Functions

In eighth grade, the concept of a **function** is introduced. Students learn that a function is a correspondence that associates each given input with exactly one output. A function is essentially a formalized construct for expressing a mathematical rule.

Functions are the focus for much of algebra and higher mathematics. Students at this level will give special attention to linear functions and will learn to recognize when a function is not linear. They learn that a linear function defines a straight line and can be expressed by the equation $y = mx + b$ where m is the slope of the line and b is the y-intercept.

The Grade 8 Common Core State Standards for Functions specify that students should–

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Thorough understanding of functions is essential for future math success. Teachers will want to offer many opportunities for understanding. The following hands-on activities will help students learn important function concepts in a meaningful way.

Mathematically proficient students look for generalized methods and shortcuts in problem-solving. For example, by paying attention to the calculation of slope as they repeatedly check whether points are on a line, students might generate the related equation. Coaching students to look for such patterns will be important.

Additionally, by drawing an analogy between functions and machines, students can further appreciate the repeated reasoning inherent in functions. Visualizing a function machine as a box open at two ends, students can appreciate the process of putting something in one end, having something happen to it in the middle, and getting something else out from the other end. The function is the machine inside the box, and it is defined by what it does to the input that you give to it. The process never changes. It works the same way every time! Students can look for and expect such logical repetition in functions.

Functions

Contents

LESSON 1

Objective

Graph linear equations; recognize a nonlinear equation.

Common Core State Standards

- **8.F.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Functions

Graphing Linear Equations

A linear function is commonly expressed as a linear equation of the form $y = mx + b$. Students learn to graph linear equations by substituting values for x into the equation, finding the corresponding y values, plotting the x-y pairs on a grid, and drawing a line through the points. Students also should know that many functions are not linear and should recognize the characteristics of nonlinear equations.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *When choosing values of x to substitute into an equation, why is it a good idea to choose at least one positive value, one negative value, and zero?*
- **Ask:** *Is it easier to see if (–3, –4) is a solution to y = 2x + 2 by using the graph or by using the table?* Have students explain their responses.
- Have students plot points on their pegboards for the equation $y = x^2 – 3$, using the values $x = –3, –2, –1, 0, 1, 2,$ and 3. Elicit that the points do not lie on a straight line; the graph is a curve. Note that squaring x is what makes the equation nonlinear. Explain that a linear equation never has x or y raised to a power other than 1. Discuss examples.

Solve It

Reread the problem with students. Have students write a paragraph describing their graphs, focusing on the directions of the graphs, where the graphs intersect each axis, and how close the graphs are to the origin.

More Ideas

For another way to teach about graphing linear equations—

- Have students use pegboards to graph $y = 3x$, $y = x \div 3$, $y = x + 3$, and $y = x – 3$. Ask students to transfer each graph to a separate transparency. Overlay the graphs on an overhead projector and have students compare the graphs.

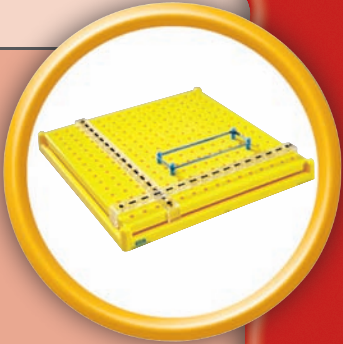
Formative Assessment

Have students try the following problem.

Which ordered pair is a solution to the linear equation y = x + 6?

A. (1, 6) **B.** (1, 7) **C.** (6, 1) **D.** (7, 1)

Try It! 30 minutes | Pairs



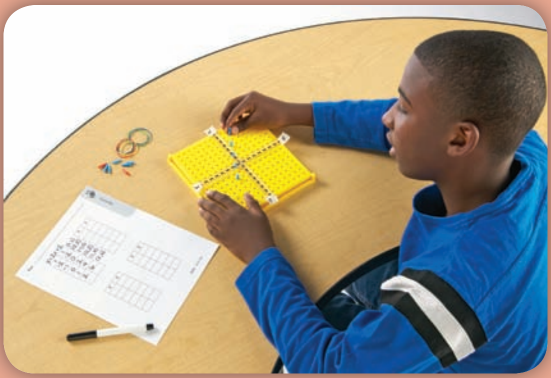
Here is a problem about graphing linear equations.

Two hikers are finding their way to camp. There are two trails they could follow. One trail can be mapped on a four-quadrant grid using the equation $y = 2x + 2$. The other trail can be mapped using the equation $y = x + 3$. If camp is located at the origin, which trail will take the hikers closer to camp?

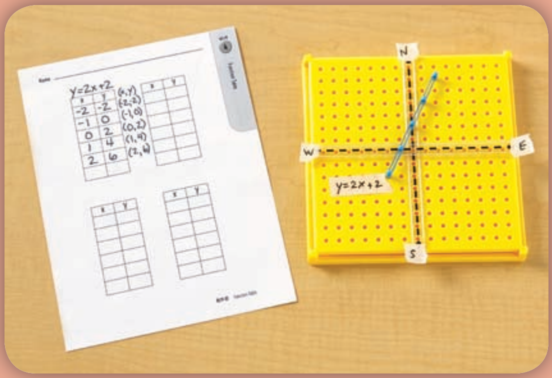
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials. Ask students to set up their pegboards for 4-quadrant graphing. Have them use masking tape to label the *x*-axis East on the right and West on the left and the *y*-axis North at the top and South at the bottom.

Materials

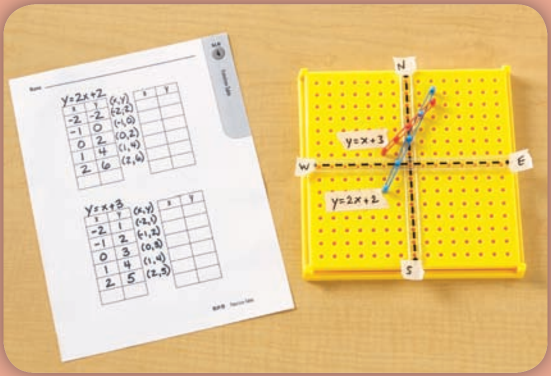
- XY Coordinate Pegboard
- Function Tables (BLM 4; 1 per pair)
- masking tape (1 six-inch strip per pair)



1. Have students write $y = 2x + 2$ at the top of the first table and fill in the x column with the values –2, –1, 0, 1, and 2. **Say:** *Substitute each value of x into the equation and find the corresponding value of y. Record the value. Then use the two values to write an ordered pair next to the table. Plot the ordered pairs on the pegboard.*



2. Ask: *Do the pegs appear to form a straight line?* Elicit that the graph for a *linear equation* is a straight line. Have students stretch a rubber band from the peg at (–2, –2) to the peg at (2, 6). Ask them to label the line $y = 2x + 2$ using a piece of masking tape.



3. Guide students through the process once again as they graph $y = x + 3$. Have students describe each graph and then compare the two graphs. **Ask:** *Does either graph pass through the origin? Is there any point that is on both graphs?*

⚠ Look Out!

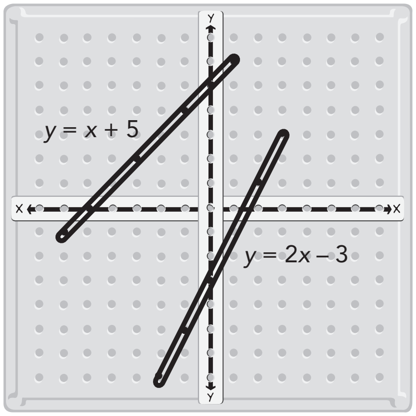
Some students might confuse the *x*- and *y*-coordinates or the *x*- and *y*-axes. Remind them that the ordered pair is arranged in alphabetical order so the *x*-coordinate is listed first.

Answer Key

Use an XY Coordinate Pegboard to graph each line. Make a table of ordered pairs for each line.

(Check students' work.)

1.



$y = x + 5$

x	1	0	–1	–3	–5
y	6	5	4	2	0

$y = 2x - 3$

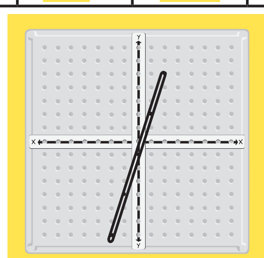
x	3	2	0	–1	–2
y	3	1	–3	–5	–7

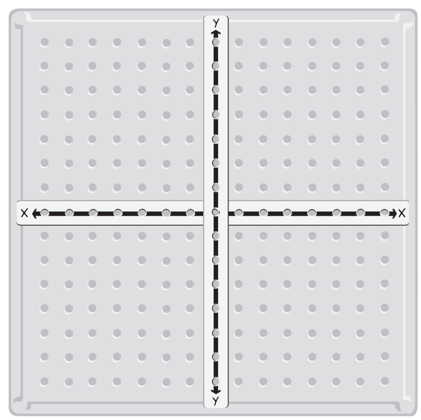
Using an XY Coordinate Pegboard, graph the line on a coordinate plane. Make a table of ordered pairs for the line.

(Check students' models.)

2. $y = 3x - 1$

x	2	1	0	–1	–2
y	5	2	–1	–4	–7





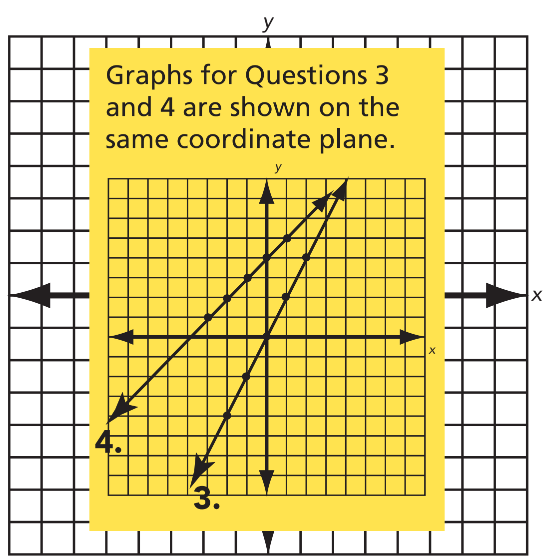
Make a table of ordered pairs for each equation. Graph and label each line on the coordinate plane.

3. $y = 2x$

x	2	1	0	–1	–2
y	4	2	0	–2	–4

4. $y = x + 4$

x	1	0	–1	–2	–3
y	5	4	3	2	1



Answer Key

Challenge! Use the equations $y = x - 1$ and $y = x^2 - 3$ to show how two points can be on the graph of a linear equation and also on the graph of a nonlinear equation.

Challenge: Students should generate points for both equations, graph the line and the curve, and note that the points (–1, –2) and (2, 1) are on both graphs.

LESSON 2

Objective

Graph functions based on input/output coordinate pairs and determine if the function is linear or not.

Common Core State Standards

- **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Functions

Linear Functions

As the mathematics experiences at the middle school level become increasingly algebraic, students begin working with the concept of linearity as part of their study of patterns and relationships. Students' ability to determine whether a function is linear or not paves the way for an understanding of more complex concepts, such as rate of change, slope, and *y*-intercept, that will ensue.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What did you plot along the x-axis? What did you plot along the y-axis? What does each peg represent? Why do we start with (0, 3)?*
- **Ask:** *How do you have to "move" in order to get from one peg to the next? (What is the pattern?)*
- **Ask:** *How long will it take Sam to save $5? $9?* **Say:** *Continue the graph to find out how much he will save over time.*
- **Ask:** *What does the word* linear *mean? Why do you think this function is called a linear function?*
- **Ask:** *How would the graph be different if Sam did not have any money when he started saving?*

Solve It

Reread the problem with students. Ask them to write the definition of a linear function and sketch an example.

More Ideas

For other ways to teach about linear equations—

- Have students create an input/output table of values for the expression $2x + 5$. Have them plot the coordinate values on a pegboard and compare the resulting graph to the expression $2x + 3$. Encourage students to talk about how the constant affects the values in each expression.
- Have students create an input/output table of values for the expression $x + 3$. Have students plot the coordinate values on a pegboard, including some negative values. Have students talk about real-life situations in which negative values could apply and situations in which they could not.

Formative Assessment

Have students try the following problem.

Which of the following functions is not linear?

A.

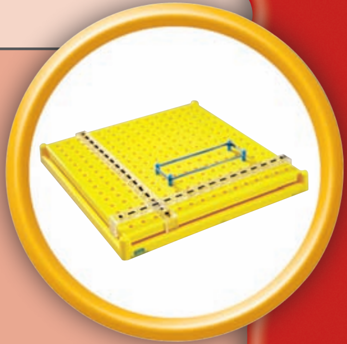
x	*y*
0	2
1	1
2	0

B.

x	*y*
-2	2
-1	5
0	8

C. $y = 3 + x^2$

D. $y = 4x + 1$



Try It!

30 minutes | Groups of 4

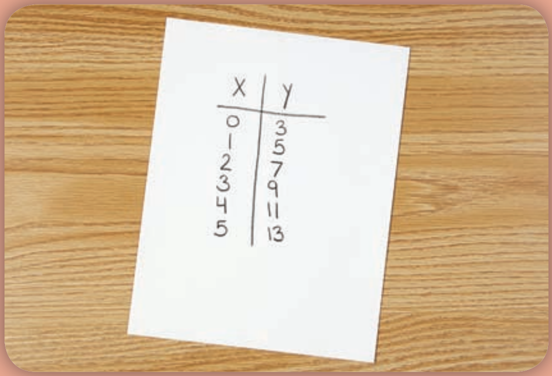
Here is a problem about linear functions.

Sam is saving his money to buy a DVD of one of his favorite movies. He already has $3 saved. If he saves $2 each week, at the end of what week will he have saved enough money to buy the DVD for $11.99?

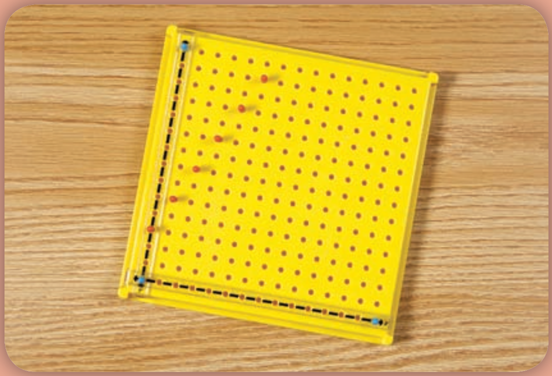
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

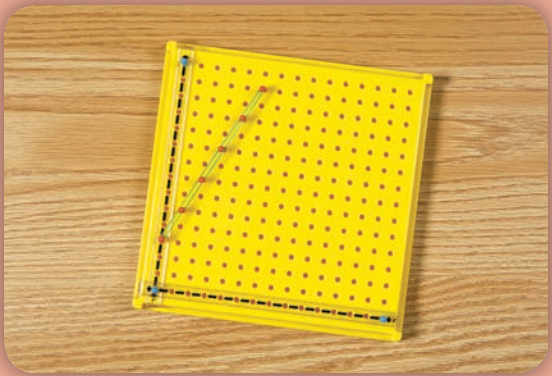
- XY Coordinate Pegboard
- paper (1 sheet per group)
- pencils (1 per group)



1. Have students create an input/output table, beginning with the coordinate pair (0, 3). Have students discover that as each *x*-coordinate increases by 1, the *y*-coordinate should increase by 2, representing the $2 increase for each week.



2. Have students set up their pegboards for Quadrant I graphing. Have them peg (0, 3) to represent the $3 Sam already has saved. Have students use the pattern of “up two, right one” to match the coordinate pairs in their input/output table.



3. Continue to plot the coordinates until the students are able to answer the question: At the end of which week will Sam have enough money to buy the DVD for $11.99?

⚠ Look Out!

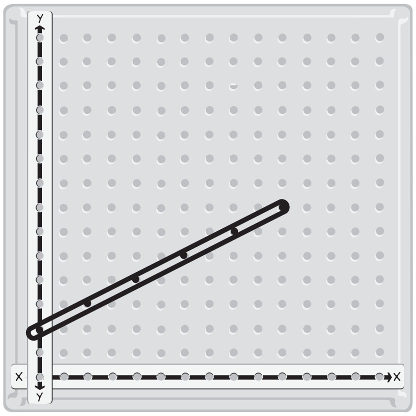
Some students may need help understanding what the new coordinate pairs mean with respect to the original problem. They may get caught up in the graphing and forget that each *x*-coordinate represents the number of weeks Sam saves his money toward the DVD purchase, and that each *y*-coordinate represents the total amount of money he has saved to date.

Answer Key

Use an XY Coordinate Pegboard to model the graph of the function. Make a table of the ordered pairs. Is the function linear? Write yes or no.

(Check students' work.)

1.



x	*y*
0	2
2	3
4	4
6	5
8	6
10	7

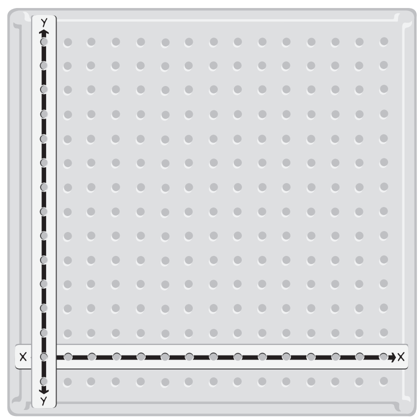
yes

Using an XY Coordinate Pegboard, graph the function. Sketch the graph of the function. Is the function linear? Write yes or no.

(Check students' models.)

2.

x	*y*
0	4
1	5
2	7
3	9
2	1
4	5



no

Determine if each function is linear. Write yes or no.

3.

x	*y*
1	2
3	4
5	6
7	8
9	11
11	13

no

4.

x	*y*
4	1
1	4
5	2
2	5
6	3
3	6

no

5.

x	*y*
0	3
1	3
2	3
3	3
2	3
4	3

yes

Answer Key

Challenge! How can you determine without graphing a set of ordered pairs if the set of ordered pairs models a linear function? Show an example.

Challenge: (Sample) Choose two ordered pairs from the table. Find the difference in the *x*-values. Find the difference in the *y*-values. Write a ratio of the difference in the *y*-values to the difference in *x*-values. Repeat this process for all combinations of ordered pairs. If all ratios are equal, the ordered pairs model a function.

LESSON 3

Functions

Lines in Slope-Intercept Form

Objective

Given a set of ordered pairs, graph the line and write an equation for it in $y = mx + b$ form.

Common Core State Standards

- **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

By now students are familiar with equations in the form of $y = mx + b$. They've learned that the slope (m) means "the rise over the run" and that the y-intercept (b) is the point at which the line intersects the y-axis. This activity will help students determine a line, in slope-intercept form, from a table of values.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Why is the* x*-value of the* y*-intercept always zero?*
- **Ask:** *What are the other points on this line? How can we use the equation to check?*
- **Ask:** *How can you use the equation to show that (6, 3) is not on the line?*

Solve It

Reread the problem with students. Ask students to explain in writing how knowing the y-intercept and the slope of a line helps them graph the line.

More Ideas

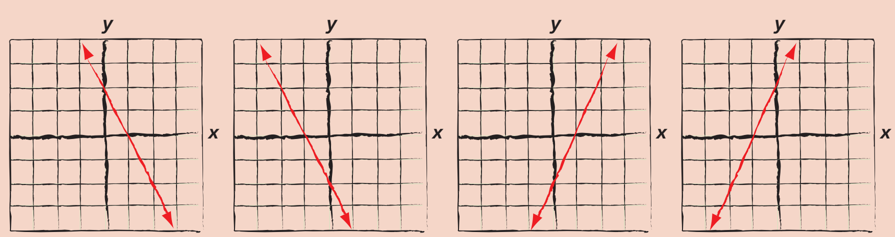
For another way to teach about slope and intercept—

- Have students work in pairs. Have them set up their pegboards for Quadrant I graphing. One student places a peg anywhere on the y-axis and another peg anywhere in the quadrant. The other student stretches a rubber band between the two pegs and then writes the equation for the line using the $y = mx + b$ format. Have students alternate roles and repeat several times.

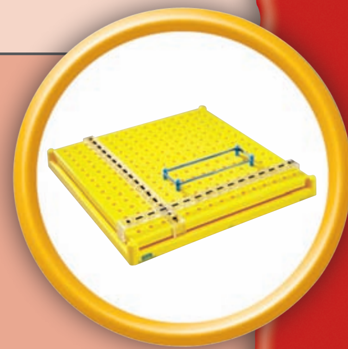
Formative Assessment

Have students try the following problem.

Which graph below is the graph of the equation $y = 2x + 2$*?*



A. B. C. D.



Try It! 20 minutes | Pairs

Here is a problem about graphing a line in slope-intercept form.

Peter would like to visit his grandmother, but he must take a taxicab to her house. (She will give him a ride home.) The table below shows the rates the cab company charges.

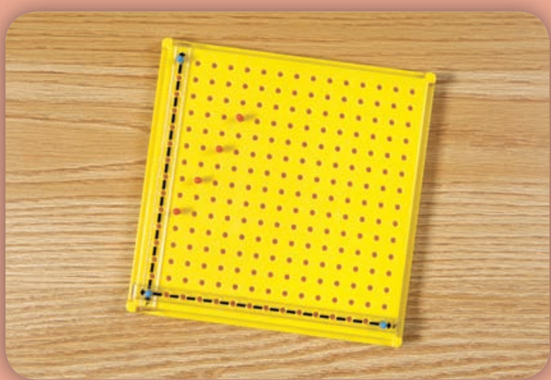
Miles	1	2	3	4
Total	\$5	\$7	\$9	\$11

How much will it cost Peter for cab fare if his grandmother lives 7 miles away?

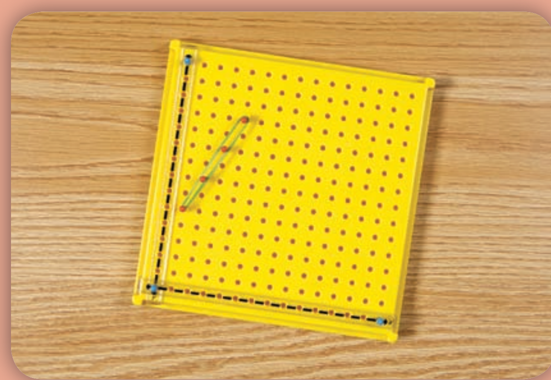
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

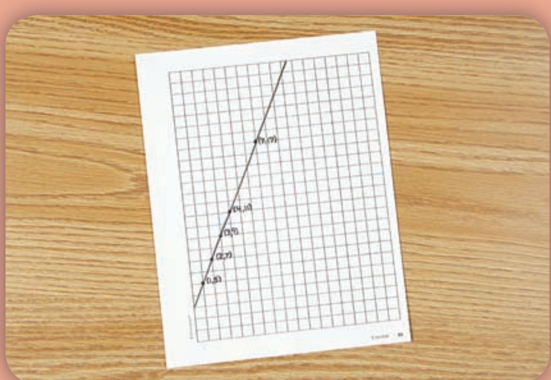
- XY Coordinate Pegboard
- $\frac{1}{4}$-Inch Grid Paper (BLM 6; 1 per student)



1. Have students set up their pegboards for Quadrant I graphing. Then have them plot the values from the table.



2. Have students stretch a rubber band from (1, 5) to (4, 11). **Ask:** *How do you determine the slope of the line?* Elicit from students that you go "up 2 and over 1." **Say:** *This means that the ratio of rise over run is 2:1.*



3. Have students transfer their graphs to the grid paper. **Ask:** *How do you determine the* y*-intercept of the line?* Make sure students understand that in order to find the *y*-intercept they must determine the value of *y* when $x = 0$. They can do this by extending the line through the *y*-axis. Have students write the equation of the line in the form of $y = mx + b$. **Ask:** *How much will it cost Peter for cab fare if his grandmother lives 7 miles away?*
Have students locate the correct value on their graph.

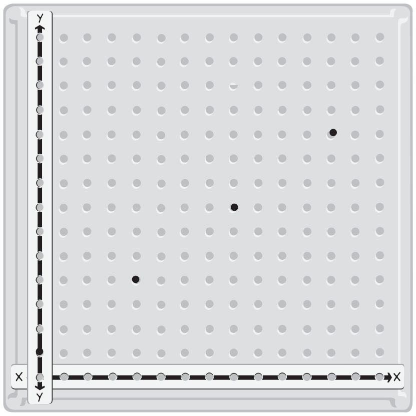
⚠ Look Out!

Some students may confuse the *y*-intercept with the *x*-intercept. Explain that when *x* is 0, the point is on the *y*-axis. That is why that point is called the *y*-intercept.

Answer Key

Use an XY Coordinate Pegboard to plot the ordered pairs. Make a table. Write the equation of the line in the form $y = mx + b$. (Check students' work.)

1.



x	y
0	1
4	4
8	7
12	10

$m = \frac{3}{4}$

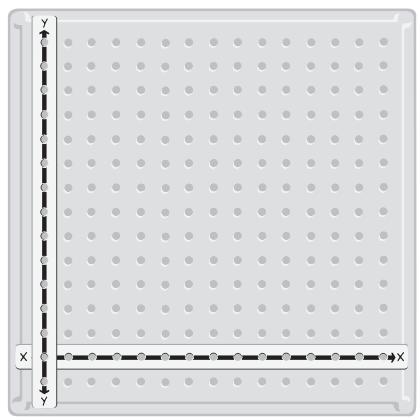
$b = 1$

$y = \frac{3}{4}x + 1$

Using an XY Coordinate Pegboard, model the line that contains the ordered pairs in the table. Sketch the model. Write the equation of the line in the form $y = mx + b$. (Check students' models.)

2.

x	y
0	1
1	4
2	7
3	10
4	13



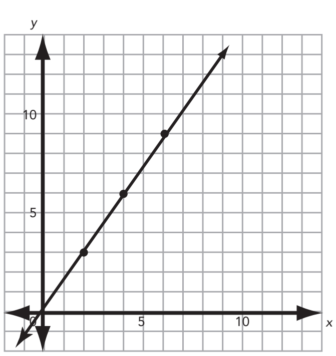
$m = 3$

$b = 1$

$y = 3x + 1$

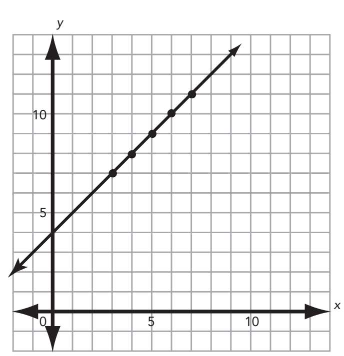
Write the equation of each line in the form $y = mx + b$.

3.



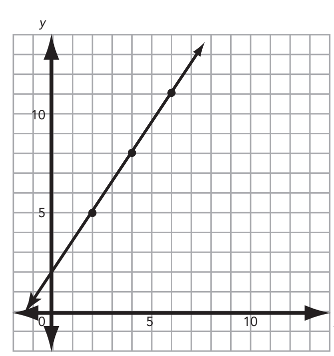
$y = \frac{3}{2}x$

4.



$y = x + 4$

5.



$y = \frac{3}{2}x + 2$

Answer Key

Challenge! Describe how to graph a line if all you know are the slope and *y*-intercept of the line. Draw a picture to help.

Challenge: (Sample) The slope and the *y*-intercept can be substituted into $y = mx + b$. Then from the *y*-intercept, use the slope to find another point on the graph and connect the points to graph the line.

LESSON 4

Functions

Symbolic Algebra

The National Council of Teachers of Mathematics states "It is essential that they [students] become comfortable in relating symbolic expressions containing variables to verbal, tabular, and graphical representation of numerical and quantitative relationships." (*Principles and Standards for School Mathematics,* page 223) To this end, this activity focuses on an introductory story and the link between its representational algebraic expression, its values in tabular form, and its graph.

Objective

Determine the link between an algebraic equation, a corresponding table of values, and the resulting graph.

Common Core State Standards

- **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *According to the story problem, what should be the value of* y*?*
- **Ask:** *What is a good rule of thumb for determining what each peg value should be based on a given table of values?*
- **Ask:** *Look at the pegboard. What is the* y*-intercept for your equation? Why isn't it 2?* Elicit from students that each hole on the *y*-axis represents $15 in order to fit the graph on the pegboard.

Solve It

Reread the problem with the students. Have them write the algebraic equation in the form of $y = mx + b$. Then have students create a table of values based on the equation. After the table is complete, have students graph the coordinates. Finally, have students read the results and indicate when (at the end of what week) Joshua will have enough money to buy the guitar.

More Ideas

For another way to teach about writing algebraic equations—

- Have students budget for something they would like to buy. **Ask:** *How much money can you put toward your purchase right now? How much can you afford to put aside each week?* **Say:** *Write an equation for your savings plan in slope-intercept format. Create a table and graph the coordinates.* **Ask:** *How long will it take you to save enough to buy what you want?*

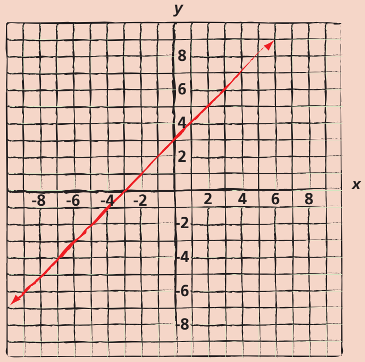
Formative Assessment

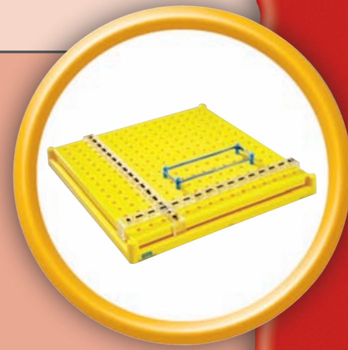
Have students try the following problem.

Which equation matches the graph?

A. $y = x + 3$ **B.** $y = 3x$

C. $y = \frac{9}{5}x + 32$ **D.** $y = 5$





Try It! 30 minutes | Pairs

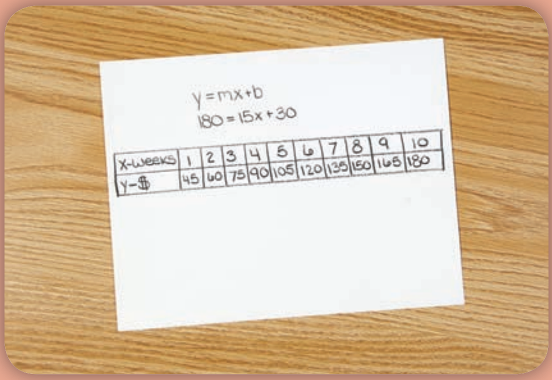
Here is a problem about solving a word problem using algebraic symbols, a table, and a graph.

Joshua wants to buy an electric guitar that costs $180. He already has saved $30. He also plans to save $15 a week toward his purchase. How many weeks must Joshua save his money before he can buy the guitar?

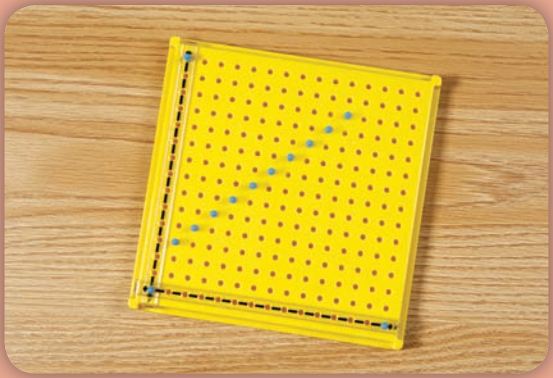
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials
- XY Coordinate Pegboard
- paper (1 sheet per pair)
- pencils (1 per pair)

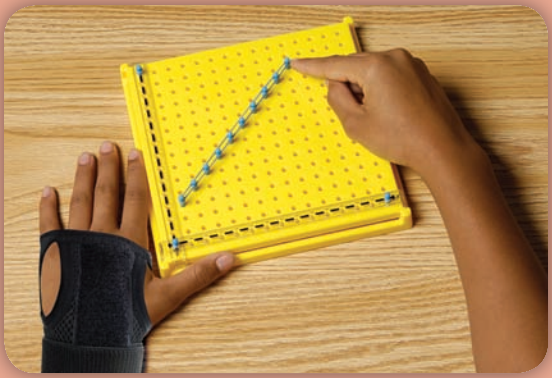
1. Help students set up an algebraic equation to represent the word problem. Have them determine what the unknown variable (*x*) will represent. Have students write the algebraic equation in slope-intercept form ($y = mx + b$).



2. Have students create a table of values for the equation, beginning with $x = 1$ and ending with $x = 10$.



3. Have students set up their pegboards for Quadrant I graphing. **Say:** *Look over the values on your table.* **Ask:** *What should each peg on the* y*-axis represent? What should each peg on the* x*-axis represent?* Elicit from the students that each *x*-value should represent one week and that each *y*-value should represent $15.

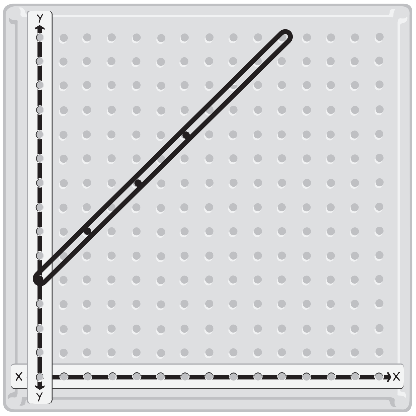


4. Ask students to connect the points with a rubber band to show the linear function. Have students point to the solution on their graphs.

Answer Key

Use an XY Coordinate Pegboard to graph the line shown on the grid. Make a table of ordered pairs for six points on the line. Write an equation for the line in the form $y = mx + b$. (Check students' work.)

1.

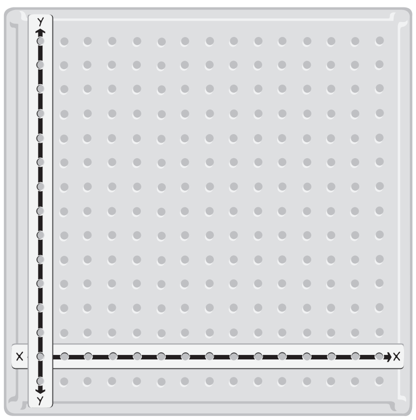


x	y
0	4
2	6
4	8
6	10
8	12
10	14

$y =$ $x + 4$

Using an XY Coordinate Pegboard, graph the line for the equation given. Sketch the model. Make a table of ordered pairs for the line. (Check students' models.)

2. $y = \frac{1}{2}x + 3$



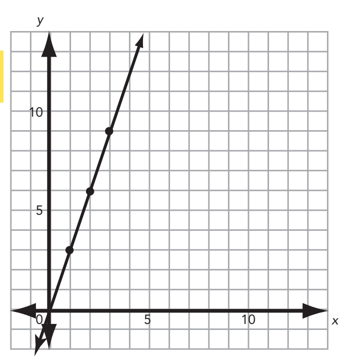
x	y
0	3
2	4
4	5
6	6
8	7
10	8

Match each representation in the top row with its equation.

3. a

x	y
0	1
1	2
2	3
3	4
4	5

4. c



5. b

x	y
0	1
2	5
4	9
6	13
8	17

a. $y = x + 1$

b. $y = 2x + 1$

c. $y = 3x$

Answer Key

Challenge! For Questions 3 and 5, what information for the equation did you get directly from the table and what information did you have to make a calculation to find? Explain.

Challenge: (Sample) The y-intercept can come directly from the table, but the slope has to be calculated.

LESSON 5

Objective

Construct a function to model a linear relationship.

Common Core State Standards

- **8.F.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Constructing Functions

In this activity, students have the opportunity to write the equation of a line that is graphed using problem-based data. This activity can be used as a quick review of rate of change and y-intercept, as well as of linearity.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is the value of* y *for* $x = 0$*? Why?*
- **Ask:** *Do you find it easier to determine the rate of change of a line from a table of values or from a graph? Explain.*
- **Ask:** *How do you know for sure that the equation is* $y = \frac{4}{3}x$*?*

Solve It

Reread the problem with students. After students have graphed the coordinate pairs and determined the equation, have them substitute values for y. **Ask:** *Day camp costs $36. How much of this fee will Joe pay, and how much will his grandfather pay?*

More Ideas

For other ways to teach about constructing functions to model linear relationships—

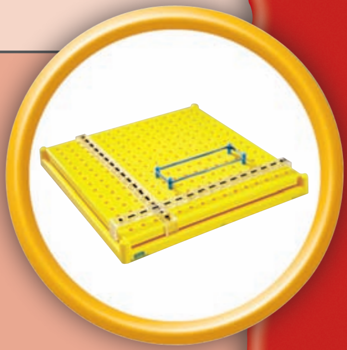
- Have students use Algeblocks® sets to represent various linear and nonlinear relationships and write equations from the representations.
- Extend the activity to include a constant by giving students the points (0, 2), (3, 4), and (6, 6) and challenging them to graph the points and write the equation of the line.

Formative Assessment

Have students try the following problem.

Jaimie is participating in a fundraiser for school. For every 5 magazines she sells, her mother will purchase 2. Write an equation to represent Jaimie's total magazine sales.

A. $y = 5x + 2$ **B.** $y = \frac{5}{2}x$ **C.** $y = \frac{7}{5}x$ **D** $y = 2x + 5$



Try It! 20 minutes | Pairs

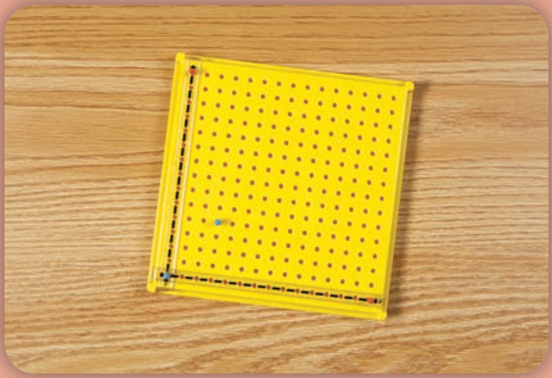
Here is a problem about constructing a function to model a linear relationship.

Joe wants to save money so that he can go to day camp this summer. His grandfather promises him that for every $3 he saves, he'll give him a dollar. Construct a model for the total amount of money Joe has for camp as a function of the amount he saves.

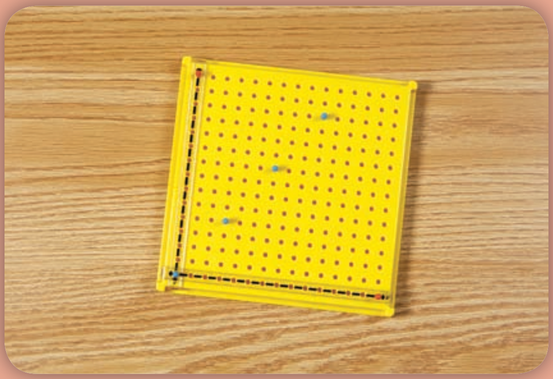
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials
- XY Coordinate Pegboard

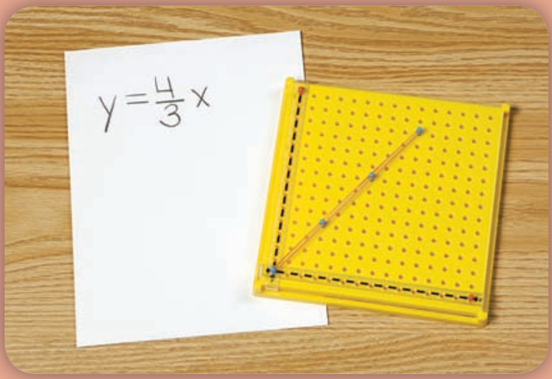
1. Discuss with students how to set up their graphs. **Say:** *The* x*-axis will represent how much money Joe has saved, and the* y*-axis will represent the total amount of money he has, including the money his grandfather gives him.*



2. Ask: *At which coordinate pair should the graph begin?* Elicit from students that the graph should start at (0, 0), because if Joe doesn't save any money, he won't receive any from his grandfather. **Ask:** *How much money will Joe have if he saves $3?* After students have stated their answer, have them peg the coordinate (3, 4).



3. Say: *Determine the rate of change based on the two coordinate pairs.* Make sure students see that the rate is "up 4, over 3." **Ask:** *How much should each peg be worth on the* x*-axis? On the* y*-axis?* Make sure students understand that each peg should be worth $3 on the *x*-axis and $4 on the *y*-axis. Have students peg an additional ordered pair.

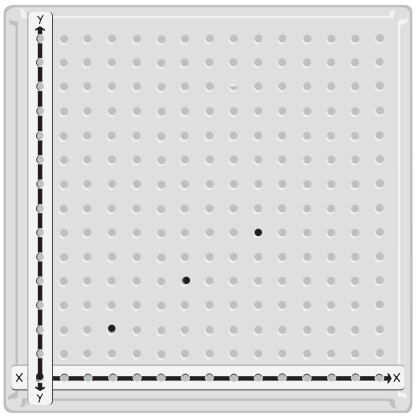


4. Have students write the equation of the line they formed.

Answer Key

Use an XY Coordinate Pegboard to model the line determined by the points shown on the grid. Write the equation for the line. Answer each question.

(Check students' work.)

1. 

equation: $y = \frac{2}{3}x$

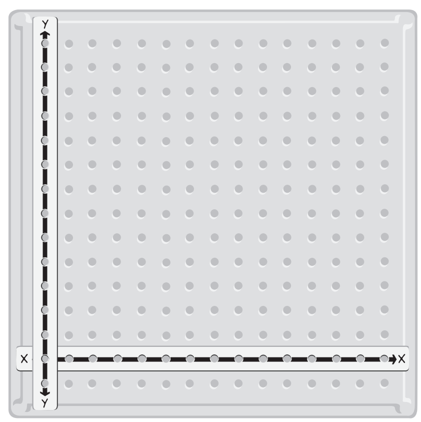
What is the ordered pair for the point on the line when x = 30? (30, 20)

What is the ordered pair for the point on the line when x = 90? (90, 60)

Using an XY Coordinate Pegboard, model the line determined by the ordered pairs given. Write an equation for the line. Answer each question.

(Check students' models.)

2. (0, 2), (1, 6), and (3, 14)



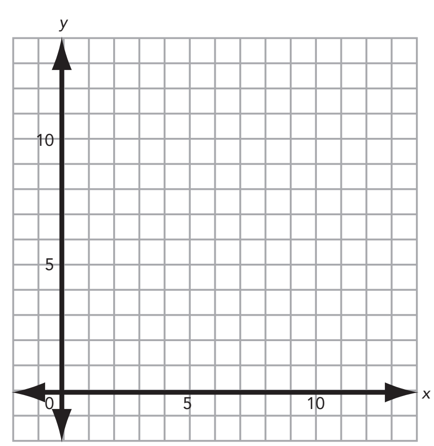
equation: $y = 4x + 2$

What is the ordered pair for the point on the line when x = 2? (2, 10)

What is the ordered pair for the point on the line when x = 6? (6, 26)

Graph the line that passes through the given points. Write the equation of the line. Answer each question.

3. (0, 1), (2, 2), and (4, 3)



equation: $y = \frac{1}{2}x + 1$

What is the ordered pair for the point on the line when x = 6? (6, 4)

What is the ordered pair for the point on the line when x = 20? (20, 11)

Answer Key

Challenge! What information do you need to write the equation for a line? After you have the equation, how can you find additional points on the line when given a value for *x*?

Challenge: (Sample) To write the equation of the line you need the rate of change and the *y*-intercept. After you have the equation, substitute a value in for *x* and solve for *y*.