LEVEL EIGHTEEN

ALGEBRA II

Developmental Mathematics

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1 LINEAR EQUATIONS

An **equation** is a mathematical statement that two algebraic expressions are equal. A **linear equation in one variable** is an equation that can be written in the form $ax + b = c$, where a, b and c are real numbers, $a \neq 0$ and x is the variable. The **variable** is any letter used to represent an unknown value. Commonly used variables are x, y and z, but any letter is acceptable.

Here are a few examples of linear equations in one variable:

$2x + 5 = 9$ $\qquad$ $3x + 4 = 2x - 3$ $\qquad$ $11(x - 5) = 4 - 7x$

Here are a few examples that are NOT linear equations in one variable:

$3x + 2y = -12$ - This is a linear equation in two variables.

$x^2 - 5x + 6 = 0$ - This is called a quadratic equation which we will study later in this level. The x^2 term is not linear.

$5x - 7$ - This is not an equation since it has no equals symbol. It is simply an algebraic expression.

A **solution** to a linear equation in one variable is any value of the variable which makes the equation a true statement.

For example $x = 2$ is a solution to the equation $2x + 5 = 9$ since when 2 is substituted in for x, the result is a true statement: $2 \cdot 2 + 5 = 4 + 5 = 9$. On the other hand, $x = 7$ is not a solution to the equation $2x + 5 = 9$. When 7 is substituted in for x, the result is not true: $2 \cdot 7 + 5 = 14 + 5 = 19 \neq 9$.

Example: Determine if $y = -4$ is a solution to the equation, $6y + 3 = 5y - 1$.
Solution: Substitute $y = -4$ into the equation and determine if the result is true.

$$\begin{aligned} 6(-4) + 3 &= 5(-4) - 1 \\ -24 + 3 &= -20 - 1 \\ -21 &= -21 \end{aligned}$$

Thus, $y = -4$ is a solution to the equation.

Example: Determine if $z = 5$ is a solution to the equation, $12 - 3z = 3$.
Solution: Substitute $z = 5$ into the equation and determine if the result is true.

$$\begin{aligned} 12 - 3(5) &= 3 \\ 12 - 15 &= 3 \\ -3 &\neq 3 \end{aligned}$$

$\rightarrow$ False statement. Thus, $z = 5$ is not a solution.

SOLVING LINEAR EQUATIONS

Now that we have discussed how to recognize a linear equation in one variable and learned how to determine if a given value is a solution to the linear equations, we will begin solving linear equations in one variable. We will need to use several different properties of equality to solve linear equations. For the properties listed below, let *a, b,* and *c* represent real numbers.

Addition Property of Equality: If $a = b$, then $a + c = b + c$

Subtraction Property of Equality: If $a = b$, then $a - c = b - c$

Multiplication Property of Equality: If $a = b$, then $a \times c = b \times c$

Division Property of Equality: If $a = b$, then $a \div c = b \div c$, provided $c \neq 0$

These properties state that an equation, where two expressions are equal, will remain equal if the same number is added, subtracted, multiplied or divided on both sides of the equation. It is important to note that when dividing both sides of an equation by a number, the number must be non-zero. Division by zero is undefined.

When solving a linear equation in one variable, we are trying to find the value of the variable. To do this, we will think of "undoing" the operations on the same side of the equation as the variable to isolate the variable on one side of the equation.

We are going to look at several examples of solving linear equations in one variable and will begin with basic examples. Many readers may know the solution to the equation simply by looking at it, but to learn how to solve more complex equations we will begin by solving basic equations.

Example 1: Solve the equation $x + 5 = 12$.
Solution: To "undo" adding 5 to the variable, subtract 5. To have the equation remain equal subtract 5 from both sides of the equation.

a.
$$\begin{array}{r} x + \cancel{5} = 12 \\ \underline{\quad -\cancel{5} \quad -5} \\ x = 7 \end{array}$$

b.
$$\begin{aligned} x + 5 &= 12 \\ x + \cancel{5} - \cancel{5} &= 12 - 5 \\ x &= 7 \end{aligned}$$

These problems can be done (a) vertically or (b) horizontally.

To check, substitute 7 into the equation for x: $7 + 5 = 12$.

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Example 2: Solve the equation $y - 14 = -8$.
Solution: "Undo" subtracting 14 by adding 14 to both sides of the equation.

a.
$$\begin{array}{r} y - \cancel{14} = -8 \\ \underline{+\cancel{14} \quad +14} \\ y = 6 \end{array}$$

b.
$$\begin{array}{l} y - 14 = -8 \\ y - \cancel{14} + \cancel{14} = -8 + 14 \\ y = 6 \end{array}$$

Check: $6 - 14 = -8$

Example 3: Solve the equation $13z = -52$.
Solution: Divide both sides by 13 to isolate the variable z.

$$\frac{13z}{13} = \frac{-52}{13}$$

$$z = -4$$

Check: $13 \times (-4) = -52$

Example 4: Solve the equation $\frac{t}{2} = 17$.

Solution: To "undo" division by 2, multiply both sides of the equation by 2.

$$2 \times \frac{t}{2} = 2 \times 17$$

$$t = 34$$

The check is left to the reader.

Example 5: Solve the equation $7x + 13 = -22$.
Solution: There are two operations we need to "undo" to isolate the x. In this example, it is simpler to "undo" the addition and then "undo" the multiplication.

$$\begin{array}{r} 7x + \cancel{13} = -22 \\ \underline{-\cancel{13} \quad -13} \\ \frac{7x}{7} = \frac{-35}{7} \\ x = -5 \end{array}$$

The check is left to the reader.

Example 6: Solve the equation $11 - 2x = 29$.
Solution:
$$\begin{array}{r} \cancel{11} - 2x = 29 \\ \underline{-\cancel{11} \qquad -11} \\ \frac{-2x}{-2} = \frac{18}{-2} \\ x = -9 \end{array}$$

Don't lose the sign on the 2 when canceling the 11's.

Check: $11 - 2(-9) = 11 + 18 = 29$

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APPLICATIONS

Recognizing Linear Equations in One Variable: Examine each of the problems 1-6 below and circle the linear equations in one variable.

1. $3x - 5 = 18$
2. $15x - 11 = 17x^2 - 23$
3. $4 = 7y + 1$
4. $3m + 2n = 45$
5. $t - 3 + 2t$
6. $8s + 19 = 2$

7. Determine if $m = -4$ is a solution to the equation $8 - 2m = 16$.

8. Determine if $x = 7$ is a solution to the equation $3x - 15 = x - 2$.

Solving Linear Equations in One Variable: For problems 9-18, solve each linear equation and check your solution.

9. $x + 5 = -19$ ____________________

10. $y - 3 = -15$ ____________________

11. $4m = 28$ ____________________

12. $\frac{t}{3} = -5$ ____________________

13. $\frac{2}{3} d = 12$ ____________________

14. $-z = 17$ ____________________

15. $2x + 5 = 19$ ____________________

16. $3 - 7y = 3$ ____________________

17. $\frac{1}{2} x - 7 = 11$ ____________________

18. $\frac{3}{5} z - \frac{1}{2} = 1$ ____________________

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2 MORE ON SOLVING LINEAR EQUATIONS

In this lesson, we will solve linear equations which may involve some simplifying and which may have the variable on both sides of the equation. The goal is still to find the value of the variable that satisfies the equation by isolating the variable on one side.

In addition to the equality properties, we will also use another property to simplify the algebraic expressions on either side of the equation called the Distributive Property. This property states that a real number multiplied by a sum is equal to the sum of that real number multiplied by each term in the sum.

Distributive Property of Multiplication:

For any real numbers a, b and c, $a\,(b + c) = a \cdot b + a \cdot c$

Example 1: Solve the linear equation $x + 7 = 2x - 5$.
Solution: Move one of the variable terms so the variable only appears on one side of the equation and then solve and check.

$$\begin{array}{rcl} x + 7 &=& 2x - 5 \\ -x & & -x \\ \hline 7 &=& x - 5 \\ +5 & & +5 \\ \hline 12 &=& x \end{array}$$

The check is left to the reader.

Example 2: Solve the linear equation $2(y + 4) = 5y - 3$.
Solution: Simplify each side completely, get the variable on one side and solve.

$$\begin{array}{rcl} 2(y + 4) &=& 5y - 3 \\ 2y + 8 &=& 5y - 3 \\ -2y & & -2y \\ \hline 8 &=& 3y - 3 \\ +3 & & +3 \\ \hline \frac{11}{3} &=& \frac{3y}{3} \\ \\ \frac{11}{3} &=& y \end{array}$$

$$2\left(\frac{11}{3} + 4\right) = 5\left(\frac{11}{3}\right) - 3$$

$$2\left(\frac{11}{3} + \frac{12}{3}\right) = \frac{55}{3} - \frac{9}{3}$$

$$2\left(\frac{23}{3}\right) = \frac{46}{3}$$

$$\frac{46}{3} = \frac{46}{3}$$

All of the examples of linear equations thus far have been examples of conditional equations. **Conditional equations** are equations that are true only for specific values of the variable. There are two other possibilities, an **identity** and a **contradiction**. An **identity equation** is an equation that is true for any value of the variable and a **contradiction** is an equation that is not true for any value of the variable.

Example 3: Solve the linear equation $4(x-1)+10=8x+6-4x$.

Solution: First, simplify each side of the equation completely.

$$4(x-1)+10 = 8x+6-4x$$
$$4x-4+10 = 4x+6$$
$$4x+6 = 4x+6 \rightarrow$$

Both sides are identical.
Any value of x will satisfy the equation.
Identity and solution are both real numbers.

Example 4: Solve the linear equation $3x-2(x-5)=x+9$.

Solution: First, simplify each side completely.

$$3x-2(x-5)=x+9$$
$$3x-2x+10=x+9$$
$$x+10=x+9$$

Now get the variable on one side of the equation.

$$\cancel{x}+10=\cancel{x}+9$$
$$\underline{-\cancel{x} \qquad -\cancel{x}}$$
$$10=9 \rightarrow$$

False statement. Contradiction.
Thus, there is no solution.

Example 5: Solve the linear equation $12-2m-3(m+2)=4m+6-m$.

Solution: First, simplify each side completely.

$$12-2m-3(m+2)=4m+6-m$$
$$12-2m-3m-6=3m+6$$
$$-5m+6 = 3m+6$$

Now get the variable on one side of the equation.

$$-\cancel{5m}+6=3m+6$$
$$\underline{+\cancel{5m} \qquad +5m}$$
$$6=8m+6$$

Now solve for m.

$$\cancel{6}-\cancel{6}=8m+\cancel{6}-\cancel{6}$$
$$0=8m$$
$$\frac{0}{8}=\frac{8m}{8}$$
$$0=m$$

The check is left to the reader.

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APPLICATIONS

Solving Linear Equations: For each of the following, solve the linear equation for the variable. Be sure to check your solution.

1. $3x + 5 = 2x - 8$
2. $4 - y = 6y + 18$
3. $2(m - 5) = -4$
4. $3(2 + 5n) = 16n - 9$
5. $w + 6 = w - 17$
6. $7t + 10 = -16$
7. $8(c - 2) = 5c - 1$
8. $15 = 1 - 7k$
9. $\frac{-2}{7}x = 18$
10. $0.25y - 0.73 = -4.36$
11. $-5(2q - 13) = 6(3 - q)$
12. $8(4d - 3) = 30(d - 1) + 2d + 6$
13. $6t - 8 = 0$
14. $5(3x - 1) = 4x - 5$

3 GENERAL STRATEGY OF PROBLEM SOLVING

The remainder of Unit A investigates using linear equations to solve different applications or word problems. Many students are overwhelmed in attempting to translate a word problem into an algebraic equation. To combat this, a strategy is laid out below to help students break the problem into doable steps.

Verbal Description or Word Problem →	**Verbal Model** or Equation in Words →	**Math Model** or Algebraic Equation
• Read the problem until you have a clear understanding. • Be able to identify important given information. • Be able to identify the unknown quantity you are trying to find. • It may be helpful to organize the information in a table, diagram, or picture.	• This step is the bridge between the word problem and the math model that you will solve. • Write an equation in words that relates unknown quantity to the information given in the problem. • Note: Some problems have an equation in words stated as a sentence in the problem.	• Label the unknown quantity (or quantities) with a variable or an expression involving that variable. • Translate the word equation using the variable, any variable expression and numbers. • Solve the equation and check the solution.

It requires practice to be confident and capable when solving word problems. So practice, practice, practice!! Also, when reading problems it is important to recognize key words that imply addition, subtraction, multiplication, division or equality. Below each operation, add to the list as many words or phrases that imply the stated mathematical operation.

Addition	Subtraction	Multiplication	Division	Equality
Sum	Difference	Product	Quotient	Equals
Total of	Subtracted	Times	Divided By	Is
Added To	Less Than	Twice	Ratio Of	Was
Combined	Decreased By	Tripled	Into	Are
More Than		Of		

Before we begin investigating different types of word problems, let's begin by practicing translating words into algebraic expressions.

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APPLICATIONS

Example 1: Convert the statement into an algebraic expression.
"Five times a number increased by thirteen"

Solution: "Times" implies multiplication $\rightarrow 5x$
"Increased by" implies addition $\rightarrow 5x + 13$

Example 2: Convert the statement into an algebraic expression.
"Two less than the quotient of six and a number"

Solution: "Less than" implies subtraction, but be careful it is reversed.
"The quotient" implies division $\rightarrow \frac{6}{y} - 2$

The Language of Mathematics: Convert each of the statements below into algebraic expressions or algebraic equations.

1. The product of 7 and a number __________________
2. Eighteen less than a number __________________
3. The total of a number and negative nine __________________
4. The ratio of a number and twenty-four __________________
5. Eleven increased by twice a number __________________
6. Triple the difference of a number and six __________________
7. Five subtracted from the quotient of a number and seventeen __________________
8. Three times a number increased by ten is twenty-five __________________
9. Four less than five times a number __________________
10. The length of a rectangle is two inches more than seven times the width. Express the length of the rectangle in terms of its width, *W*.

    ____

4 PROBLEM SOLVING: NUMBER RELATIONSHIPS

Now we are ready to begin solving word problems. We will begin with word problems that are very closely related to the translating of statements into algebraic expressions discussed in the previous lesson.

Example 1: The sum of five times a number and twelve is forty-two. Find the number.

Solution: With this type of problem we begin the problem solving strategy at the second step. The statement in the problem is actually an equation in words. Now, we need to label our variable and convert the statement into an algebraic equation.

Let x = the unknown number.
"Sum" implies addition of $5x$ and 12 which equals 42.
$5x + 12 = 42$

Now solve and check:

$5x + 12 = 42 \longrightarrow 5x = 30 \longrightarrow x = 6$

Check: $5 \cdot 6 + 12 = 30 + 12 = 42$

Example 2: The difference of five and two times a number is the same as the sum of the number and twenty. Find the number.

Solution: Again, the equation in words is stated in the problem.

Let n = the unknown number
Equation: $5 - 2n = n + 20$

Now solve and check:

$5 - 2n = n + 20 \longrightarrow 5 - 3n = 20 \longrightarrow -3n = 15 \longrightarrow n = -5$

Check: $5 - 2(-5) = 5 + 10 = 15 = (-5) + 20$

Example 3: The sum of two consecutive integers is thirty-seven. Find the two numbers.

Solution: Equation in words: 1st number + 2nd number = 37
Now, we need to write the two consecutive integers using a variable and an expression involving that variable. An example of two consecutive numbers is 6, 7 or 23, 24. In each case the second number is 1 more than the first. Two unknown consecutive numbers could be represented by x = 1st number and $x + 1$ = 2nd number.

Equation: $x + (x + 1) = 37$

Now solve and check:

$x + (x + 1) = 37 \longrightarrow 2x + 1 = 37 \longrightarrow 2x = 36 \longrightarrow x = 18$

Check: $18 + 19 = 37$

Thus, the two consecutive numbers are 18 and 19.