

This diagnostic test consists of two parts, **Fundamentals** and **Problem Solving**. If your student can solve nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems, then the Art of Problem Solving text **Prealgebra** would only serve as a review for your student. We recommend using the following process to assess whether or not your student has already mastered the contents of the book:

Step 1: The student should attempt all of the questions below without a calculator and without any help. There is no time limit.

Step 2: Check the student's responses using the answer key at the end of this document.

Step 3: The student should be given a second chance on the problems that they answered incorrectly.

Fundamentals

1. **Exponent basics.** Compute:

(a) 2^5

(b) $(-5)^2 - 4^2$

(c) $3^0 + 3^{-2}$

2. **Exponent laws.** Express each of the following as a power of 2:

(a) $2^4 \cdot 2^3$

(b) $\frac{2^8}{2^4}$

(c) $(2^4)^3$

3. **Number Theory basics.**

(a) Find the least common multiple of 54 and 24.

(b) Find the greatest common divisor of 288 and 684.

(c) Find the least common multiple of 72, 120, and 28.

4. **Fraction arithmetic.** Compute:

(a) $\frac{2}{3} + \frac{7}{6} - \frac{1}{4}$

(b) $\frac{3}{4} \cdot \frac{8}{5} \cdot \frac{7}{6}$

(c) $10 \div \frac{2}{3}$

(d) $6\frac{1}{4} - 3\frac{1}{3}$

5. **Fraction and Decimal conversion.** Convert the following decimals to fractions in simplest form:

(a) 0.625

(b) 3.64

(c) $0.\overline{8}$

Convert the following fractions to decimals:

(d) $\frac{13}{4}$

(e) $\frac{7}{20}$

(f) $\frac{5}{6}$

6. **Linear Equations.** Solve each of the following equations:

(a) $3r - 4 = 16 - 7r$

(b) $\frac{2x - 3}{5} = \frac{4 - 3x}{7}$

(c) $2 - \frac{t}{4} = 3 \left(5 - \frac{t}{6} \right)$

7. **Ratio and Rates.**

(a) The ratio of boys to girls at a summer camp is 4 to 5. If the total number of students at the camp is 108, then how many boys are at the camp?

(b) The ratio of teachers to students in a particular school is 1 to 11. The ratio of female students to the total number of students is 4 to 9. If there are 396 female students, then how many teachers are there?

(c) A train is traveling 1 mile every 75 seconds. If the train continues at this rate, then how far will it travel in two hours?

8. **Square Roots.** Simplify each of the following:

(a) $\sqrt{81}$

(c) $\sqrt{1\frac{7}{9}}$

(e) $\sqrt{75} + \sqrt{27} - \sqrt{192}$

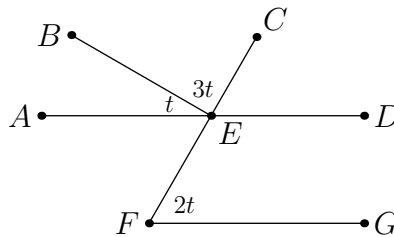
(b) $\sqrt{144}$

(d) $\frac{\sqrt{120}}{\sqrt{3}}$

(f) $3\sqrt{18} + 7\sqrt{8}$

9. Angles.

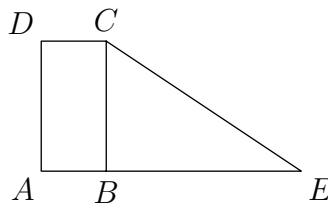
- (a) In the diagram below, \overline{AD} and \overline{CF} intersect at point E , and $\overline{AD} \parallel \overline{FG}$. We have $\angle CEB = 3\angle AEB$ and $\angle EFG = 2\angle AEB$, as shown. Find the measure of $\angle CED$ in degrees.



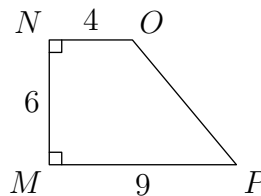
- (b) What is the measure of each interior angle of a regular polygon with 9 sides?

10. Area.

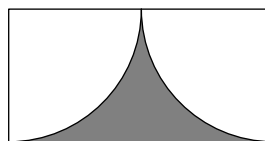
- (a) In the diagram below, rectangle $ABCD$ and right triangle BCE have the same area. Find the ratio AB/AE .



- (b) Find the area of $MNOP$ in the diagram below.



- (c) Two quarter-circles are drawn inside a rectangle as shown below. The two quarter-circles meet at a point on a side of the rectangle. If the radius of each quarter-circle is 6 inches, then what is the area of the shaded region?



11. Pythagorean Theorem.

- (a) Bill walks $\frac{1}{4}$ mile south, then $\frac{3}{4}$ mile east, and finally another $\frac{3}{4}$ mile south. How many miles is he, in a direct line, from his starting point?
- (b) The bases of a 39-foot pole and a 15-foot pole are 45 feet apart, and both poles are perpendicular to the ground. The ground is flat between the two poles. What is the length of the shortest rope that can be used to connect the tops of the two poles?
- (c) The base of an isosceles triangle is 24 and the area of the triangle is 60. What is the length of one of the equal sides?

12. Quadrilaterals.

- (a) If $EFGH$ is a parallelogram and $\angle E = 41^\circ$, then find the other angles of the parallelogram.
- (b) One base of a trapezoid has length 8 inches and the height of the trapezoid is 4 inches. If the trapezoid's area is 80 square inches, then what is the length of the other base of the trapezoid?
- (c) The lengths of the diagonals of a rhombus are 10 inches and 24 inches. What are the perimeter and the area of the rhombus?

13. Basic Statistics.

- (a) Compute the average (mean), median, and mode of the following numbers:

83, 84, 100, 91, 92, 96, 84.

- (b) Suppose you average 82 on your first 7 tests in a class. What must you score on the eighth test to raise your average to 84?

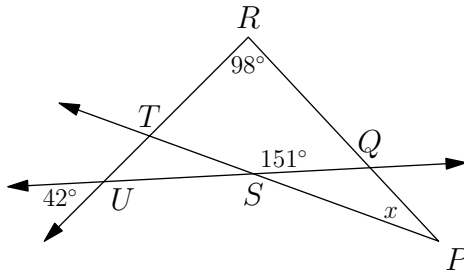
14. Counting and Probability Fundamentals.

- (a) How many multiples of 4 are between 103 and 217?
- (b) A shopkeeper sells house numbers. She has a large supply of the digits 1, 2, 7, and 8, but no other digits. How many different three-digit house numbers could be made using only the digits in her supply?
- (c) Suppose that we roll two fair 6-sided dice. What is the probability that the two numbers rolled sum to 5?

Problem Solving

- 15. What is the value of the sum $5 + 10 + 15 + \cdots + 95 + 100$?
- 16. Express $2^5 \cdot 8^3 \cdot 16^2$ as a power of 4.
- 17. The four-digit number $A55B$ is divisible by 36. What is the sum of A and B ?

18. Find the value of x in the diagram below.



19. Rebecca walks 100 feet in a straight line. She then turns 20 degrees to her left and walks another 100 feet, and then turns 20 degrees to her left again. She continues this pattern until she reaches the point where she started. How far did she walk?
20. In rectangle $ABCD$, point X is the midpoint of \overline{AD} and Y is the midpoint of \overline{CD} . What fraction of the area of the rectangle is enclosed by $\triangle AXY$?

Don't look at the next page until you've attempted all the problems!

The answers to Do You Know Prealgebra are below.

1. (a) 32
(b) 9
(c) $1\frac{1}{9}$ or $\frac{10}{9}$
2. (a) 2^7
(b) 2^4
(c) 2^{12}
3. (a) 216
(b) 36
(c) 2520
4. (a) $\frac{19}{12}$ or $1\frac{7}{12}$
(b) $\frac{7}{5}$
(c) 15
(d) $2\frac{11}{12}$ or $\frac{35}{12}$
5. (a) $\frac{5}{8}$
(b) $\frac{91}{25}$ or $3\frac{16}{25}$
(c) $\frac{8}{9}$
(d) 3.25
(e) 0.35
(f) $0.8\bar{3}$
6. (a) $r = 2$
(b) $x = \frac{41}{29}$
(c) $t = 52$
7. (a) 48 boys
(b) 81 teachers
(c) 96 miles

8. (a) 9
(b) 12
(c) $\frac{4}{3}$ or $1\frac{1}{3}$
(d) $2\sqrt{10}$
(e) 0
(f) $23\sqrt{2}$
9. (a) 60°
(b) 140°
10. (a) $\frac{1}{3}$
(b) 39 square units
(c) $72 - 18\pi \text{ in}^2$ (students may approximate this as 15.45 in^2)
11. (a) $\frac{5}{4}$ miles ($1\frac{1}{4}$ and 1.25 are acceptable)
(b) 51 feet
(c) 13
12. (a) $\angle G = 41^\circ, \angle F = \angle H = 139^\circ$
(b) 32 inches
(c) Perimeter: 52 inches, Area: 120 in^2
13. (a) Average: 90, Median: 91, Mode: 84
(b) 98
14. (a) 29
(b) 64
(c) $\frac{1}{9}$
15. 1050
16. 4^{11}
17. 8
18. 11°
19. 1800 feet
20. $\frac{1}{8}$