

Objective

Multiply with fractions, including a fraction with a whole number and a fraction with a fraction.

Common Core State Standards

- **5.NF.5a** Interpret multiplication as scaling (resizing), by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Number and Operations—Fractions

Multiply with Fractions

As students learn how to multiply with fractions, they continue to use the idea of multiplication as repeated addition. The use of models helps them discover why multiplying with fractions results in a product that is less than one or both factors. This is surprising to students because it is different from multiplying whole numbers where the product is greater than the factors (except when a factor is 0 or 1).

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- **Ask:** How can you use repeated addition in this problem?
- **Ask:** Why is the product $\frac{1}{2} \times \frac{1}{3}$ less than $\frac{1}{3}$? Explain the answer in terms of scaling. Elicit that multiplying $\frac{1}{2}$ by any number means you are making the number half as large.

Solve It

Reread the problem with students. Point out that the "of" in $\frac{1}{2}$ of $\frac{1}{3}$ means to multiply, so it can be replaced by the multiplication symbol.

More Ideas

For other ways to teach multiplying with fractions—

- Extend the lesson by using Fraction Tower® Equivalency Cubes to solve this problem: 2 friends share $\frac{3}{4}$ quart of lemonade. What fraction of the lemonade does each friend drink, if they share it equally? Remind students that $\frac{3}{4}$ is $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, so half of $\frac{3}{4}$ is half of $\frac{1}{4}$ taken 3 times. Have students use two $\frac{1}{8}$ pieces to build a tower equal to $\frac{1}{4}$. Have them show that $\frac{1}{8}$ is $\frac{1}{2}$ of $\frac{1}{4}$ and that $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ is $\frac{3}{8}$, so $\frac{1}{2} \times \frac{3}{4}$ is $\frac{3}{8}$.
- Provide different scenarios to give students practice in multiplying fractions and whole numbers, such as $\frac{2}{5} \times 4$ and $\frac{3}{8} \times 3$. Have students use Deluxe Rainbow Fraction® Squares to model and solve the problems.

Formative Assessment

Have students try the following problem.

Kelly wants to decrease a recipe by $\frac{2}{3}$. How much less milk should she use if the recipe calls for 6 cups of milk?

- A. $\frac{1}{6}$ cup B. $\frac{1}{3}$ cup C. 2 cups D. 4 cups

Try It! 20 minutes | Groups of 4

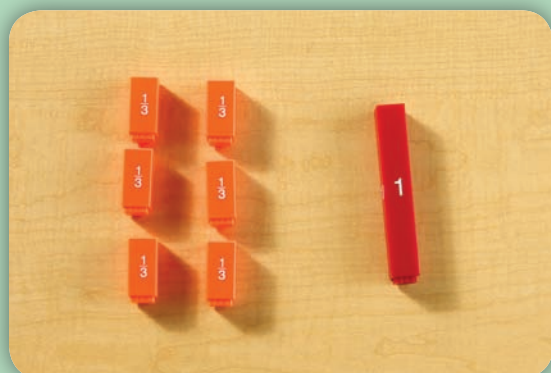
Here is a problem about multiplying with fractions.

Helen has two hot dogs. She cuts each hot dog into thirds to feed to her son, Matt. Matt eats all but $\frac{1}{2}$ of the last piece. How much of a hot dog is left over?

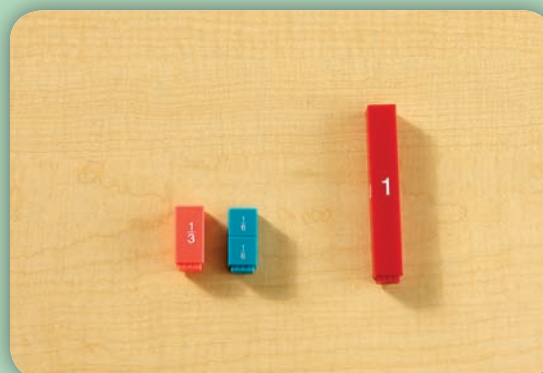
Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Equivalency Cubes, paper, and pencils to students.

Materials

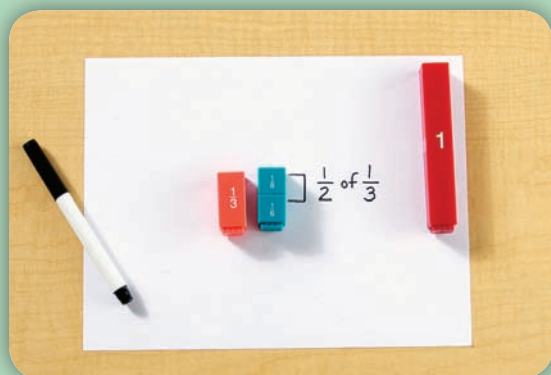
- Fraction Tower® Equivalency Cubes (2 sets per group)
- paper (1 sheet per group)
- pencils (1 per group)



1. Say: You can use repeated addition to solve the problem. Since there are 6 hot dog pieces, you can add $\frac{1}{3}$ six times to represent the total. Have students use six $\frac{1}{3}$ pieces to model the problem. Have students group the pieces and compare them to a whole.



2. Say: Matt eats all but $\frac{1}{2}$ of the last piece. Find the fraction that represents the amount of a hot dog that Matt did not eat. Have students place a $\frac{1}{3}$ piece on the table and then build an equivalent tower using two pieces of the same size. **Ask:** Why does the equivalent tower need to have two pieces? Be sure students realize that the tower must be equivalent to the $\frac{1}{3}$ piece.



3. Ask: How can you use the equivalent tower to find $\frac{1}{2}$ of $\frac{1}{3}$? Have students use a grouping symbol to show that each piece in the tower represents $\frac{1}{2}$ of $\frac{1}{3}$. **Ask:** What is $\frac{1}{2}$ of $\frac{1}{3}$?

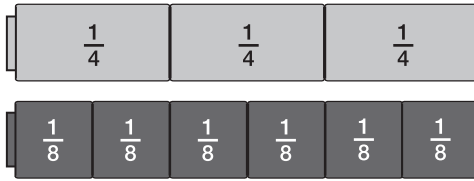
Look Out!

Watch for students who compare the $\frac{1}{2}$ piece to the $\frac{1}{3}$ piece when trying to find $\frac{1}{2}$ of $\frac{1}{3}$. Explain that the equivalent tower needs to show $\frac{1}{3}$ divided into halves. To emphasize this connection, have them trace around the $\frac{1}{3}$ piece and then use the divisions in the $\frac{2}{6}$ tower to divide the diagram into halves. Have them shade $\frac{1}{2}$ of the diagram and then compare it to the $\frac{2}{6}$ tower.

Use Fraction Towers to model the equivalent fractions. Complete the number sentence.

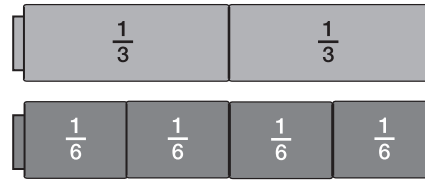
(Check students' work.)

1. What is $\frac{1}{2}$ of $\frac{3}{4}$?



$$\frac{1}{2} \times \frac{3}{4} = \underline{\frac{3}{8}}$$

2. What is $\frac{1}{4}$ of $\frac{2}{3}$?



$$\frac{1}{4} \times \frac{2}{3} = \underline{\frac{2}{12} = \frac{1}{6}}$$

Using Fraction Towers, model each product. Sketch the model. Write a number sentence for each product.

3. What is $\frac{1}{2}$ of $\frac{5}{6}$?

$$\underline{\frac{5}{12}}$$

4. What is $\frac{1}{3}$ of $\frac{3}{4}$?

$$\underline{\frac{3}{12} = \frac{1}{4}}$$

(Check students' models.)

Find each product.

5. $\frac{1}{2} \times \frac{1}{6}$

$$\underline{\frac{1}{12}}$$

6. $\frac{1}{2} \times \frac{2}{5}$

$$\underline{\frac{2}{10} = \frac{1}{5}}$$

7. $\frac{1}{5} \times \frac{5}{6}$

$$\underline{\frac{5}{30} = \frac{1}{6}}$$

8. $\frac{1}{2} \times \frac{3}{5}$

$$\underline{\frac{3}{10}}$$

9. $\frac{1}{3} \times \frac{6}{8}$

$$\underline{\frac{6}{24} = \frac{1}{4}}$$

10. $\frac{1}{2} \times \frac{3}{10}$

$$\underline{\frac{3}{20}}$$

Answer Key

Challenge! Explain why $\frac{1}{2} \times \frac{2}{3}$ is less than 1. Draw a picture to help.

Challenge: (Sample) I am finding one-half of something that is already smaller than 1.

[illegible]

Use Fraction Towers to model the equivalent fractions. Complete the number sentence.

1. What is $\frac{1}{2}$ of $\frac{3}{4}$?



$$\frac{1}{2} \times \frac{3}{4} = \underline{\hspace{2cm}}$$

2. What is $\frac{1}{4}$ of $\frac{2}{3}$?



$$\frac{1}{4} \times \frac{2}{3} = \underline{\hspace{2cm}}$$

Using Fraction Towers, model each product. Sketch the model. Write a number sentence for each product.

3. What is $\frac{1}{2}$ of $\frac{5}{6}$?

4. What is $\frac{1}{3}$ of $\frac{3}{4}$?

Find each product.

5. $\frac{1}{2} \times \frac{1}{6}$

6. $\frac{1}{2} \times \frac{2}{5}$

7. $\frac{1}{5} \times \frac{5}{6}$

8. $\frac{1}{2} \times \frac{3}{5}$

9. $\frac{1}{3} \times \frac{6}{8}$

10. $\frac{1}{2} \times \frac{3}{10}$

Name _____

Challenge! Explain why $\frac{1}{2} \times \frac{2}{3}$ is less than 1. Draw a picture to help.

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are approximately 20 lines visible. The paper has a slight shadow on the right side, suggesting it's resting on a surface.