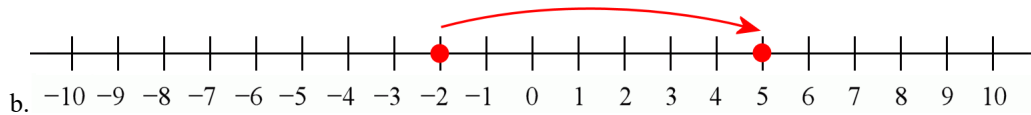
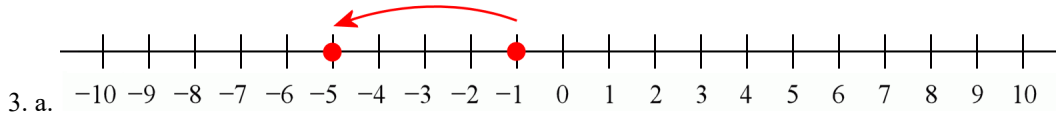
Question #	Max. points	Student score
19a	2 points	
19b	1 point	
20	8 points	
21	2 points	
22a	2 points	
22b	1 point	
<i>subtotal</i>		/ 49
<b>Ratios, Proportions, and Percent</b>		
23	4 points	
24a	1 point	
24b	2 points	
24c	1 point	
24d	1 point	
25a	1 point	
25b	2 points	
26	2 points	
27	2 points	
28a	2 points	
28b	2 points	
29	2 points	
30	2 points	
31	2 points	
32	Proportion: 1 point Solution: 2 points	
33	2 points	
<i>subtotal</i>		/ 31
<b>SUBTOTAL FOR THE FIRST FOUR SECTIONS:</b>		<b>/118</b>

Question #	Max. points	Student score
<b>Geometry</b>		
34a	2 points	
34b	2 points	
35	3 points	
36	2 points	
37	2 points	
38	2 points	
39a	1 points	
39b	3 points	
40a	2 points	
40b	2 points	
41	2 points	
42	3 points	
43a	2 points	
43b	2 points	
44a	2 points	
44b	2 points	
45a	2 points	
45b	1 point	
46a	1 point	
46b	2 points	
<i>subtotal</i>		/ 40
<b>The Pythagorean Theorem</b>		
47	2 points	
48	2 points	
49	2 points	
50	3 points	
<i>subtotal</i>		/9

Question #	Max. points	Student score
<b>Probability</b>		
51	3 points	
52a	2 points	
52b	1 point	
52c	1 point	
52d	1 point	
53	3 points	
54	3 points	
<i>subtotal</i>		/14
<b>Statistics</b>		
55	2 points	
56a	1 point	
56b	2 points	
56c	2 points	
57	2 points	
58a	1 point	
58b	1 point	
58c	1 point	
58d	3 points	
<i>subtotal</i>		/15
<b>SUBTOTAL FOR THE LAST FOUR SECTIONS:</b>		<b>/78</b>
<b>TOTAL</b>		<b>/196</b>

## Integers

- Answers will vary. Check the student's answer.  $-15 + 10 = -5$ . For example: A fish swimming at a depth of 15 ft rose 10 ft, and now it is 5 ft below the surface. Or, Mary owed her mom \$15. She paid back \$10 of her debt, and now she only owes her mom \$5. Or, the temperature was  $-15^\circ$ . It rose 10 degrees and now the temperature is  $-5^\circ$ .
- Answers will vary. Check the student's answer.  $4 \cdot (-2) = -8$ . For example: A certain ion has a charge of  $-2$ . Four such ions have a charge of  $-8$ . Or, four students bought ice cream for \$2 each, but none of them had any money with them. Each of them borrowed \$2 from a teacher. Now, their total debt is \$8. Or, a stick reaches 2 m below the surface of the lake. If we put four such sticks end-to-end, they will reach to the depth of 8 m below the surface.



4. a. 2    b. -1    c. 25    d. 24    e. -12    f. 12

5.  $|-5 - (-15)| = |10| = 10$ .

6. a.  $-1/8$     b.  $-1/4$     c.  $4 \frac{1}{5}$

## Rational Numbers

7. a.  $1 \frac{1}{28}$     b.  $45.\overline{83}$   
 c. 0.00077    d. 0.0144  
 e.  $1 \frac{4}{5}$     f.  $-6 \frac{2}{7}$   
 g.  $-0.2$  or  $-1/5$     h. 4

See below full solutions for 7. g. and 7. h. since they involve both a fraction and a decimal.

<p>g. <math>-\frac{1}{6} \cdot 1.2</math></p> <p>If we use fraction arithmetic, this becomes:</p> $= -\frac{1}{6} \cdot \frac{12}{10}$ $= -\frac{1}{6} \cdot \frac{6}{5} = -\frac{1}{5}$ <p>If we use decimal arithmetic, we get</p> $-\frac{1}{6} \cdot 1.2 = 1.2 \div (-6) = -0.2$	<p>h. <math>-\frac{2}{5} \div (-0.1)</math></p> <p>If we use decimal arithmetic, this becomes</p> $-0.4 \div 0.1 = 4 \text{ (because } 4 \cdot 0.1 = 0.4\text{)}.$ <p>With fraction arithmetic, we get</p> $-\frac{2}{5} \div \left(-\frac{1}{10}\right)$ $= \frac{2}{5} \cdot \frac{10}{1} = 4$
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8. a. 1748/10,000    b.  $-483/100,000$     c.  $2 \frac{43928}{1,000,000}$

9. a.  $-0.0028$     b. 24.93    c. 7.01338

10. a. 0.53846    b.  $1.\overline{81}$

11.

a.  $1.2 \cdot 25 = 30$

Answers will vary. Check the student's answer. For example:

The price of a pair of scissors costing \$25 is increased by 20%. The new price is \$30.

Or, a line segment that is 25 cm long is scaled by a scale factor 1.2, and it becomes 30 cm long.

Or, the lunch break, which used to be 25 minutes long, is increased by  $\frac{1}{5}$ . Now it is 30 minutes long.

b.  $(\frac{3}{5}) \div 4 = (\frac{3}{5}) \cdot (\frac{1}{4}) = \frac{3}{20}$ .

Answers will vary. Check the student's answer. For example:

There is  $\frac{3}{5}$  of a large pizza left, and four people share it equally. Each person gets  $\frac{3}{20}$  of the original pizza.

Or, a plot of land that is  $\frac{3}{5}$  square mile is divided evenly into four parts. Each of the parts is  $\frac{3}{20}$  square mile

$= \frac{15}{100}$  sq. mi.  $= 0.15$  sq. mi.

## Algebra

12.

a. $15s - 10$	b. $5x^4$	c. $3a + 3b - 6$
d. $1.02x$	e. $2w - 4$	f. $-3.9a + 0.5$

13.

a. $7x + 14$ $= 7(x + 2)$	b. $15 - 5y$ $= 5(3 - y)$	c. $21a + 24b - 9$ $= 3(7a + 8b - 3)$
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14.

a. $2x - 7 = -6$ $2x = 1$ $x = \frac{1}{2}$	b. $2 - 9 = -z + 4$ $-7 = -z + 4$ $-11 = -z$ $z = 11$
c. $120 = \frac{c}{-10}$ $-1200 = c$ $c = -1200$	d. $2(x + \frac{1}{2}) = -15$ $2x + 1 = -15$ $2x = -16$ $x = -8$
e. $\frac{2}{3}x = 266$ $2x = 798$ $x = 399$	f. $x + 1\frac{1}{2} = \frac{3}{8}$ $x = \frac{3}{8} - 1\frac{1}{2}$ $x = \frac{3}{8} - \frac{12}{8} = -\frac{9}{8} = -1\frac{1}{8}$

15. From the formula  $d = vt$  we can find that  $t = d/v$ . In this case,  $t = 0.8 \text{ km}/(12 \text{ km/h}) = 0.0\overline{6} \text{ h} = 0.0\overline{6} \text{ h} \cdot (60 \text{ min/h}) = \underline{4 \text{ minutes}}$ . This is reasonable because the distance he ran is fairly short.

16. a. The equation that matches the situation is  $\frac{4p}{5} = 48$ .

b.  $\frac{4p}{5} = 48$

$$4p = 240$$

$$p = 60$$

The original price was \$60.

17. Let  $w$  be the width of the rectangle. The student can write any of the equations below:

- $2w + 2 \cdot 55 = 254$
- $2w + 110 = 254$
- $w + w + 55 + 55 = 254$
- $w + w + 110 = 254$
- $w + 55 + w + 55 = 254$

A solution of the equation:


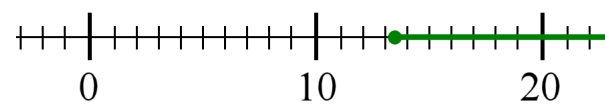
$$2w + 110 = 254$$

$$2w = 144$$

$$w = 72$$

The rectangle is 72 cm wide.

18.

<p>a. <math>3x - 7 &lt; 83</math>  <math>3x &lt; 90</math>  <math>x &lt; 30</math></p> 	<p>b. <math>2x - 16.3 \geq 10.5</math>  <math>2x \geq 26.8</math>  <math>x \geq 13.4</math></p> 
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19. a. Let  $n$  be the number of boxes. The cost of the boxes with the discount is  $15n - 25$ .

The inequality is  $15n - 25 \leq 150$ . Solution:

$$15n - 25 \leq 150$$

$$15n \leq 175$$

$$n \leq 11.67$$

b. The solution means that you can buy 11 boxes at most.

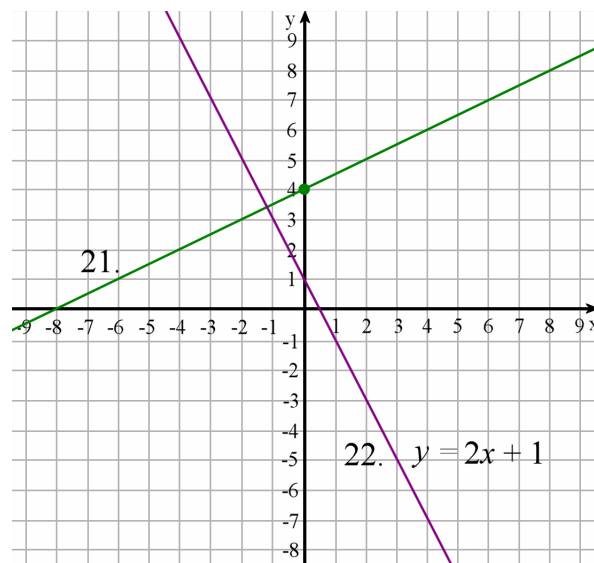
20.

<p>a.</p> $9y - 2 + y = 5y + 10$ $10y = 5y + 10 + 2$ $5y = 12$ $y = 12/5 = 2 \frac{2}{5}$	<p>b.</p> $2(x + 7) = 3(x - 6)$ $2x + 14 = 3x - 18$ $2x - 3x = -18 - 14$ $-x = -32$ $x = 32$
<p>c.</p> $\frac{y + 6}{-2} = -10$ $y + 6 = 20$ $y = 14$	<p>d.</p> $\frac{w}{2} - 3 = 0.8$ $\frac{w}{2} = 3.8$ $w = 7.6$

21. See the graph on the right.

22. a. See the graph on the right.

b. The slope is  $-2$ .



## Ratios, Proportions, and Percent

23.

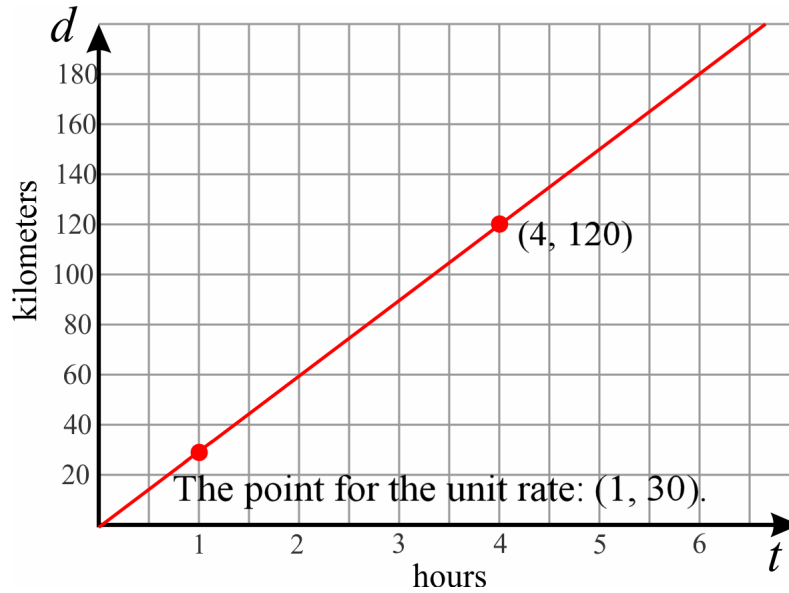
a. Lily paid \$6 for  $\frac{3}{8}$  lb of nuts.

$$\frac{\$6}{\frac{3}{8} \text{ lb}} = \$6 \cdot \frac{8}{3} \text{ per lb} = \$16 \text{ per lb}$$

b. Ryan walked  $2 \frac{1}{2}$  miles in  $\frac{3}{4}$  of an hour.

$$\frac{2 \frac{1}{2} \text{ mi}}{\frac{3}{4} \text{ h}} = \frac{5}{2} \cdot \frac{4}{3} \text{ mi/h} = \frac{20}{6} \text{ mi/h} = 3 \frac{1}{3} \text{ mi/h}$$

24. The graph below shows the distance covered by a moped advancing at a constant speed.



- a. The speed of the moped is 30 km/h.
- b. See the image above. The point is (4, 120), and it signifies that after driving 4 hours, the moped has covered 120 km.
- c.  $d = 30t$
- d. See the image. It is the point (1, 30).
25. a. The Toyota Prius gets better gas mileage, because it gets  $565 \text{ mi}/11.9 \text{ gal} \approx 47.48 \text{ mi/gal}$  whereas a Honda Accord gets  $619 \text{ mi}/17.2 \text{ gal} \approx 35.99 \text{ mi/gal}$ .
- b. The cost of driving 300 miles with a Toyota Prius is  $300 \text{ mi} \cdot (11.9 \text{ gal}/565 \text{ mi}) \cdot \$3.19/\text{gal} \approx \$20.16$ .  
The cost of driving 300 miles with a Honda Accord is  $300 \text{ mi} \cdot (17.2 \text{ gal}/619 \text{ mi}) \cdot \$3.19/\text{gal} \approx \$26.59$ .  
The difference is  $\$26.59 - \$20.16 = \underline{\$6.43}$ .
26. She can withdraw  $\$2,500 \cdot 0.08 \cdot 3 + \$2,500 = \$600 + \$2,500 = \underline{\$3,100}$ .
27. After the 15% price increase, the ticket costs  $1.15 \cdot \$10 = \$11.50$ .  
Then, the price decreased by 25% is  $0.75 \cdot \$11.50 = \$8.625 \approx \underline{\$8.63}$ .
28. a. The percentage of increase was  $(72,000 - 51,500)/51,500 \approx \underline{39.8\%}$ .
- b. She will have  $1.398 \cdot 72,000 = 100,656 \text{ visitors} \approx \underline{101,000 \text{ visitors}}$ .
29. Let  $r$  be the amount of rainfall in the previous month. Then,  $1.35r = 10.5 \text{ cm}$ , from which  $r = 10.5 \text{ cm}/1.35 \approx \underline{7.8 \text{ cm}}$ .
30. The side of the enlarged square is  $(4/3) \cdot 15 \text{ cm} = 20 \text{ cm}$ . Its area is  $20 \text{ cm} \cdot 20 \text{ cm} = \underline{400 \text{ cm}^2}$ .
31. There are two basic ways to calculate the distance on the map from the distance in reality. One way is that we first convert the given distance, 8 km, into centimeters, which are units used on the map, and then multiply by the ratio 1:50,000.
- $8 \text{ km} = 8,000 \text{ m} = 800,000 \text{ cm}$ . The distance on the map is  $800,000 \text{ cm} \cdot (1/50,000) = \underline{16 \text{ cm}}$ .
- Another way is to convert the ratio so that it uses common measuring units. The ratio 1:50,000 signifies that 1 cm on the map is 50,000 cm in reality. From this, we can write  $1 \text{ cm} = 50,000 \text{ cm} = 500 \text{ m} = 0.5 \text{ km}$ . So 1 cm on the map corresponds to 0.5 km in reality. The given distance of 8 km corresponds to  $8 \text{ km} \div (0.5 \text{ km}/1 \text{ cm}) = \underline{16 \text{ cm}}$ .

32. Proportions vary as there are several different ways to write the proportion correctly. Here are four of the correct ways. Besides these four, you will get four more by switching the right and left sides of these four equations.

$\frac{600 \text{ ml}}{554 \text{ g}} = \frac{5000 \text{ ml}}{x}$	$\frac{554 \text{ g}}{600 \text{ ml}} = \frac{x}{5000 \text{ ml}}$	$\frac{5000 \text{ ml}}{600 \text{ ml}} = \frac{x}{554 \text{ g}}$	$\frac{600 \text{ ml}}{5000 \text{ ml}} = \frac{554 \text{ g}}{x}$
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The key point is that in each of the correct ways,  $x$  ends up being multiplied by 600 ml in the cross-multiplication. If  $x$  ends up being multiplied by 554 g or 5,000 ml in the cross-multiplication, the proportion is set up incorrectly.

Here is the solution process for one of the proportions above. Each of the others has the same final solution,  $x = 4,617 \text{ g}$ .

$$\frac{600 \text{ ml}}{554 \text{ g}} = \frac{5000 \text{ ml}}{x}$$

$$600 \text{ ml} \cdot x = 554 \text{ g} \cdot 5000 \text{ ml}$$

$$x = \frac{554 \text{ g} \cdot 5000 \text{ ml}}{600 \text{ ml}}$$

$$x = 4,617 \text{ g}$$

33. A farmer sells potatoes in sacks of various weights. The table shows the price per weight.

<b>Weight</b>	5 lb	10 lb	15 lb	20 lb	30 lb	50 lb
<b>Price</b>	\$4	\$7.50	\$9	\$12	\$15	\$25

- a. These two quantities are *not* in proportion. For example, looking at the cost of potatoes for 5 lb and for 20 lb, the weight increases four-fold, but the cost increases only three-fold (from \$4 to \$12). Or, when the weight increases three-fold from 5 lb to 15 lb, the price does not increase three-fold but, from \$4 to \$9.

Another way to see that is in the beginning of the chart, the weights increase by 5 lb up to 20 lb, but the cost does not increase by the same amount. Instead, the cost increases first by \$3.50, then by \$1.50, then by \$3.

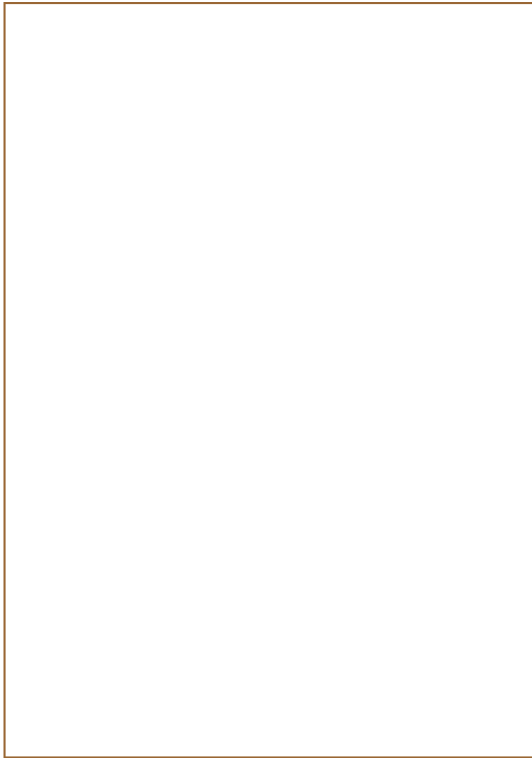
- b. There is no need to answer this, since the quantities are not in proportion.



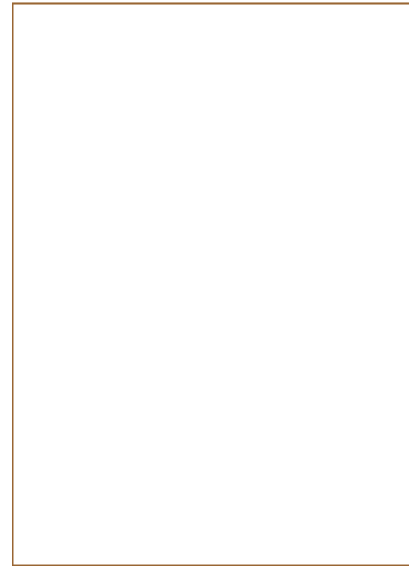
## Geometry

34. a. The given rectangle measures 7 cm by 10 cm. We multiply those by 45 to get the true dimensions:  
 $7 \text{ cm} \cdot 45 = 315 \text{ cm} = 3.15 \text{ m}$  and  $10 \text{ cm} \cdot 45 = 450 \text{ cm} = 4.5 \text{ m}$ .  
The area is  $A = 3.15 \text{ m} \cdot 4.5 \text{ m} = 14.175 \text{ m}^2$ .
- b. The dimensions of the room at a scale 1:60 will be  $45/60 = 3/4$  of the dimensions of the room drawn at a scale of 1:45 so the scale drawing will be smaller than the drawing given in the problem. The width is  $(3/4) \cdot 7 \text{ cm} = 5.25 \text{ cm}$  and the height is  $(3/4) \cdot 10 \text{ cm} = 7.5 \text{ cm}$ .

Or, you can divide the actual dimensions by 60 to get  $315 \text{ cm} \div 60 = 5.25 \text{ cm}$  and  $450 \text{ cm} \div 60 = 7.5 \text{ cm}$ .



Scale 1:45



Scale 1:60

35. We simply multiply the given dimensions (which are in inches) by the ratio 3 ft/1 in, essentially multiplying them by 3:

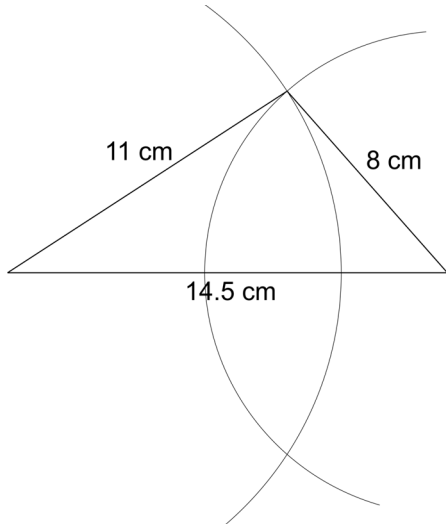
$$4 \frac{1}{4} \text{ in} \cdot (3 \text{ ft}/1 \text{ in}) = 12 \frac{3}{4} \text{ ft} \text{ or } 12 \text{ ft } 9 \text{ in} \text{ and}$$

$$3 \frac{1}{2} \text{ in} \cdot (3 \text{ ft}/1 \text{ in}) = 10 \frac{1}{2} \text{ ft} \text{ or } 10 \text{ ft } 6 \text{ in}.$$

36.  $A = \pi \cdot (8 \text{ cm})^2 \approx 201 \text{ cm}^2$ .

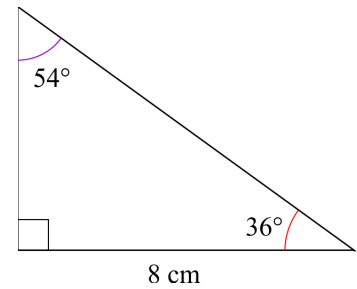
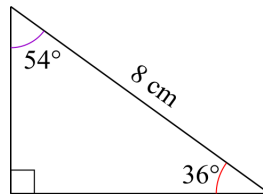
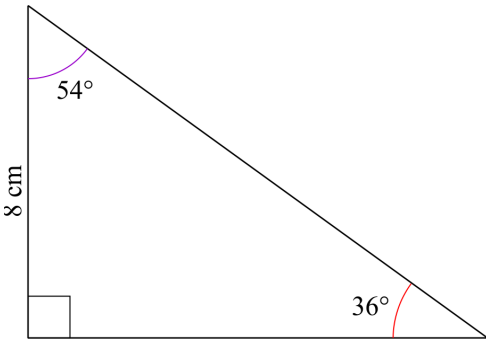
37.  $C = \pi \cdot 2 \cdot 9 \text{ in} = 56.5 \text{ in}$ .

38. Check the student's drawing. The image below is not to true scale, but the student's drawing of a triangle should have the same shape as the triangle below. It will just be larger.



39. a. No, it doesn't.

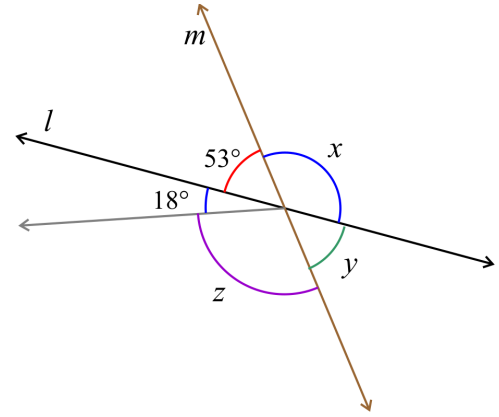
b. The three given angles determine the shape of the triangle. The 8-cm side can be opposite of any of the given angles, so we get three different triangles. The images below are not to true scale but are smaller than in reality. They give you an idea of what the three different triangles look like. Check the student's drawings.



40. a. There are several ways to write an equation for  $x$ .

(1) Since angle  $x$  and the  $53^\circ$  angle are supplementary,  $x + 53^\circ = 180^\circ$ , from which  $x = 180^\circ - 53^\circ = 127^\circ$ .

(2) Angle  $y$  and the  $53^\circ$  angle are vertical angles, so  $y = 53^\circ$ . Then, angle  $x$  and angle  $y$  are supplementary, so we can write the equation  $x + 53^\circ = 180^\circ$ , and once again  $x = 180^\circ - 53^\circ = 127^\circ$ .



b. There are several ways to write an equation for  $z$ .

(1) Since angles  $y$ ,  $z$ , and the  $18^\circ$  angle lie along the same line (line  $l$ ), their measures sum up to  $180^\circ$ , and we can write  $y + z + 18^\circ = 180^\circ$ . Since  $y$  and the  $53^\circ$  angle are vertical angles,  $y = 53^\circ$  and we can substitute that for  $y$  in the equation to get:

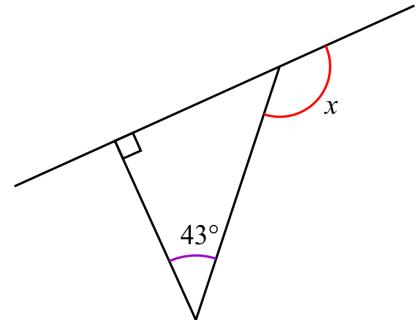
$$\begin{aligned} 53^\circ + z + 18^\circ &= 180^\circ \\ z &= 180^\circ - 53^\circ - 18^\circ \\ z &= 109^\circ \end{aligned}$$

(2) Or, since the combination angle  $z + 18^\circ$  and  $x$  are vertical angles,  $z + 18^\circ = x$ . From part (a) we know that  $x = 127^\circ$  so the equation becomes:

$$\begin{aligned} z + 18^\circ &= 127^\circ \\ z &= 127^\circ - 18^\circ \\ z &= 109^\circ \end{aligned}$$

41. The triangle in the image has an angle of  $43^\circ$  and a right angle, so its third angle is  $180^\circ - 43^\circ - 90^\circ = 47^\circ$ .

The unknown angle  $x$  supplements the third angle of the triangle, so its measure is  $180^\circ - 47^\circ = 133^\circ$ .



42. a. The cross section is a rectangle.

b. The cross section is a triangle.

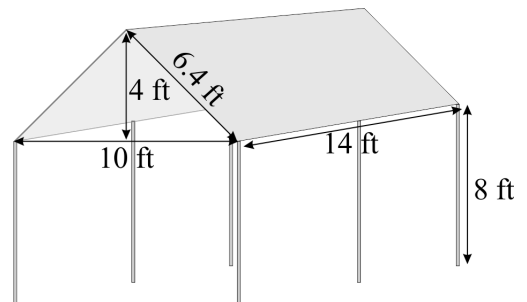
c. The cross section is a trapezoid.

43. a. The roof or top part of the entire canopy is a triangular prism. The base of the prism is a triangle with a base side of 10 ft and a height of 4 ft, so its area is  $10 \text{ ft} \cdot 4 \text{ ft} / 2 = 20 \text{ ft}^2$ .

The volume is  $V = A_b \cdot h = 20 \text{ ft}^2 \cdot 14 \text{ ft} = 280 \text{ ft}^3$ .

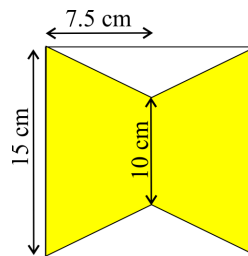
b. The bottom part is a rectangular prism, and its volume is  $10 \text{ ft} \cdot 14 \text{ ft} \cdot 8 \text{ ft} = 1,120 \text{ ft}^3$ .

The total volume is  $280 \text{ ft}^3 + 1,120 \text{ ft}^3 = 1,400 \text{ ft}^3$ .



44. a. One way is to use the formula for the area of a trapezoid. See the image on the right.  
The area of one trapezoid is  $A = (10 \text{ cm} + 15 \text{ cm})/2 \cdot 7.5 \text{ cm} = 93.75 \text{ cm}^2$ .

Then, the area of the two trapezoids is  $2 \cdot 93.75 \text{ cm}^2 = 187.5 \text{ cm}^2$ .



Another way is to subtract the area of the two white triangles from the area of the entire 15 cm by 15 cm square.

The area of one triangle is  $15 \text{ cm} \cdot 2.5 \text{ cm}/2 = 18.75 \text{ cm}^2$ .

The area of the two trapezoids is then  $15 \text{ cm} \cdot 15 \text{ cm} - 2 \cdot 18.75 \text{ cm}^2 = 187.5 \text{ cm}^2$ .

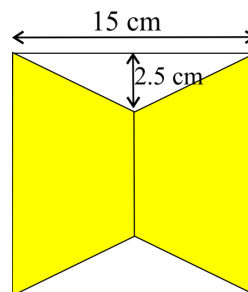
- b. The trapezoids cover  $187.5/(15 \cdot 15) \approx 83.3\%$  of the entire square.

45. a.  $V = A_b \cdot h = \pi \cdot (6 \text{ cm})^2 \cdot 4.5 \text{ cm} = 508.938 \text{ cm}^3 \approx \underline{509 \text{ cm}^3}$ .

- b. The volume is 509 ml = 0.509 L.

46. a. Since 1 ft = 12 in, 1 cubic foot has  $12 \text{ in} \cdot 12 \text{ in} \cdot 12 \text{ in} = \underline{1,728 \text{ in}^3}$ .

- b.  $3 \frac{1}{4} \text{ ft} = 3 \cdot 12 \text{ in} + 3 \text{ in} = 39 \text{ in}$ . The volume is  $V = (39 \text{ in})^3 = \underline{59,319 \text{ in}^3}$ .



## The Pythagorean Theorem

47. a. Its area is  $5 \text{ m}^2$ , because  $(\sqrt{5} \text{ m})(\sqrt{5} \text{ m}) = (\sqrt{5} \text{ m})^2 = 5 \text{ m}^2$ .

- b. It is  $\sqrt{45} \text{ cm} \approx 6.7 \text{ cm}$ .

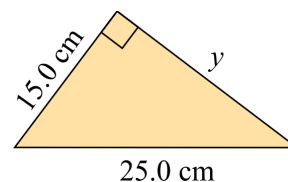
48.  $57^2 + 76^2 \stackrel{?}{=} 95^2$   
 $3,249 + 5,776 \stackrel{?}{=} 9,025$   
 $9,025 = 9,025$

Yes, the lengths 57 cm, 95 cm, and 76 cm do form a right triangle.

49. Using the Pythagorean Theorem, we can write the equation

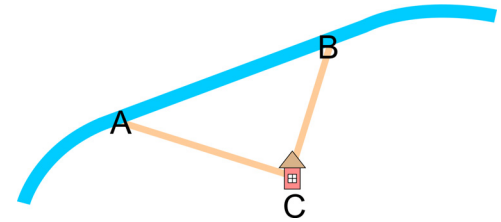
$$\begin{aligned} 15^2 + y^2 &= 25^2 \\ 225 + y^2 &= 625 \\ y^2 &= 400 \\ y &= \sqrt{400} = 20 \text{ (We ignore the negative root.)} \end{aligned}$$

The side is 20 cm long.



50. Let  $x$  be the length AB. Since ABC is a right triangle, applying the Pythagorean Theorem we get:

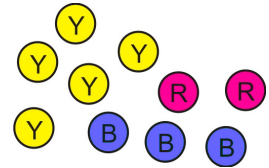
$$\begin{aligned}
 120^2 + 110^2 &= x^2 \\
 14400 + 12100 &= x^2 \\
 x^2 &= 26,500 \\
 x &= \sqrt{26,500} = 162.788 \text{ m (We ignore the negative root.)}
 \end{aligned}$$



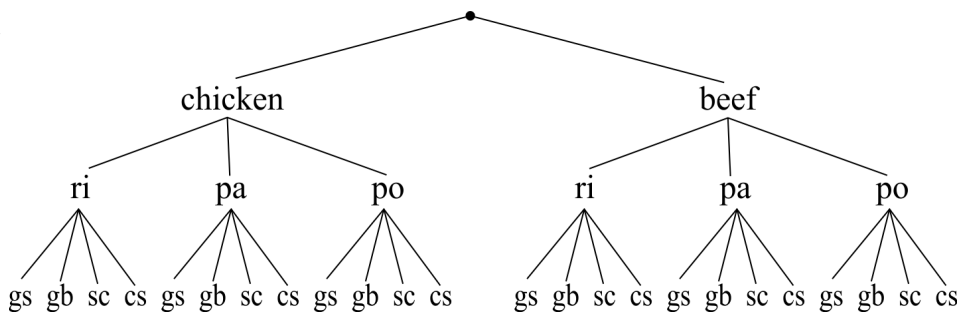
You will walk  $120 \text{ m} + 110 \text{ m} = 230 \text{ m}$ . Your friends walk  $163 \text{ m}$ . So, you will walk  $230 \text{ m} - 163 = \underline{67 \text{ m}}$  more than your friends.

## Probability

51. a.  $P(\text{not red}) = 8/10 = 4/5$   
 b.  $P(\text{blue or red}) = 5/10 = 1/2$   
 c.  $P(\text{green}) = 0$



52. a.



- b.  $P(\text{beef, rice, coleslaw}) = 1/24$   
 c.  $P(\text{no coleslaw nor steamed cabbage}) = 12/24 = 1/2$   
 d.  $P(\text{chicken, green salad}) = 3/24 = 1/8$

53. None of the conclusions (a), (b), or (c) are valid.

(a) This die is unfair.

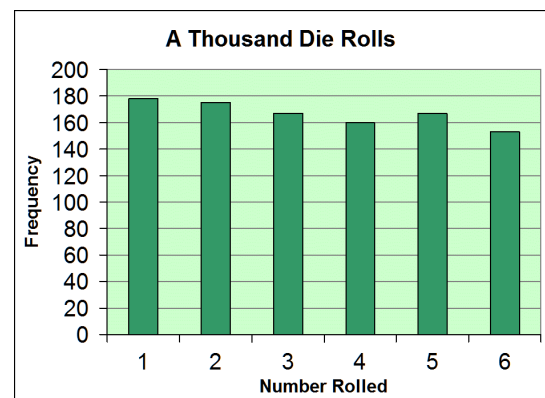
Not valid. In a repeated random experiment, the frequencies for the various outcomes do vary, and the variability seen in the chart is definitely within normal variation. In fact, the data comes from running a computer simulation that uses random numbers, and the simulation was run 1,000 times.

(b) On this die, you will always get more 1s than 6s.

Not valid. The die is not necessarily unfair. A normal die could produce the frequencies seen in the chart.

(c) Next time you roll, you will not get a 4.

Not valid. Rolling a die is a random experiment and you might get 4 the next time you roll.



54. We can toss 10 coins (or a single coin 10 times) to simulate 10 children being born. Let heads = girl, and tails = boy (or vice versa).

Then, repeat that experiment (tossing 10 coins) hundreds of times. Observe how many of those repetitions include 9 heads and one tail, which means getting 9 girls and one boy. The relative frequency is the number of times you got 9 heads and one tail divided by the number of repetitions, and gives you an approximate value for the probability of 9 girls and 1 boy in 10 births.

## Statistics

55. Cindy's sampling method is **biased**. She chose students from her class, which means that all the other students in the college didn't have a chance to be selected in her sample. For a sampling method to be unbiased, every member of the population has to have an equal chance of being selected in the sample. (Her method would work if she was only studying the students in her class.)
56. Four people are running for mayor in a town of about 20,000 people. Three polls were conducted, each time asking 150 people who they would vote for. The table shows the results.

	Clark	Taylor	Thomas	Wright	Totals
Poll 1	58	19	61	12	150
Poll 2	68	17	56	9	150
Poll 3	65	22	53	10	150

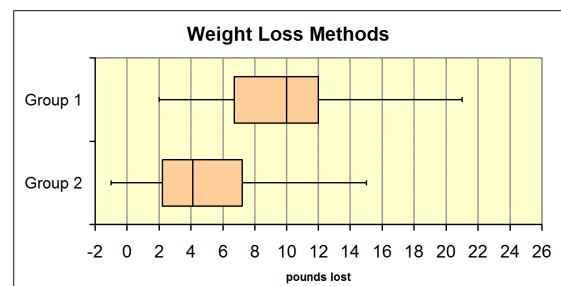
- a. Based on the polls, we can predict Clark to be the winner of the election. He is leading in two of the three polls.
- b. To estimate how many votes Thomas will get, use the average percentage of votes he got in the three polls. You can calculate that as  $((61 + 56 + 53) \div 3) / 150$  or as  $(61/150 + 56/150 + 53/150) \div 3$ . Either way, you will get 37.78%. This gives us the estimate that he would get  $0.3778 \cdot 8,500 \approx 3,200$  votes in the actual election.
- c. We will gauge how much off the estimate of 3,200 votes is by using the individual poll results.  
 Based on poll 1, we would estimate Thomas to get  $(61/150) \cdot 8,500 \approx 3,460$  votes.  
 Based on poll 2, we would estimate Thomas to get  $(56/150) \cdot 8,500 \approx 3,170$  votes.  
 Based on poll 3, we would estimate Thomas to get  $(53/150) \cdot 8,500 \approx 3,000$  votes.

Looking at the highest and lowest numbers (3,000 and 3,460), we can gauge that our estimate of 3,200 votes might be off by a few hundred votes.

57. The total number of people Gabriel surveyed is  $45 + 57 + 18 = 120$ .  
 Of those,  $45/120 = 37.5\%$  support building the highway.  
 This gives us the estimate that  $0.375 \cdot 2,120 = 795 \approx 800$  households in the community would support building the highway.

Opinion	Number
Support the highway	45
Do not support it	57
No opinion	18

58. a. Group 1 appears to have lost more weight.  
 b. Group 1 appears to have a greater variability in the amount of weight lost.  
 c. The person gained 1 pound.  
 d. Yes, the method used with group 1 is significantly better than the other.



The median weight loss for Group 1 is 10 lb whereas for Group 2 only a little over 4 lb. The difference in the medians is about 6 lb. The interquartile ranges are about 5 lb for both Groups. The difference in the medians (about 6 lb) is more than one time the interquartile range (about 5 lb), which shows us that the difference is significant.