BUILDING PYRAMIDS

GEOMETRY • PATTERNS/FUNCTIONS • LOGIC

- Looking for patterns
- Interpreting data

Getting Ready

What You'll Need

Cuisenaire Rods, at least 2 sets per group

Isometric dot paper, pages 101–102 (optional)

Calculator, 1 per group (optional)

Overhead Cuisenaire Rods and/or 1-centimeter grid paper transparency (optional)

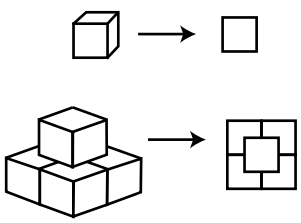
Overview

Children use Cuisenaire Rods to build models of pyramids. They record data about each structure, look for patterns, and make conjectures. In this activity, children have the opportunity to:

- discover, extend, and describe both geometric and numerical patterns
- draw conclusions from discovered patterns
- find relationships between geometric constructions and the number of white rods needed to build them
- discover and describe properties of a pyramid

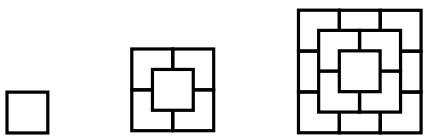
The Activity

The drawings at the right show overhead views of the pyramids. For example:

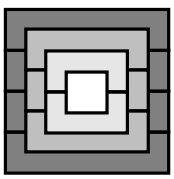


Introducing

- Display these three structures made from white rods arranged in square layers.



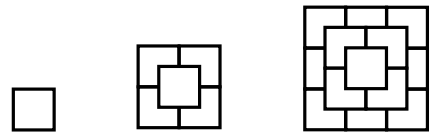
- Ask children to predict what the next structure would look like.
- When children suggest a pyramid of 1 white rod on top of 4 white rods on top of 9 white rods on top of 16 white rods, point out that you are running out of white rods. Ask volunteers to suggest other ways to build the fourth structure.
- Accept any answers that call for different-colored rods in combinations that build a pyramid equivalent to the fourth white-rod structure. The equivalent pyramid at the right has 4 purple rods, 3 light green rods, 2 red rods, and 1 white rod.



On Their Own

How many white Cuisenaire Rods would you need to build a solid pyramid that is 20 layers high?

- Work with a group to build pyramids from white rods.
- Here are overhead views of some solid pyramids made with white rods.
- Build each of these pyramids and record how many white rods are needed to build each.



1 layer 2 layers 3 layers

- Build larger and larger pyramids and record the number of white rods needed for each. When you run out of white rods, you can substitute rods of other colors that are equivalent to the number of white rods you need.
- Look for patterns in your pyramids and in your data. Use this information to predict the number of white rods needed to build a 20-layer pyramid.

The Bigger Picture

Thinking and Sharing

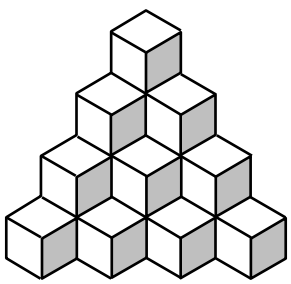
Have children discuss what they discovered while constructing their pyramids.

Use prompts such as these to promote class discussion:

- Did you use different combinations of rods to build your pyramids? Why?
- What did you notice as you built larger and larger pyramids?
- What patterns did you notice in the data?
- How many layers did you need to add before you could make predictions about the final answer?
- How did you find the total number of white rods needed?
- Could you predict the number of white rods needed for any pyramid using the information you have gathered? Explain.

Extending the Activity

1. Ask children to investigate triangular pyramids of the type shown here and to determine the numbers of white rods needed to build them.
2. Have children use isometric dot paper to record a side view of each construction.



Teacher Talk

Where's the Mathematics?

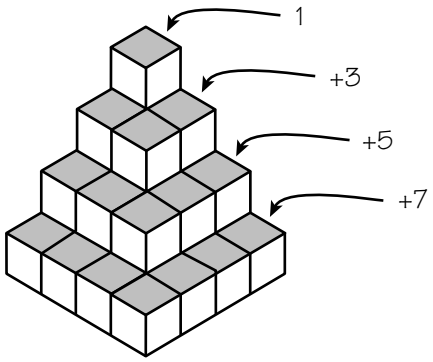
This activity helps children make connections between the geometric patterns of ever enlarging pyramids and the numerical patterns that reflect the enlargements. It also provides children with a visual portrayal of the set of square numbers (1, 4, 9, 16, ...).

Since children quickly run out of white rods, they need to substitute rods of other colors for white rods in order to build their pyramids. Children discover that a square base of five yellow (25 white) rods would be topped by square layers of four purple (16 white) rods, three light green (9 white) rods, two red (4 white) rods, and finally, one white rod. Some children will organize this information in a chart that might have one, two, or all three of the columns shown below, in which layer one is at the top of the pyramid and layer twenty is at the base of the pyramid.

Layer number	Number of white rods in the layer	Total number of white rods needed
1	1	1
2	4	5
3	9	14
4	16	30
5	25	55
6	36	91
7	49	140
8	64	204
9	81	285
10	100	385
11	121	506
12	144	650
13	169	819
14	196	1,015
15	225	1,240
16	256	1,496
17	289	1,785
18	324	2,109
19	361	2,470
20	400	2,870

One pattern that children might notice is that the number of white rods in a layer is the number of the layer multiplied by itself. For example, the ninth layer has 9 x 9, or 81, white rods. Another way to express the idea of multiplying a number by itself is to say that the number has been *squared*. The reason for calling the process of multiplying a number by itself *squaring* may be seen in the fact that each layer of the pyramid is a square. This shows the connection between 9 x 9 and the formation of a square from nine blue rods.

Another pattern that children might point out is that the numbers in the second column increase by successive odd numbers. Moving from 1 to 4, the increase is 3; moving from 4 to 9, the increase is 5; moving from 9 to 16, the increase is 7. This pattern of 3, 5, 7, 9, ... may be seen more dramatically if children look at the growing pyramid from above and arrange the square layers so that all of the layers share a corner, as shown here.



Looking at the pyramid this way will make it easier for children to see that the second layer has three white rods more than the top layer; the third layer has five white rods more than the second layer, and so forth.

To find the total number of white rods needed to build the pyramid, some children may simply add the numbers in the second column of the chart, whereas other children might create the third column that keeps a running total of white rods needed for each successive pyramid. The numbers in this third column are found by adding the number of rods in the last layer to the last total in the third column. All of this record keeping will be greatly facilitated by the use of a calculator. An advantage to creating the third column of numbers is that children can refer to this chart and quickly find, for example, how many white rods would be needed to make a pyramid eight layers high. If children have not made the third column, they would have to stop and add $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$ before they could give the answer.

By the end of this activity, most children will be able to generalize that a pyramid with N layers would require $1^2 + 2^2 + 3^2 + ... + N^2$ white rods. Becoming familiar with the notion of squaring, making data charts, and looking for patterns in these charts will be useful skills that children can transfer to their study of algebra.

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