

Objective

Estimate the irrational square root of given positive integers.

Common Core State Standards

■ 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

The Number System

Approximating Square Roots

By now, students have studied various types of rational and irrational numbers, including square roots. In previous grades, students learned how to find the principal square root of numbers that produce a positive rational number. In this lesson, students will estimate the square roots of numbers that produce positive irrational numbers.

Try it! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- Ask: Look at a 1 × 1 square. How many squares make up a 1 × 1 square? Elicit 1. What is the square root of 1?
- Ask: Look at a 2 × 2 square. How many squares make up a 2 × 2 square? Elicit 4. What is the square root of 4? Continue similarly for another example or two.
- Ask: What is the next square after 16? (25) What is the square root of 25? Continue through 64 and its square root (8), if necessary.
- Say: Use a calculator to find the square root of 55. Compare it to your estimate. Is it between 7 and 7.5?

Solve It

Reread the problem with students. Have students build squares to determine between which integers the square of their target number falls. Have students roughly estimate the square root, using the relative distance between the squares of two integers.

More Ideas

For other ways to teach about squares and square roots—

- Have students create a table of all integers from 1 to 30 and estimate the square roots of the numbers using Color Tiles.
- Have students find the approximate location of $\sqrt{3}$ on a Folding Number Line.

Formative Assessment

Have students try the following problem.

Which of the following is the best estimate for the square root of 45?

- A. between 4 and 5 but closer to 4
- B. between 4 and 5 but closer to 5
- C. between 6 and 7 but closer to 6
- D. between 6 and 7 but closer to 7

Try It! 30 minutes | Pairs

Here is a problem about estimating square roots.

Mr. Malcom has enough 1' by 1' carpet tiles left over from a building project to cover an area of 55 square feet. He has offered to donate the tiles to a neighborhood preschool. The preschool wants to carpet a square play area with the tiles. What is the largest square they can make?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.





1. Have students build up the following squares with their Color Tiles: 1×1 , 2×2 , 3×3 , 4×4 , 5×5 , 6×6 , 7×7 , and 8×8 . Have students determine the area of each of the squares as they work. (Note: Students may not have Color Tiles in the proportions shown here. The layering is done for clarity.)



3. Ask: Do you think the square root is greater or less than 7.5? Explain. Elicit from students that the square root is less than 7.5 because 55 is closer to 49 than to 64.

2. Now have students determine which squares the number of carpet tiles falls between.
Ask: Between which squares does 55 fall?
Elicit from students that it falls between 7 (the square is 49) and 8 (the square is 64).

Look Out!

Materials

Color Tiles (65 per pair)
paper (1 sheet per pair)
pencils (1 per pair)

Some students may not be able to articulate how to estimate square roots without hearing others explain it first. Approximating square roots should improve with practice. Point out to students that the word *root* in mathematics means "the answer." For a given number of tiles, the square root of that number will give you the dimensions of the square that you can build with that number of tiles.





Use Color Tiles to estimate the square root of the given number. Fill in the blanks. Write a sentence about the estimate of the square root.



Using Color Tiles, model square numbers to help you estimate the given square root. Sketch the model. Write the estimate and justify it.

(Check students' models.)

2. √76



Estimate each square root. Write the two numbers the square root is between and circle the number it is closer to.

4. $\sqrt{45}$ **5.** $\sqrt{33}$ **3.** $\sqrt{15}$ between 3 and 4, but between 6 and 7. but between 5 and 6, but closer to 4 closer to 7 closer to 6 **7.** $\sqrt{20}$ **6.** $\sqrt{65}$ **8.** $\sqrt{50}$ between 8 and 9, but between 4 and 5, but between 7 and 8, but closer to 8 closer to 4 closer to 7

Answer Key

Challenge! Explain how you decide which two numbers the value of a square root is between.

Challenge: (Sample) Find the greatest square number that is less than the number under the radical symbol. Find the least square number that is greater than the number under the radical symbol. Decide which square number is closer to the number under the radical symbol.





Use Color Tiles to estimate the square root of the given number. Fill in the blanks. Write a sentence about the estimate of the square root.

1. √28

It is closer t	√28 is betw	ar	28 is betwe	
to	veen and	nd	en the square numbers of	

Using Color Tiles, model square numbers to help you estimate the given square root. Sketch the model. Write the estimate and justify it.

2. √76

76 is between the square numbers of

_____ and _____.

 $\sqrt{76}$ is between _____ and _____.

It is closer to _____.

Estimate each square root. Write the two numbers the square root is between and circle the number it is closer to.

 3. √15
 4. √45
 5. √33

 6. √65
 7. √20
 8. √50

Name	
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Challenge! Explain how you decide which two numbers the value of a square root is between.