

THE MORE, THE BETTER

GEOMETRY • NUMBER



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video clip

- Comparing
- Classifying
- Polygons
- Fractional equivalence
- Spatial visualization

Getting Ready

What You'll Need

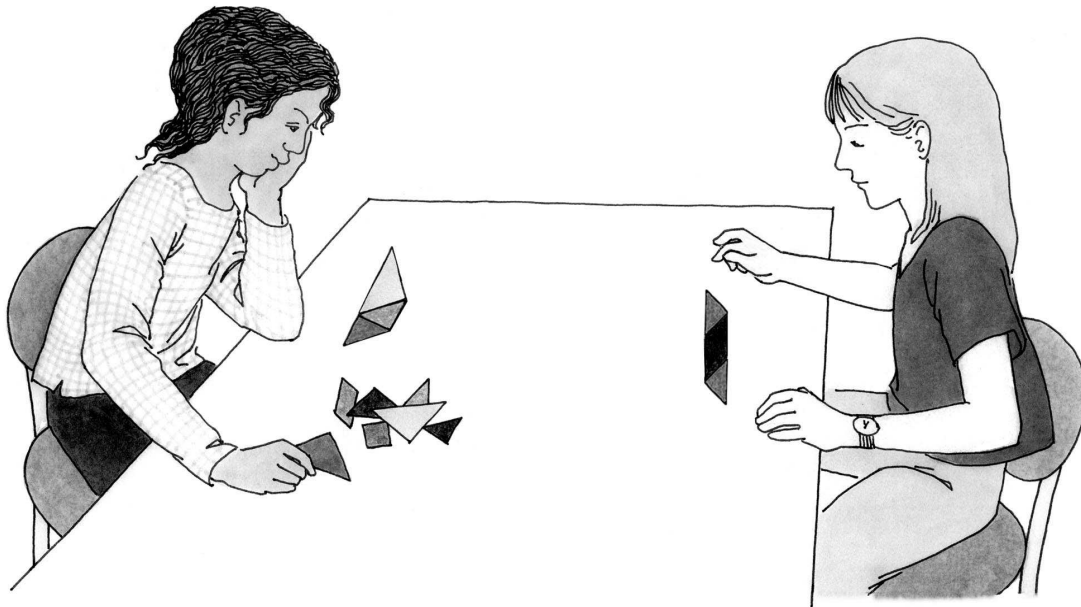
Tangrams, 1 set per child

7-piece Tangram Squares, page 99,
1 per group (optional)

Overview

Children search for all the convex Tangram shapes that can be made with different numbers of pieces that represent the same fractional part of the seven-piece Tangram square. In this activity, children have the opportunity to:

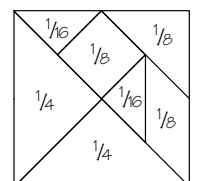
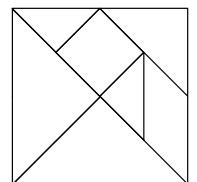
- ◆ create and compare a variety of polygons
- ◆ recognize convex and concave polygons
- ◆ assign fractional amounts to each Tangram piece
- ◆ use equivalence
- ◆ work with fractions as an area model



The Activity

Introducing

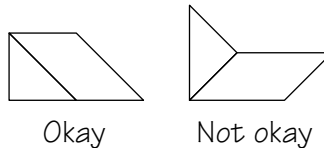
- ◆ Show children a square made from the seven Tangram pieces.
- ◆ Tell children that the 7-piece square has a value equal to 1. Ask them to find the fractional value of each piece.
- ◆ Trace the square, showing the outline of each piece.
- ◆ As volunteers give the value of each piece and explain their reasoning, record the fractional amount in the appropriate place on the tracing.



On Their Own

How can you make a Tangram shape that has a certain area?

- Work with a group. Each of you make a shape that has no indents and whose area is $\frac{1}{2}$ the area of the square made with all 7 Tangram pieces.



- Compare your shapes. Record only the ones that are different.
- Count the number of pieces used.
- Make as many more shapes as you can that are also $\frac{1}{2}$ the area of the 7-piece square, but each time use a different number of pieces. Find the least number of pieces possible, the most number of pieces, and all the possibilities in-between.
- Do this activity several more times. Each time, instead of $\frac{1}{2}$, select one of these fractional amounts to be the area of your shape: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{4}$.
- Look at your groups' solutions. Make a list of what your group observes.

The Bigger Picture

Thinking and Sharing

Discuss one fractional amount at a time. Have volunteers share the shapes they found and what they noticed.

Use prompts such as these to promote class discussion:

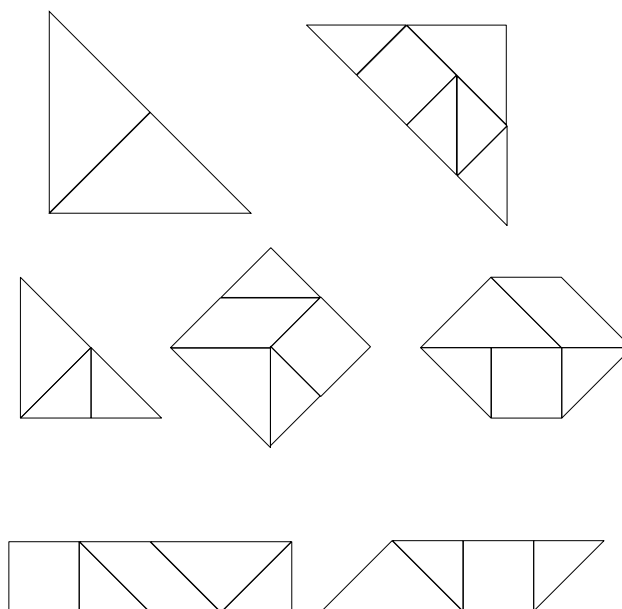
- ◆ How did you decide which Tangram pieces to use for your shapes?
- ◆ How did you know your shape was the correct fractional amount of the 7-piece square?
- ◆ Were some fractional amounts easier to create than others?
- ◆ Which fractional amount could be made in the most ways? Why?
- ◆ Which fractional amount could be made in the fewest ways? Why?
- ◆ Did you ever find it impossible to build a shape with no indents that represented the fractional amount you needed? Explain.

Extending the Activity

1. Give children the labeled Tangram square on p. 99 or have them make their own. Show them how to represent the square with a number sentence such this: $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{4} + \frac{1}{4} = 1$. Now have children write number sentences for each of the shapes they made when they did the activity.
2. Have children repeat the activity but allow concave shapes.

As they build shapes whose areas are a specific fractional amount of the area of the 7-piece square, children deepen their understanding of equivalence. They also focus on a geometric attribute—convex vs. concave—often used to describe, identify, and classify polygons.

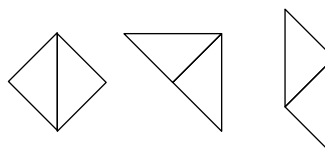
One-half of the area can be shown in more ways than any other fractional amount. And, while there is only one way to show one-half with two pieces, there are many ways to show it with five pieces. Children might start with the two large Tangram triangles, then substitute two small triangles for one of them, thereby making a shape with three pieces. Or, they may substitute instead one medium triangle and two small triangles to produce a four-piece shape that is either a triangle or one of the quadrilaterals shown below.



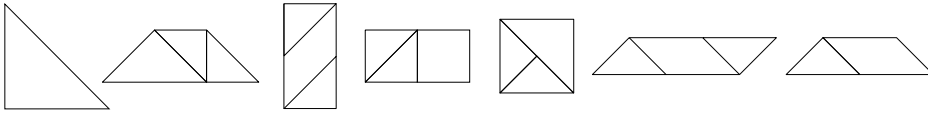
Some children think in terms of shapes only, such as 1 medium triangle equals 2 small triangles or 1 large triangle equals the 3 smaller triangles. Others use the fractional names of the pieces. For example, they will refer to the large triangles as the $\frac{1}{4}$ -pieces, the medium triangle, the square, and the parallelogram as the $\frac{1}{6}$ -pieces, and the two small triangles as the $\frac{1}{16}$ -pieces.

It is not necessary for every group to have investigated every fraction before starting a class discussion. Here are some solutions for each of the other fractional amounts.

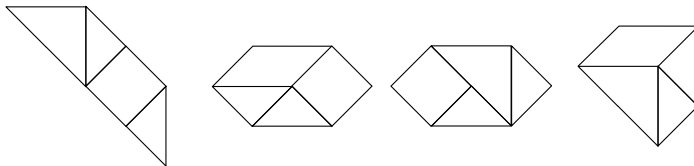
Shapes with $\frac{1}{6}$ the area of the 7-piece Tangram square



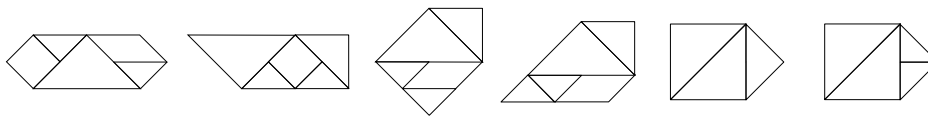
Shapes with $\frac{1}{4}$ the area of the 7-piece Tangram square



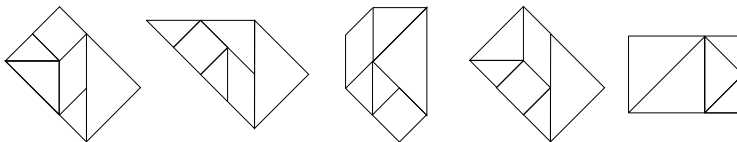
Shapes with $\frac{3}{8}$ the area of the 7-piece Tangram square



Shapes with $\frac{5}{8}$ the area of the 7-piece Tangram square



Shapes with $\frac{3}{4}$ the area of the 7-piece Tangram square



Another way to focus on the data is to have the class organize the data in a chart like this.

Fractional Amount	Number of Pieces					
	1	2	3	4	5	6
$\frac{1}{2}$		✓	✓	✓	✓	
$\frac{1}{6}$		✓				
$\frac{1}{4}$	✓	✓	✓			
$\frac{3}{8}$				✓		
$\frac{5}{8}$			✓	✓	✓	
$\frac{3}{4}$					✓	✓

This format helps children find missing shapes or discover why some are impossible. Although the parallelogram, the square, and the medium triangle together represent $\frac{3}{8}$ of the area of the 7-piece square, all the shapes that can be made with them are concave. Therefore, in this activity, $\frac{3}{8}$ can only be represented with four pieces. Similar reasoning can explain why $\frac{3}{4}$ cannot be represented with fewer than five pieces. Considering $\frac{5}{8}$ first as $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$, then as $\frac{1}{4} + \frac{1}{4} + \frac{1}{8}$, gives the minimum number of pieces that works. Substituting the small triangles for one medium triangle confirms that $\frac{1}{8} = \frac{2}{16}$ at the same time as it produces an acceptable five-piece shape.