

# LINEAR ALGEBRA

LECTURE 1: VECTOR SPACES

1.1. DEFINITION. A vector space  $V$  over a field  $F$  is a set  $V$  with two binary operations, addition and scalar multiplication, satisfying the following axioms:

- (V1)  $(u + v) + w = u + (v + w)$
- (V2)  $u + v = v + u$
- (V3)  $u + 0 = u = u + 0$
- (V4)  $u + (-u) = 0 = u + (-u)$
- (V5)  $(\alpha u) + (\beta u) = (\alpha + \beta)u$
- (V6)  $\alpha(u + v) = \alpha u + \alpha v$
- (V7)  $(\alpha\beta)u = \alpha(\beta u)$
- (V8)  $1u = u = 1u$



1.2. PROPOSITION. Let  $V$  be a vector space over a field  $F$ . Then the set of all scalar multiples of a fixed vector  $u \in V$ , denoted by  $\langle u \rangle$ , is a subspace of  $V$ .