

## PROBLEM 1

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function satisfying

$$f(x+y) = f(x) + f(y) \quad \text{for all } x, y \in \mathbb{R}.$$

Suppose that  $f$  is continuous at  $x=0$ .

(a) Show that  $f$  is linear, i.e.,  $f(x) = cx$  for some constant  $c$ .

(b) Show that  $f$  is continuous everywhere.

(c) Show that  $f$  is differentiable everywhere.

(d) Show that  $f$  is a linear transformation.

(e) Show that  $f$  is a linear map.

(f) Show that  $f$  is a linear transformation.