LESSON 2

Geometry

Constant Perimeter and Changing Area

Objective

Observe and generalize about the area enclosed within a constant perimeter.

Common Core State Standards

- **6.G.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

The concepts of perimeter and area are often misunderstood (and sometimes confused) by students who tend to lack real-world experience, such as tiling floors, painting walls, and fencing gardens. This activity gives students hands-on experience with the concept of constant perimeter and addresses the effect that a change in dimension has on the enclosed area.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What was the area of the first "garden" you made with the fencing? What was its perimeter?*
- **Ask:** *What were the areas of the other gardens you made? What were the perimeters?*
- **Ask:** *What conclusion(s) can you draw about perimeter and maximum area (of a rhombus) from this lesson?* (The square has the maximum area.)

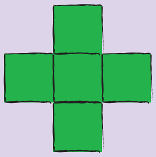
Solve It

Reread the problem with students. Ask them to explain in writing the maximum area possible for a rhombus with a given perimeter. Have them include sketches.

More Ideas

For other ways to teach the concept of constant perimeter and maximum area—

Length	Width	Area
5	1	
4	2	
3	3	



- Have students use Color Tiles to create rectangles with a perimeter of 12 units. Have them use a T-chart to record the areas of the rectangles. Then have students create other shapes with a perimeter of 12 units (see example). What is the area of each?
- Have students use the XY Coordinate Pegboard to create a rectangle with a perimeter of 16 units. Then have them create different rectangles with the same perimeter until they have made all of the possible rectangles. Which rectangle has the greatest area? Which has the smallest area?

Formative Assessment

Have students try the following problem.

What is the largest rectangular area that can be enclosed with 24 feet of fencing?

A. 11 sq. ft **B.** 14 sq. ft **C.** 24 sq. ft **D.** 36 sq. ft

Try It! 20 minutes | Pairs

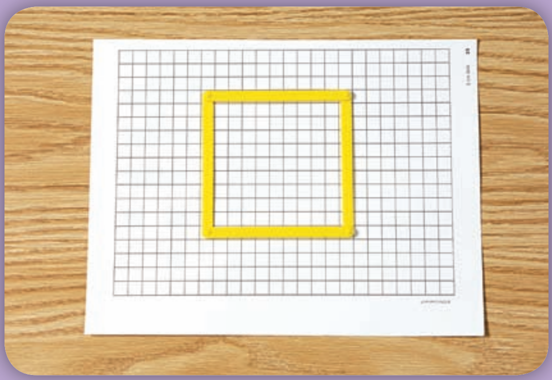
Here is a problem about finding the maximum area that can be contained by a quadrilateral with a given perimeter.

Jesse and Keshia have four 10-foot pieces of edging. They want to use the uncut pieces to enclose a vegetable garden. Keshia wants the garden to be a rhombus so they can avoid the birdbath. Jesse wants a square garden so that it will enclose as large an area as possible. Keshia insists that it doesn't matter. As long as the perimeter is still 40 feet, the area it encloses will always be the same. Who is right?

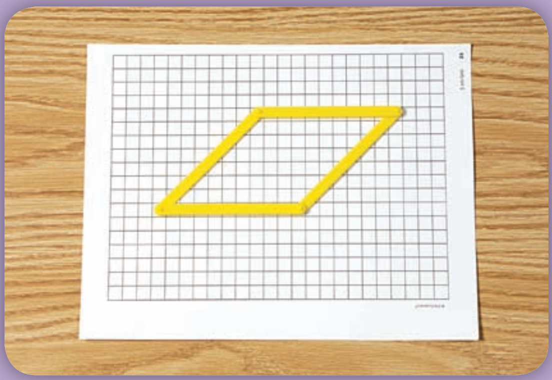
Introduce the problem. Then have students do the activity to solve the problem. Distribute AngLegs and grid paper to students.

Materials

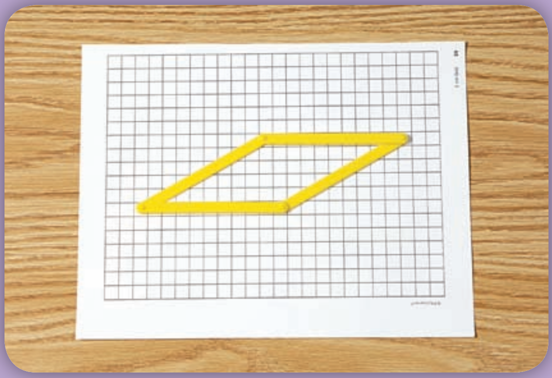
- AngLegs® (4 yellow)
- Centimeter Grid Paper (BLM 10; 1 per pair)



1. Have students construct a rectangle using 4 yellow (10 cm) AngLegs. Have the students place the figure on the grid paper. The figure will be a square, 10 cm on a side. Have students use the formula for the area of a parallelogram ($A = bh$) to find its area.



2. Now have students "collapse" the parallelogram to a height of 8 cm without moving the base. They should use the formula to find the area of the new parallelogram.



3. Have students continue reducing the height of the parallelogram to 6 cm and then to 4 cm. Each time, they should calculate the area of the new parallelogram they form.

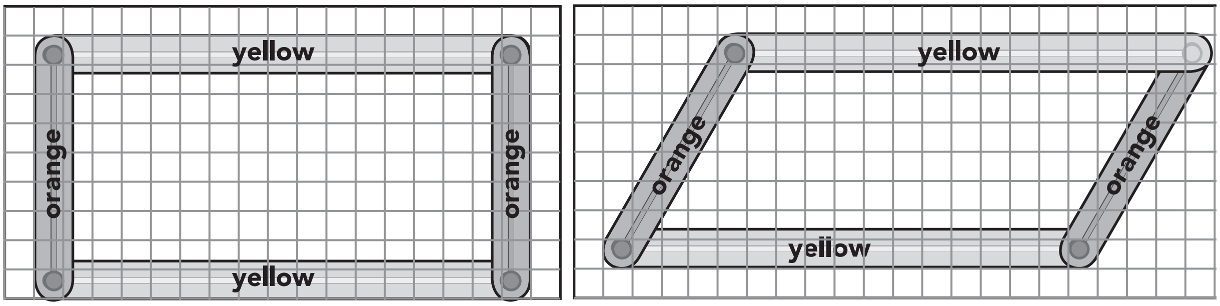
⚠ Look Out!

Students may need to review the definitions of *parallelogram* and *rhombus*. They also may need to review the formula for finding the area of a parallelogram. Also, explain to students that the AngLegs have a groove running lengthwise down the midpoint of each leg. Encourage them to place the AngLegs on the grid paper so that the groove coincides with the lines of the grid whenever possible. This will make it easier for them to determine the dimensions of the various parallelograms.

Answer Key

Use AngLegs® and grid paper to model the shapes shown. Find the perimeter of each shape. Find the area of each shape. (Check students' work.)

1.



Perimeter of rectangle 52 units

Area of rectangle 153 sq units

Perimeter of parallelogram 52 units

Area of parallelogram 136 sq units

Using AngLegs and grid paper, model two shapes that have the given perimeter, but different areas. Name the area of each shape.

2. 50 units

Figure 1

Figure 2

Check students' models; answers will vary depending on models.

Area of Figure 1 ______________

Area of Figure 2 ______________

Find the perimeter and area of each figure.

3.

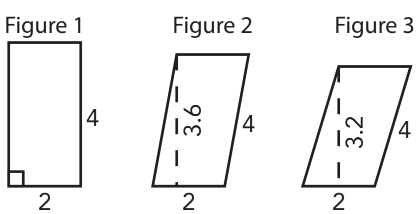


Figure 1: 12 units; 8 units²;
Figure 2: 12 units; 7.2 units²;
Figure 3: 12 units; 6.4 units²

4.

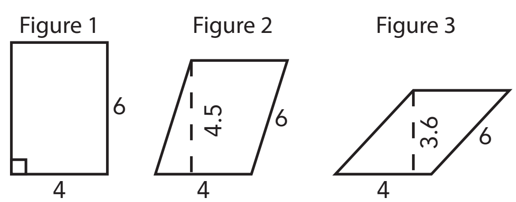


Figure 1: 20 units; 24 units²;
Figure 2: 20 units; 18 units²;
Figure 3: 20 units; 14.4 units²

Answer Key

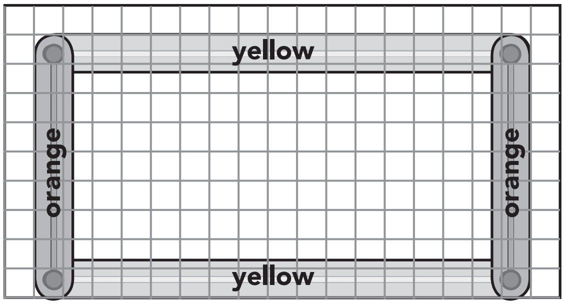
Challenge! How can a rectangle with side lengths of 6 and 10 have a different area than a parallelogram with side lengths of 6 and 10? What do you know about their perimeters? Draw a picture to help.

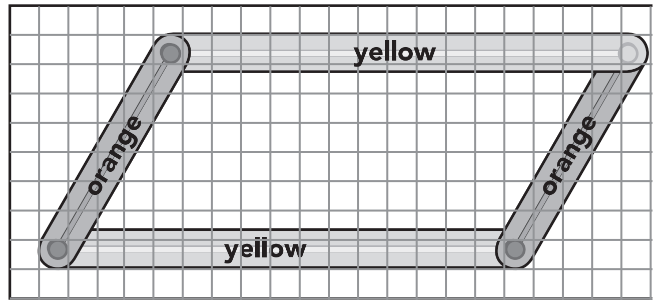
Challenge: (Sample) In the rectangle, the height is equal to one of the side's lengths. In the parallelogram, the height is not equal to one of the side's lengths. So, when you multiply length times height for the rectangle, you are multiplying different numbers than you do when you multiply the length times the height for the parallelogram. Both shapes have the same perimeters because the lengths of their sides are congruent.

Name ______________________________

Use AngLegs® and grid paper to model the shapes shown. Find the perimeter of each shape. Find the area of each shape.

1.





Perimeter of rectangle ________ units

Area of rectangle ___________ sq units

Perimeter of parallelogram ________ units

Area of parallelogram ___________ sq units

Using AngLegs and grid paper, model two shapes that have the given perimeter, but different areas. Name the area of each shape.

2. 50 units

Figure 1

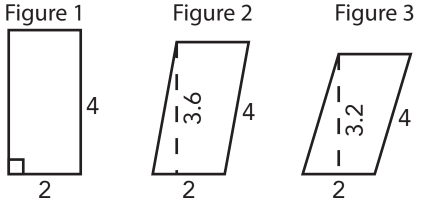
Figure 2

Area of Figure 1 ________________

Area of Figure 2 __________________

Find the perimeter and area of each figure.

3. Figure 1 Figure 2 Figure 3



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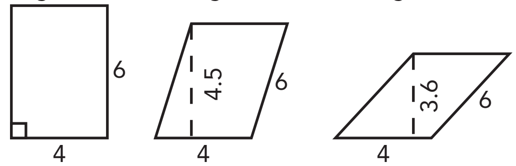
4. Figure 1 Figure 2 Figure 3



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Name ___________________________________

Challenge! How can a rectangle with side lengths of 6 and 10 have a different area than a parallelogram with side lengths of 6 and 10? What do you know about their perimeters? Draw a picture to help.

Name ______________________________