

PROBLEMS

1. **Problem 1**

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Assume that f is continuous at $x=0$. Prove that f is linear, i.e., $f(x) = cx$ for some constant $c \in \mathbb{R}$.

2. **Problem 2**

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$. Assume that f is not identically zero. Prove that $f(x) = e^{cx}$ for some constant $c \in \mathbb{R}$.

3. **Problem 3**