Ratios and Proportional Relationships

A **ratio** is a comparison of two quantities by division. In normal usage, the "ratio of *a* to *b*" is written $\frac{a}{b}$, *a:b*, or *a* to *b*. For example, if you have 3 tickets to a movie and your friend has 4, the ratio of your tickets to your friend's is $\frac{3}{4}$, 3:4, or 3 to 4.

Ratio comparisons can be part-to-part (red apples to green apples) or part-to-whole (red apples to all the apples in the basket). Further, ratios behave like fractions. For example, the ratio $\frac{4}{6}$ can be reduced to $\frac{2}{3}$.

Rate is usually defined as a ratio that compares quantities having different units of measure. A *unit rate* is expressed as a part-to-one ratio: If you pay $45 for 5 movie tickets, you pay a unit rate of $9 per ticket (a ratio of 9:1).

Proportion is a statement showing that two ratios are equivalent. Proportional relationships can help us find missing information and solve problems. In the apple example, if the ratio of red apples to green apples is 4:5 and there are 12 red apples in the basket, a proportion can be set up and solved to find out how many green apples are in the basket.

The Grade 6 Common Core State Standards for Ratios and Proportional Relationships specify that students should–

- Understand ratio concepts and use ratio reasoning to solve problems.

Students will find that life is filled with opportunities to apply proportional reasoning. Proportional reasoning is required to calculate the cost of a food item when knowing the price per pound, to determine a weekly pay rate given an hourly wage, or to determine a distance represented on a map using the map scale.

The following hands-on activities enable teachers to help students learn the concepts of ratios and proportional reasoning in a rich and meaningful way. Because the concepts are closely related, they can be confusing for students. It is important that students develop a true understanding of ratios and proportions and not just a rote awareness of how to compute with them. Teachers will want to help students discover relationships among specific units and make sense of the quantities they are computing.

Ratios and Proportional Relationships

Contents

LESSON 1

Objective

Use ratios to represent relationships.

Common Core State Standards

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, *"The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

Ratios

Students use ratios to show various relationships between quantities, including whole to part, part to whole, and part to part. With an understanding of ratio, students can engage in proportional reasoning, which is a major component of a student's foundation in math.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How many yellow rods did you use? What is the ratio of yellow to orange? What is the ratio of orange to yellow? Are these ratios the same? Why or why not?*
- **Ask:** *How many red rods make one orange rod? What is the ratio of red to orange? Orange to red? What is the ratio of white to yellow? Yellow to white?*
- **Ask:** *Is the order important when we describe the ratio?*

Solve It

Reread the problem with students. As they represent and identify the ratios, have them write the ratios in words and in numbers. Help them identify whether the ratio is whole to part, part to whole, or part to part.

More Ideas

For other ways to teach ratios—

- Use Two-Color Counters to solve problems such as *The ratio of frogs to ducks in a pond is 5 to 3. Write three ratios to represent this situation.* Have students represent the frogs with the yellow side of the counters and the ducks with the red side of the counters. **Say:** *The total number of ducks and frogs is 8, so the ratio of frogs to the total number of ducks and frogs is 5 to 8.* Guide students to write other ratios.
- Extend the lesson using Cuisenaire® Rods to generate equal ratios. Provide a scenario such as *A trail mix recipe calls for 1 cup of raisins and 3 cups of peanuts. Find the amounts of raisins and peanuts in three different-size batches of this recipe.* Have students build trains on the Centimeter Grid (BLM 10) for each ratio.

Formative Assessment

Have students try the following problem.

There are 3 parrots, 7 parakeets, and 2 finches at a pet store. What is the ratio of parakeets to birds?

A. 12 to 7 **B.** 7:5 **C.** $\frac{7}{12}$ **D.** 5 to 7

Try It!

20 minutes | Groups of 4

Here is a problem about ratios.

An animal habitat includes 3 snakes, 2 alligators, and 5 lizards. What are three ratios you can use to describe the relationships between these animals?

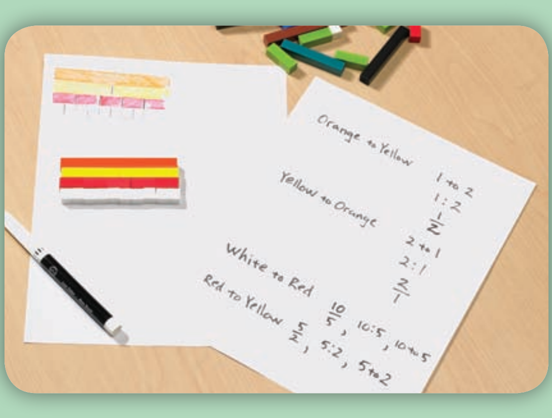
Introduce the problem. Then have students do the activity to solve the problem. Distribute the Cuisenaire Rods, paper, and pencils to students. Explain that ratios compare wholes to parts, parts to wholes, and parts to parts.

Materials

- Cuisenaire® Rods (1 set per group)
- paper (2 sheets per group)
- pencils (1 per group)
- colored pencils (1 set per group)

1. Have students select an orange rod from their Cuisenaire Rods set. Ask them to create a one-color train the same length as the orange rod using only two rods. **Ask:** *Which color did you use?*

2. Have students draw and color their representation. **Ask:** *How many yellow rods does it take to make a train the same length as the orange rod?* Remind students that this comparison of two quantities by division is called a ratio. A ratio comparing two quantities, a and b ($b \neq 0$), can be written in three ways: $\frac{a}{b}$, a:b, or a to b.



3. Instruct students to build more one-color trains the same length as the orange rod. Have them re-create these on their paper and then write the different ratios; e.g., yellow to white, white to orange, and red to yellow. After students display the ability to identify ratios, have them apply their learning to the Try It! question.

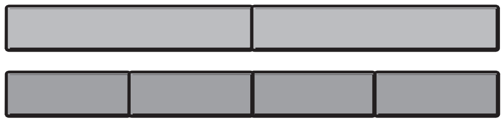
⚠ Look Out!

Some students may write 7 to 3 or 5 to 5 when writing the ratios for animals to snakes or lizards to animals. Stress that snakes and lizards are parts of the total number of animals, so they must include them in the whole when comparing a whole to a part or a part to a whole. Have students write ratios for other whole-to-part or part-to-whole comparisons to make sure they understand the concept.

Answer Key

Use Cuisenaire Rods to model the ratios shown.
Write the ratio three different ways. (Check students' work.)

1.



2:4

2 to 4

$\frac{2}{4}$

Using Cuisenaire Rods, model the ratio. Sketch the model.
Write the ratio two more ways.

2. 1:3

1 to 3

$\frac{1}{3}$

3. $\frac{2}{5}$

2:5

2 to 5

Write each ratio two more ways.

4. 2:3

2 to 3

$\frac{2}{3}$

5. 1 to 5

1:5

$\frac{1}{5}$

6. 3:4

3 to 4

$\frac{3}{4}$

7. $\frac{2}{1}$

2:1

2 to 1

8. 6:1

6 to 1

$\frac{6}{1}$

9. $\frac{2}{8}$

2:8

$\frac{2}{8}$

Answer Key

Challenge! How are the ratios 3 to 7 and 7 to 3 different? Use a diagram to help.

Challenge: (Sample) The ratios are making different comparisons. That's why they are not the same.

LESSON 2

Objective

Use proportions to represent relationships.

Common Core State Standards

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
- **6.RP.2** Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*
- **6.RP.3b** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Proportions

Students use their knowledge of ratios to represent and solve proportions. They use models to find equivalent ratios and use ratios to solve a proportion. They learn that a proportion is a statement that two ratios are equal. These activities develop proportional thinking, which students use to solve problems involving rates, unit conversions, and functions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What ratio is modeled with the rods?*
- **Ask:** *How many green rods would you need to create a train the same length as 15 red rods? How do you know? What operation did you use?*
- **Ask:** *Are the two ratios equal? How do you know?* **Say:** *If two ratios are equal, we call the relationship a* proportion.
- **Ask:** *How many red rods does the proportion require for each green rod?* Elicit that the answer is $1\frac{1}{2}$ (or $\frac{3}{2}$). Tell students that this ratio— $\frac{3}{2}$ red rods per green rod—is called the *unit rate* for the problem.

Solve It

Reread the problem with students. Have students write each proportion represented by the models. Emphasize that only one solution works for the problem, but that any two equal ratios define a proportion.

More Ideas

For other ways to teach proportions—

- Use Pattern Blocks. Suggest to students that a supplier sells 3 blue rhombuses for every 2 red trapezoids. Have students build a blue hexagon on top of a red hexagon to show the 3-to-2 correspondence. Have students determine how many blue rhombuses are sold for every 6 red trapezoids.
- Have students generate equal ratios in tables and use the ratio tables to form proportions. Students work in pairs using Two-Color Counters to form the equal ratios.

Formative Assessment

Have students try the following problem.

The ratio of benches to trees in a park is 2:9. If there are 18 trees, how many benches are in the park?

A. 4 benches **B.** 7 benches **C.** 14 benches **D.** 9 benches

Try It!

30 minutes | Groups of 4

Here is a problem about proportions.

A company makes charms for bracelets. For every 3 hearts, it makes 2 diamonds. If the company makes 15 hearts, how many diamonds does it make?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Cuisenaire Rods, paper, and pencils. Explain that a proportion is formed by two equivalent ratios. Write on the board:

$$\frac{\textit{3 hearts}}{\textit{2 diamonds}} = \frac{\textit{15 hearts}}{\textit{? diamonds}}$$

Materials

- Cuisenaire® Rods (1 set per group)
- paper (2 sheets per group)
- pencils (1 per group)

1. Have students work with the rods to find those that have a 3:2 ratio (e.g., red to green). Observe as students create a train of 3 red rods and build a one-color train using only 2 rods underneath the red train. Have students create a representation of their trains on a separate sheet of paper and write the corresponding ratio.

2. Instruct students to keep the red and green trains. Have them construct a train using 9 red rods to the right of those trains. Have students create their representation on their paper and write the corresponding proportion to be solved.

3. Have students complete the train and their representation. Tell students that a statement that says two ratios are equal is called a *proportion*. Talk with students about "solving a proportion." Equal fractions are equal ratios, so solving a proportion is the same as finding an equivalent fraction. Have students go back and solve the Try It! question.

⚠ Look Out!

Some students might have trouble writing the proportion correctly. Emphasize that the comparison is down or across. Set up a framework for students to fill in:

$$\frac{\square\ \textit{hearts}}{\square\ \textit{diamonds}} = \frac{\square\ \textit{hearts}}{\square\ \textit{diamonds}}$$

or

$$\frac{\square\ \textit{hearts}}{\square\ \textit{hearts}} = \frac{\square\ \textit{diamonds}}{\square\ \textit{diamonds}}$$

Have them fill in 3:2 down and 3:2 across to emphasize the pattern.

Answer Key

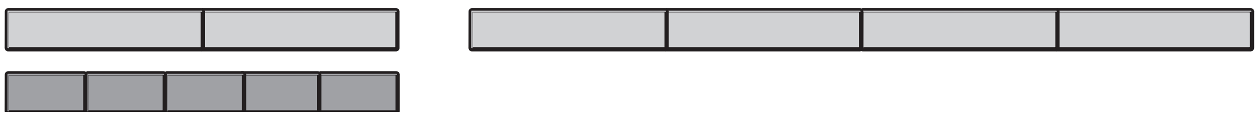
Use Cuisenaire Rods to model each proportion. Then solve the proportion using the rods.

(Check students' work.)

1. $\frac{1}{6} = \frac{2}{?}$

$\frac{1}{6} = \frac{2}{12}$

2. $\frac{2}{5} = \frac{4}{?}$



$\frac{2}{5} = \frac{4}{10}$

Using Cuisenaire Rods, model each proportion. Sketch the model.Then solve the proportion.

3. $\frac{3}{2} = \frac{?}{6}$

$\frac{3}{2} = \frac{9}{6}$

4. $\frac{?}{8} = \frac{1}{4}$

$\frac{2}{8} = \frac{1}{4}$

Solve each proportion.

5. $\frac{2}{3} = \frac{6}{9}$

6. $\frac{1}{3} = \frac{3}{9}$

7. $\frac{5}{6} = \frac{20}{24}$

8. $\frac{4}{5} = \frac{12}{15}$

9. $\frac{4}{8} = \frac{1}{2}$

10. $\frac{2}{5} = \frac{10}{25}$

Answer Key

Challenge! What question do you ask yourself to solve Question 6? What question do you ask yourself to solve Question 9? How do the problems and questions differ?

Challenge: (Sample) How do you get from 3 to 9? How do you get from 8 to 2? The problems differ in that in Question 6 the greater number is the denominator of the second fraction. This means that the answer for 6 is related to multiplication and the answer for 9 is related to division.

LESSON 3

Objective

Determine a ratio and use the ratio to solve a proportion.

Common Core State Standards

- **6.RP.3b** Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Ratio and Proportion: Finding the Ratio

Students have dealt with ratios and proportions, in the form of fraction notation, in previous lessons and grades. For example, 2 to 5 is expressed as 2:5 in ratio notation and $\frac{2}{5}$ in fraction notation. A *proportion* is a statement of equality between two ratios. In this lesson, students will use a proportion to determine an unknown quantity.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is a ratio? Give an example.*
- **Ask:** *What is a proportion? Give an example.*
- **Ask:** *How did you determine the ratio of pints to square feet covered? How did you set up your proportion?*
- **Ask:** *How did you solve the proportion?*

Solve It

Reread the problem with students. Have them determine the ratio between pints of paint and square feet covered. Have students use their ratio to determine the unknown value of a proportion.

More Ideas

For other ways to teach about ratio and proportion—

- Have students repeat this activity by creating the same rectangles on their XY Coordinate Pegboard.
- Have students extend this activity by using proportions to convert the pints mentioned in the story problem into quarts or gallons.

Formative Assessment

Have students try the following problem.

What are the two ratios in the proportion shown here? What is the value of x?

$$\frac{1}{4} = \frac{x}{8}$$

A. $\frac{1}{4}, \frac{x}{8}$; 2 **B.** $\frac{x}{4}, \frac{8}{4}$; 8 **C.** $\frac{1}{x}, \frac{4}{8}$; 4 **D.** $\frac{4}{1}, \frac{8}{x}$; 2

Try It! 20 minutes | Groups of 3

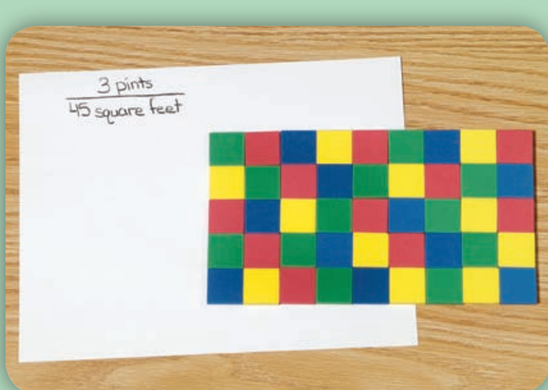
Here is a problem about ratios and proportions.

Felicia is painting the deck of a porch with a textured paint that adds traction to slippery surfaces. She remembers that on a previous project it took 3 pints of paint to cover an area of 45 square feet. What was the ratio of pints to square feet for that job? How many pints will she need for this porch, which has an area of 75 square feet?

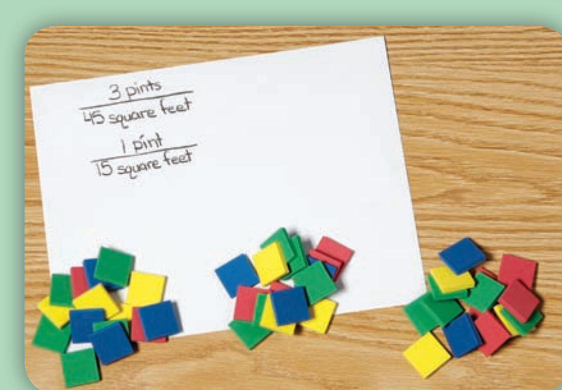
Introduce the problem. Then have students do the activity to solve the problem. Distribute Color Tiles to students.

Materials

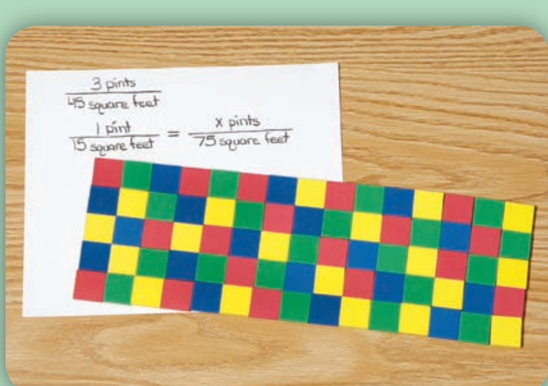
- Color Tiles (75 per group)



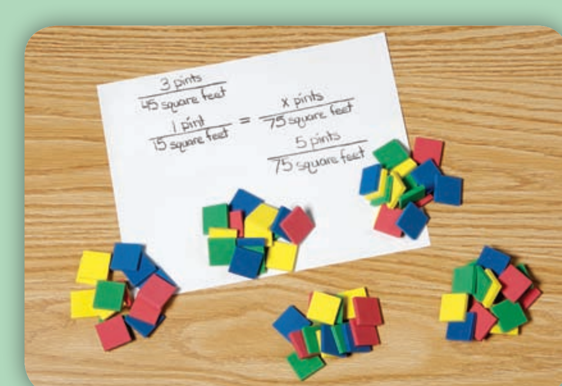
1. Have students make a "porch" surface with 45 Color Tiles. **Ask:** *How many pints of paint are needed to cover 45 square feet? Write this information as a ratio.*



2. Ask: *Based on this ratio, how many square feet will 1 pint of paint cover?* Have students divide their tiles into three equal groups. **Say:** *Write this information as a ratio.*



3. Ask: *What is the area, in square feet, of the porch being painted?* Have students make a "porch" with 75 Color Tiles.



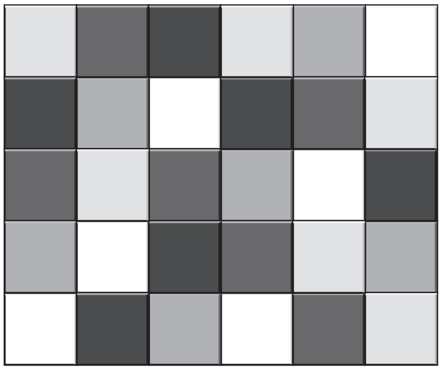
4. Ask: *If 1 pint of paint covers 15 square feet, how many pints will Felicia need to cover 75 square feet?* Have students write a proportion reflecting this information and solve the problem.

Answer Key

Use Color Tiles to model the rectangle shown. Use the ratio represented to write and solve a proportion for the problem.

(Check students' work.)

1. 5 yards of fabric made a banner that was 30 square feet.



How many yards of fabric are needed to make a banner that is 120 square feet?

$\frac{5}{30} = \frac{x}{120}$

20

Using Color Tiles, model a rectangle for the ratio described. Sketch the model. Then write a proportion to solve the problem.

2. 4 cups of flour made 28 dozen cookies.

How many cups of flour are needed to make 35 dozen cookies?

$\frac{4}{28} = \frac{x}{35}$

5

Solve each proportion.

3. $\frac{7}{1} = \frac{49}{7}$

4. $\frac{8}{10} = \frac{28}{35}$

5. $\frac{2}{3} = \frac{12}{18}$

6. $\frac{2}{12} = \frac{10}{60}$

7. $\frac{3}{7} = \frac{12}{28}$

8. $\frac{1}{3} = \frac{12}{36}$

Answer Key

Challenge! If you use a proportion to solve a scale factor problem related to a map, given the scale factor is 1 in. = 12 mi, how do you use the actual distance of 180 miles to find the distance on the map? Write the proportion.

Challenge: (Sample) The 180 miles must be placed in the same position of the fraction as the 12 miles was placed, but in the other fraction; $\frac{1}{12} = \frac{?}{180}$. The numerators in the fractions represent the distances on the map and the denominators in the fractions represent the actual distances.