LESSON 2

## Objective

Estimate irrational square roots.

## Common Core State Standards

- **8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

The Number System

# Irrational Square Roots

The square root, $r$, of a number, $x$, is the number such that $r^2 = x$. This also can be written as $\sqrt{x} = r$. In previous work, students have found the principal square roots of numbers. Students need to understand that not all square roots are rational numbers. When a square root is a nonterminating decimal with no repeating pattern, it is irrational. Irrational square roots must be estimated.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *What is the square root of 2? Why is $\sqrt{2}$ irrational? Why must $\sqrt{2}$ be estimated?*
- **Ask:** *How can $\sqrt{2}$ be estimated? Is $\sqrt{2}$ closer to 1 or 2? How did you make the estimate more accurate?*
- **Ask:** *What is the best range for the estimate of $\sqrt{2}$ on the number line?* Have students show the 1.414–1.415 interval. **Ask:** *Can you ever find the exact location?* Emphasize that the exact location is impossible to find because $\sqrt{2}$ is irrational.

## Solve It

Reread the problem with students. Have students draw the square with the diagonal and label it $\sqrt{2}$. Then have them draw the section of the number line with the best range for the estimate indicated.

## More Ideas

For other ways to teach about estimating irrational square roots—

- Have students make a table to find the integers between which the square root of 2 falls using tenths, hundredths, and thousandths increments.
- Have students use their calculators and the Folding Number Line to find the best range for the estimate of $\sqrt{3}$, $\sqrt{5}$, and $\sqrt{7}$.

## Formative Assessment

Have students try the following problem.

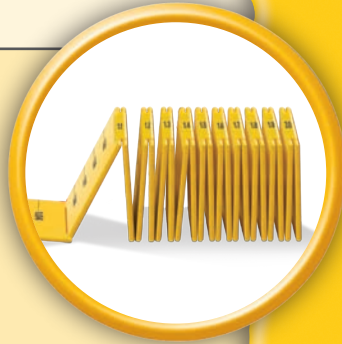
*A picture frame has a square opening that measures 3 inches on each side. The diagonal length is $\sqrt{18}$ inches. Which is the best estimate of this length?*

**A.** between 4 and 5, but closer to 5

**B.** between 4.1 and 4.2, but closer to 4.2

**C.** between 4.23 and 4.24, but closer to 4.24

**D.** between 4.24 and 4.25, but closer to 4.24



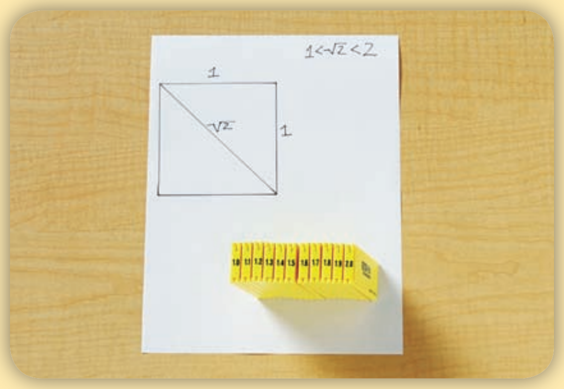
## Try It! 15 minutes | Groups of 4

Here is a problem about estimating irrational square roots.

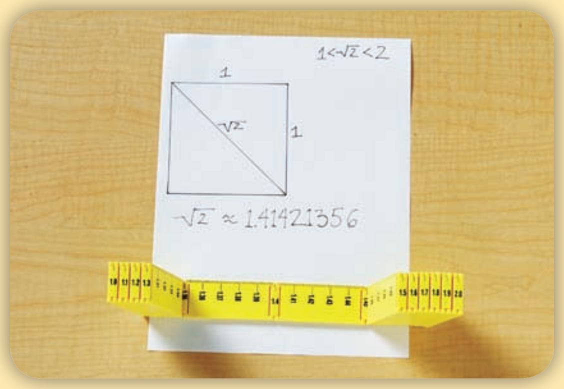
*Carol drew a square with side lengths of 1 inch. Then she drew the diagonal and wanted to know its length. She calculated that it is $\sqrt{2}$. What is the approximate length of the diagonal?*

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

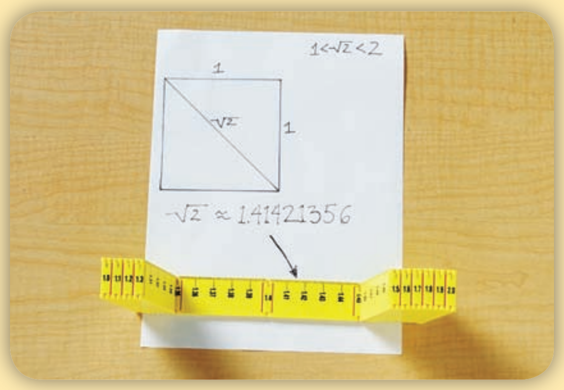
### Materials
- Folding Number Line
- Calculator
- paper (1 sheet per group)
- pencils (1 per group)




**1. Ask:** *Between what two whole numbers is $\sqrt{2}$ found?* Note that 2 is between the two perfect squares 1 and 4 so $\sqrt{2}$ is between $\sqrt{1}$ and $\sqrt{4}$, or between 1 and 2. **Ask:** *Which number line should you use to locate $\sqrt{2}$?* Elicit that the 1–2 number line is the one they should use.




**2.** Have students use their calculators to find $\sqrt{2}$. Explain that a nonterminating number such as this, with no repeating pattern, is an irrational number. **Ask:** *How close to 1 is $\sqrt{2}$?* Have students look at the tenths place in the number to see where $\sqrt{2}$ would be located on the Folding Number Line. Have students find the 1.4–1.5 interval on the line.




**3. Ask:** *How can we make our estimate more accurate? Which hundredths interval should we locate?* Have students find the 1.41–1.42 interval on the line. **Ask:** *On which thousandths interval is $\sqrt{2}$ located?* Students should find the 1.414–1.415 interval.

### ⚠ Look Out!

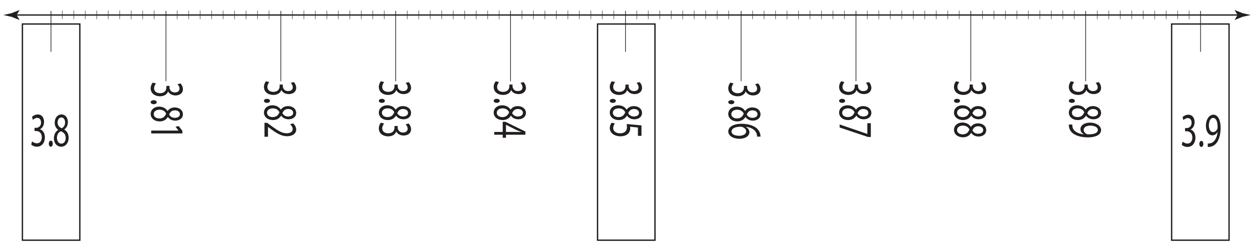
Students sometimes divide by 2 to find the square root. Reinforce the definition of square root, explaining that dividing by 2 is not the same.

**Answer Key**

## Use the Folding Number Line to estimate the square root. Fill in the blanks.

(Check students' work.)

**1.** $\sqrt{15}$




$\sqrt{15}$ is between the whole numbers 3 and 4.

A better estimate is between 3.8 and 3.9.

A better estimate is between 3.87 and 3.88.

A better estimate is between 3.872 and 3.873.

## Using the Folding Number Line, estimate the square root. Fill in the blanks.

**2.** $\sqrt{38}$

$\sqrt{38}$ is between the whole numbers 6 and 7.

A better estimate is between 6.1 and 6.2.

A better estimate is between 6.16 and 6.17.

A better estimate is between 6.164 and 6.165.

## Give the tenths interval on which the irrational number falls.

**3.** $\sqrt{75}$ 8.6–8.7

**4.** $\sqrt{56}$ 7.4–7.5

**5.** $\sqrt{117}$ 10.8–10.9

**6.** $\sqrt{48}$ 6.9–7.0

## *Answer Key*

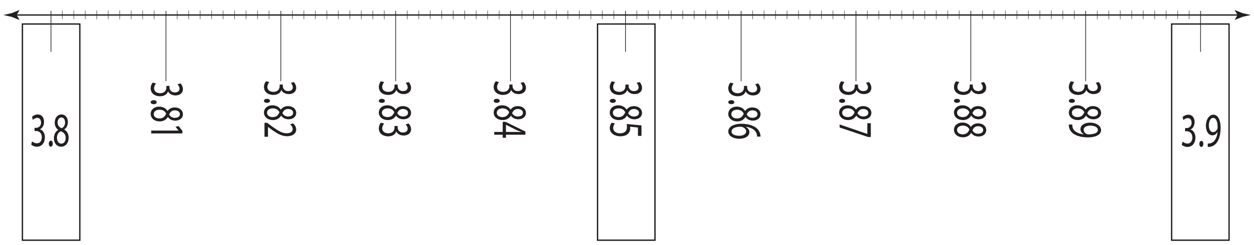
**Challenge!** Using the Folding Number Line, show $\sqrt{17}$ and explain why increasing the number of decimal places in the endpoints of a range makes the estimate of an irrational square root more accurate.

Challenge: (Sample) $\sqrt{17} = 4.123$. Increasing the number of decimal places for the endpoints of a range makes the range smaller. It limits the possibilities for the square root to that smaller range, which means the estimate is more accurate.

Name ______________________________

**Use the Folding Number Line to estimate the square root. Fill in the blanks.**

**1.** $\sqrt{15}$




$\sqrt{15}$ is between the whole numbers ______ and ______.

A better estimate is between ______ and ______.

A better estimate is between ______ and ______.

A better estimate is between ______ and ______.

**Using the Folding Number Line, estimate the square root. Fill in the blanks.**

**2.** $\sqrt{38}$

$\sqrt{38}$ is between the whole numbers ______ and ______.

A better estimate is between ______ and ______.

A better estimate is between ______ and ______.

A better estimate is between ______ and ______.

**Give the tenths interval on which the irrational number falls.**

**3.** $\sqrt{75}$ ______

**4.** $\sqrt{56}$ ______

**5.** $\sqrt{117}$ ______

**6.** $\sqrt{48}$ ______

Name ___________________________________

**Challenge!** Using the Folding Number Line, show $\sqrt{17}$ and explain why increasing the number of decimal places in the endpoints of a range makes the estimate of an irrational square root more accurate.