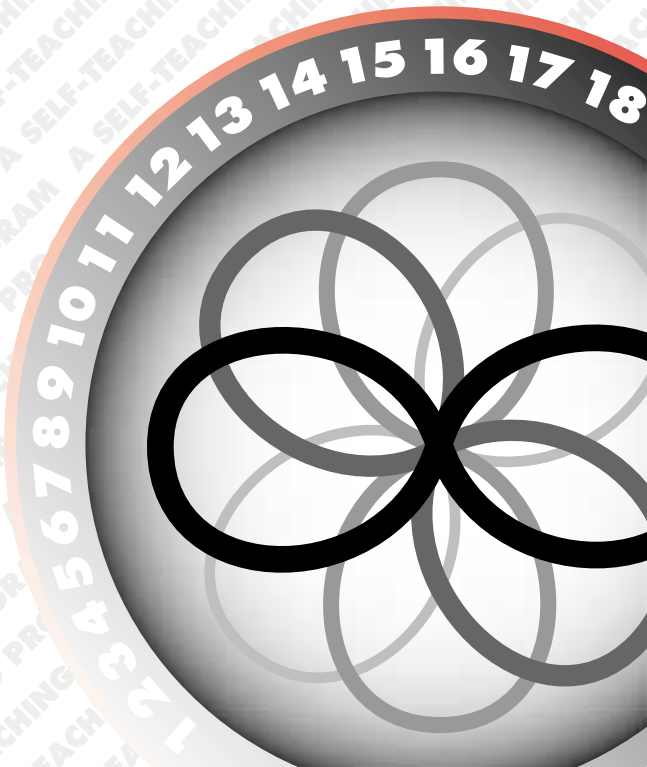
OVERVIEW

LEVEL-BY-LEVEL PROGRESSION

A SELF-TEACHING PROGRAM

Developmental Mathematics



OBJECTIVES

The main function of mathematics in education is to facilitate the development of the student's thinking. By nature, mathematics is uniquely endowed to be a medium in the learning process. Mathematics is much more than memorizing facts and carrying out computations; it is also how those facts are derived and how those computations are developed. The main focus of *Developmental Mathematics* is to cultivate independent thinking. We have used many methods and techniques to achieve this goal.

Understanding is the vehicle through which thinking develops. The student must comprehend the mathematical vocabulary he or she uses and be aware of the ideas underlying the facts and rules he or she learns. Any program which does not provide for such comprehension and awareness has subsequently eliminated thinking as an objective. This is why understanding is a main dimension of *Developmental Mathematics*.

The best condition for learning is that in which the student takes on an active role. The more responsible the student becomes, the more effective and permanent his or her learning is. *Developmental Mathematics* has been researched, designed and written to be a self-teaching medium. We aim to cultivate independent thinking through learning and understanding.

Together with the main two objectives we have outlined, there are certain unique features and highlights that we summarize in the following notes.

RESEARCH AND FIELD TESTING

Developmental Mathematics is the product of many years of research. It began in 1955, when the author was working on his Ph.D. at the University of Birmingham in England. In the United States research continued, starting in 1974 with a comprehensive testing program aimed at diagnosing the American student's learning of basic mathematics. *Developmental Mathematics* was then written and tested in many schools as a solution to the problems detected throughout the study.

The author and his assistants worked with students, and the materials were written and rewritten to ensure the students' success in independent learning.

BACK TO BASICS

Developmental Mathematics is a number-learning system that covers all of the foundations of arithmetic. It is intended to be a basic foundation upon which all future mathematical knowledge is built. It starts with counting and progresses through the concepts, facts and computational skills with whole numbers, decimals and fractions, into the beginnings of algebra and geometry. We have to remember that even with the major changes that have come about in school mathematics during the last few decades, 80% to 90% of the mathematics taught in kindergarten through eighth grade remains the application of basic arithmetic.

As is the case with any facet of human development, success in the future requires a healthy and sound foundation in the student's early years. *Developmental Mathematics* is devoted to creating a healthy and sound foundation in mathematics.

LOGICAL STRUCTURE

The program is not set up by school grades, therefore the lessons are arranged to follow the natural and logical progression of mathematical knowledge. After extensive analysis of the content, beginning with the elements of counting, into the foundations of algebra, we have built a structure that ensures growth in thinking and working with numbers. The program is presented in a series of workbooks, referred to as "Levels." Each level is composed of "Units," and each unit is made of "Lessons." Each lesson must be completed successfully before the student moves onto the next lesson. The student must master a unit before starting the next unit, and a level before starting the next level. A complete list of the levels and the mathematical content covered in each level is given on Pages 11 and 12. The program, as it is structured in this manner, has proven to be very effective in building a sound foundation of basic mathematics, as well as in saving time in the learning process. Our goal is to prepare the student for algebra in six to eight years, with a background that guarantees him or her future success.

COMPONENTS

As a self-teaching tool, the program includes the following components.

1. Assignments

On these pages the subject matter is presented through pictorial displays, discussions, questions and exercises. The presentations are simple so that it is easy for the student to learn on his or her own. Lesson by lesson, unit by unit, and level by level, the student develops his or her thinking and ability to work with numbers. Mathematical knowledge expands, and the capacity to learn independently gradually develops and matures.

2. Diagnostic Tests

At the end of each unit, we provide a "Unit Test" with items that deal with each lesson taught in the unit. An examination of the student's performance on the test will reveal whether a particular deficiency exists. Mastery of each unit is required before the student can start the next unit.

For each level, we provide a "Diagnostic Test" that covers the content taught in the entire level. The items in the test deal with each lesson of each unit in the level. When the student completes the level, he or she takes the Diagnostic Test. An examination of the student's responses on the test reveals whether there is any deficiency that must be addressed before he or she starts the next level. The Diagnostic Test may also be used in placing a student on the program to determine an accurate starting point.

3. Progress Charts

A chart is provided for the educator or parent to follow the student's progress and record completion dates of assignments and achievements on lessons as shown by the Diagnostic Tests.

4. Instruction Guides

To help educators and parents monitor the student's course of study, each of the levels 1–9 is accompanied by an "Instruction Guide," specifically written to advise the educator or parent of how a particular level fits as an integral part of the *Developmental Mathematics* program.

Unit by unit, we examine the educational outcome. Lesson by lesson, we explain the mathematical content and the methods used to teach it. Page by page, we discuss the content presented and give the answers to each question and exercise.

5. Solution Manuals

Each of the levels 10–16 is accompanied by an identical copy in which answers to the exercises and problems are printed in color. It is designed for educators and parents who want to spot answers immediately. They are of special value to parents in following their child's daily work.

6. Mathematics Placement and Scoring System *(MPASS)*

Mathematics Programs Associates (MPA) has developed an automated computerized version of the *Developmental Mathematics* placement and scoring framework, available on disk and on the World Wide Web. Visit out internet distributor at www.greatpyramid.com and find the placement *(MPASS)* mechanism within the mathematics section of the product module. You can also learn more about MPA and *Developmental Mathematics.*

FOUR-STAGE TEACHING STRATEGY

To help knowledge develop in a manner that guarantees the student's continual growth, we have adopted a four-stage teaching strategy in each lesson.

First Stage

We represent the mathematical concept, fact or skill under discussion with a pictorial model. These representations replace the concrete materials that are normally used with young students to help them visualize the situation. Students can easily learn with pictures in the same way they learn with manipulative materials.

Every concept, fact and skill can, and should be, presented concretely and pictorially. This makes mathematics physically real to the young learner. Working with the pictorial model, the student is guided in concise steps to grasp the idea underlying the concept, fact or skill. The examples presented take the student to complete understanding. While some examples are solved entirely, others are presented with little or no guidance to the student.

Second Stage

We reinforce the understanding that emerges in the first stage. Without the assistance of a concrete or pictorial model, the student is guided to use numbers only to carry out the step-by-step process that he or she has learned. Our aim is to make the concept, fact or skill abstractly significant. The student is assigned examples to study and exercises to carry out in the same step-by-step manner as in the first stage.

In the first and second stages, ideas emerge and mature, and understanding begins and gradually becomes deep and sound.

Third Stage

Although the student understands, his responses are slow in the first two stages. In the third stage, we strive to achieve quick recall and speedy performance of computational skills. The student is assigned many examples, which he or she carries out in the same step-by-step manner as learned in the first two stages. The more the student practices, the quicker his or her response is reached. The student's recall of facts becomes easier, and his or her computations become more accurate and speedy.

Fourth Stage

In the fourth stage, we train the student to apply the newly acquired knowledge to real-life situations (problem-solving). Here the student is guided to read the problem and analyze it. He or she must then choose the concept that applies and select the appropriate concept, fact or skill. He or she then carries out the steps one by one in the order that leads to the answer. The ability to apply knowledge is the highest level in the learning process.

In adopting this four-stage strategy, we allow the student enough time to practice and learn the concepts, facts and skills. Mastery is essential, not only of the facts and skills, but also of the concepts and ideas underlying them.

SAMPLE LESSON

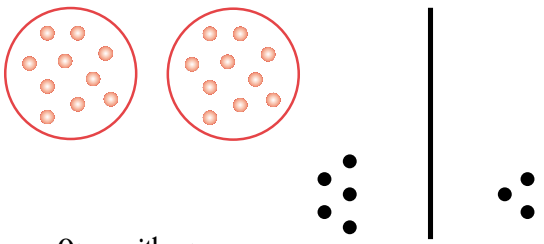
To illustrate the four-stage strategy, below we present the work the student completes in one lesson. We have chosen Lesson #5, "Adding Ones to a Tens-and-Ones Number" of Level 5, "Tens & Ones: Simple Addition and Subtraction." Using an example from the workbook, we present the four stages, showing the student's work and the learning outcomes achieved.

Stage 1

The objective is to help the student physically see the two-step process: adding ones to ones and then adding tens to the answer.

5 **ADDING ONES**

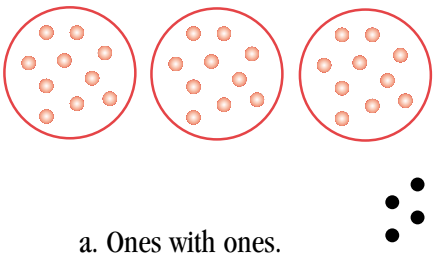
1. You have 25 items.
   You want to add 3 items.



a. Ones with ones. 5 + 3 = 8

b. Altogether. 20 + 8 = 28

2. You have 34 items.
   You want to add 5 items.



a. Ones with ones. _____

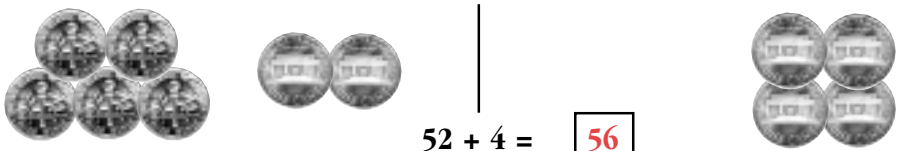
b. Altogether. _____

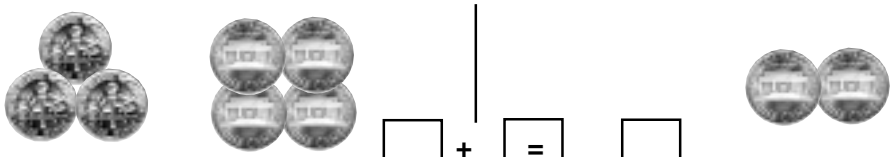
There are two examples presented pictorially. The "ten" is represented by a set of 10 items and the "ones" by single items. Colors are also used as a medium to assist in developing the line of thought.

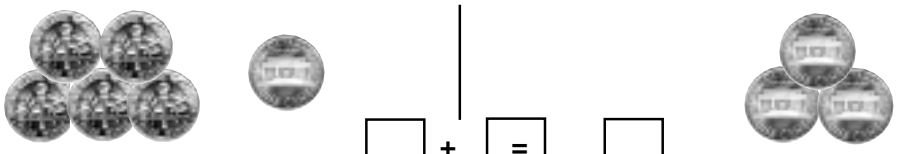
The dime/cent relationship is used graphically. The dimes are used to represent tens, and the pennies are used to represent ones.

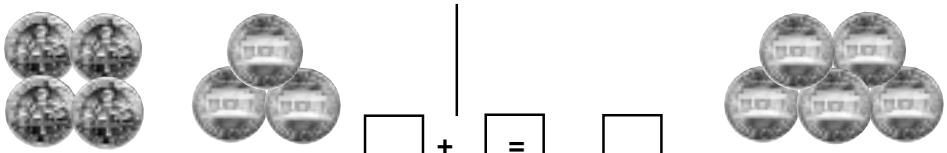
As the student completes the examples, the thought process becomes clearer, and the understanding involved gets deeper. As he or she reaches the end of the page, his or her skill has acquired a meaningful visual image.

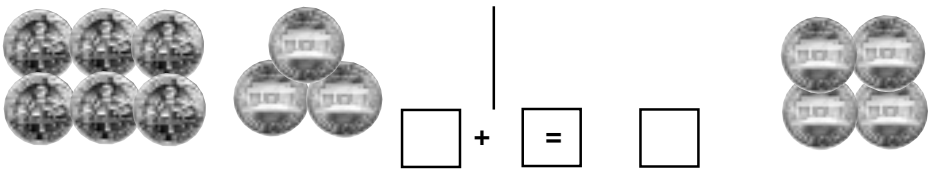
Write in the boxes.

3. 

4. 

5. 

6. 

7. 

Stage 2

The objective here is to make the lines of thought that have emerged in the first stage acquire meaningful abstract understanding. For this purpose, the pictorial representations are removed, and the numerical data is presented in the same step-by-step manner established in Stage 1. The student writes down the steps as he or she carries them out. In this manner, the steps are mastered.

As the student completes this assignment, his or her understanding of the lines of thought involved reaches maturity. This starts at the beginning of the first stage and gradually develops through the four stages to reach a higher abstract level.

Do it step by step.

1. 63 + 2

a. Ones with ones. _____

b. Altogether. _____

2. 32 + 4

a. _____

b. _____

3. 28 + 1

a. _____

b. _____

4. 71 + 5

a. _____

b. _____

5. 25 + 3

a. _____

b. _____

6. 42 + 2

a. _____

b. _____

7. 31 + 6

a. _____

b. _____

8. 86 + 2

a. _____

b. _____

9. 64 + 4

a. _____

b. _____

10. 83 + 6

a. _____

b. _____

11. 41 + 3

a. _____

b. _____

Stage 3

On this page, the student is assigned addition computations to perform in two steps: adding ones to ones and then adding tens to the answer. As the student completes the exercises, his or her competency *gradually* increases in speed and accuracy. Students differ in the number of exercises they complete before reaching mastery. While one student may only need 10 or 15 exercises, another may need all of the exercises assigned.

The student who has mastered the 36 addition facts with the ones learned in Level 2 can successfully work out the computations in his or her head.

Write the answer.

1. 35 + 1 = ___	2. 63 + 2 = ___	3. 24 + 5 = ___
4. 21 + 6 = ___	5. 52 + 4 = ___	6. 96 + 1 = ___
7. 47 + 2 = ___	8. 85 + 3 = ___	9. 74 + 4 = ___
10. 26 + 3 = ___	11. 15 + 2 = ___	12. 36 + 1 = ___
13. 72 + 6 = ___	14. 82 + 7 = ___	15. 28 + 1 = ___
16. 31 + 5 = ___	17. 22 + 5 = ___	18. 44 + 2 = ___
19. 83 + 3 = ___	20. 16 + 2 = ___	21. 62 + 7 = ___
22. 74 + 5 = ___	23. 47 + 2 = ___	24. 31 + 8 = ___
25. 11 + 6 = ___	26. 23 + 5 = ___	27. 55 + 2 = ___
28. 22 + 2 = ___	29. 44 + 4 = ___	30. 63 + 3 = ___
31. 74 + 3 = ___	32. 84 + 4 = ___	33. 92 + 7 = ___
34. 56 + 2 = ___	35. 12 + 7 = ___	36. 13 + 3 = ___
37. 14 + 4 = ___	38. 11 + 8 = ___	39. 12 + 2 = ___
40. 16 + 3 = ___	41. 77 + 2 = ___	42. 22 + 7 = ___

Stage 4

The student has just learned, to the mastery level, a particular computational skill. He or she knows the steps to be carried out and is aware of the idea underlying each step. He or she is accurate and quick.

The computation has many applications in life. As a part of making mathematics a tool for coping with life, we present the student with a selection of situations in which this particular addition skill is used.

The student is required to first analyze the story and then answer the question.

Write a number sentence.

1.


In all,
how many cents?

_____

2.


5 birds


3 birds

In all,
how many birds?

_____

3.


32 stamps


6 stamps

In all,
how many stamps?

_____

4.


21 apples


7 apples

In all,
how many apples?

_____

PERSONAL INFORMATION

While verbal expression is the normal medium for presenting a story, we also use pictorial presentations to avoid the problems of reading. In this manner, the student forms a clear visual image of the content in the story. The color helps in drawing a clear image.

Through the four stages, the student slowly moves from the lowest level of knowledge to the highest level of mastery.

Now we can be assured that "Adding Ones to a Tens-and-Ones Number" is a computational skill that has been built into the student's background. He or she is ready to move on to the next lesson.

5.  42¢  7¢

In all, how many cents?

_____

6.  21¢  8¢

In all, how many cents?

_____

7.  12 eggs  6 eggs

In all, how many eggs?

_____

8.  15 crayons  4 crayons

In all, how many crayons?

_____

9.  5 cards  1 card

In all, how many cards?

_____

SCOPE AND SEQUENCE

Developmental Mathematics is a complete number-learning system, in which the subject matter is structured logically, starting with counting, through the concepts, facts and computational skills with whole numbers, decimals and fractions into the beginnings of algebra and geometry.

The following is a listing of the topics covered within the different levels. An examination of these levels will show the sequential nature of the program. The sequence follows the growth of numbers as concepts and substance to think and operate with, both in mental and in written form.

Level 1. Ones: Concepts and Symbols

- Concepts of numbers one, two, three,..., nine
- Reading the symbols 1, 2, 3,..., 9
- Writing the symbols 1, 2, 3,..., 9

Level 2. Ones: Addition Concepts and Basic Facts

- Concept of addition
- 36 addition facts with ones
- Reading and writing addition number sentences

Level 3. Ones: Subtraction Concepts and Basic Facts

- Concept and symbol of subtraction
- 36 subtraction facts with ones
- Reading and writing subtraction number sentences
- Concept and symbol of zero
- 19 addition facts with zero
- 19 subtraction facts with zero

Level 4. Tens: Concepts, Addition and Subtraction Facts

- Concepts and symbols of ten, twenty,..., ninety
- 36 addition facts with tens
- 45 subtraction facts with tens

Level 5. Tens and Ones: Simple Addition and Subtraction

- Concept and symbols of a tens-and-ones number
- Adding without grouping
- Subtracting without exchange

Level 6. Tens and Ones: Adding with Grouping

- 45 addition facts involved in grouping
- Adding with grouping in horizontal form
- Adding with grouping in vertical form

Level 7. Tens and Ones: Subtracting with Exchange

- 45 subtraction facts involved in exchange (borrowing)
- Subtracting with exchange in horizontal and vertical forms

Level 8. Multiplication: Concepts and Basic Facts

- Concept and symbol of multiplication
- 64 basic multiplication facts
- 1 x N, N x 1, 0 x N, N x 0
- 100 multiplication facts

Level 9. Division: Concepts and Basic Facts
- Concept and symbol of division
- 64 basic division facts
- N ÷ 1, N ÷ N, 1 ÷ N
- 90 division facts

Level 10. Hundreds and Three-Unit Numbers: Concepts, Addition and Subtraction Skills
- Concept and symbol of hundreds
- Extending addition facts and skills to hundreds
- Concept and symbol of a three-unit number
- Extending addition and subtraction skills to three-unit numbers

Level 11. Three-Unit Numbers: Multiplication and Division Skills
- Multiplying by ones, i.e., 7 x 25, 5 x 127
- Multiplying by tens, i.e., 20 x 20, 30 x 30
- Multiplying by a two-unit number, i.e., 23 x 37
- Dividing by ones, i.e., 785 ÷ 5, 798 ÷ 5
- Dividing by two-unit number, i.e., 398 ÷ 25

Level 12. Thousands and Large Numbers: Concepts and Skills
- How a four-unit number is formed from its components
- Extending the place value system to four, five and six places
- Extending addition and subtraction skills
- Extending multiplication and division skills

Level 13. Decimals, Fractions and the Metric System: Concepts and Basic Skills
- Extending place value to tenths, hundredths,...
- Extending the four operations to decimals
- Metric measures of length, capacity, and weight

Level 14. Fractions: Concepts and Skills
- Concepts and symbols of fractions and mixed numbers
- Adding and subtracting fractions of the same unit
- A whole number multiplied by a fraction or a mixed number
- Dividing a whole number by a fraction
- Dividing by a fraction of the same unit

Level 15. Fractions: Advanced Skills
- Changing into a smaller or a larger fractional unit
- Adding and subtracting fractions of different units
- Dividing by a fraction or a mixed number
- Multiplying by a fraction or a mixed number

Level 16. Special Topics: Ratio, Percent, Graphs and More
- Ratio
- Percent
- Graphs
- Proportion
- Probability
- Number Theory

HOW TO PLACE THE STUDENT ON THE PROGRAM

The first task is to determine the level at which the student should begin the program. Please remember that the program is non-graded, which means that a student may have to be placed at a higher or lower level than expected for his or her age or grade. In our experience with thousands of students, we have found that, in most cases, the student must be taken back to a lower level than his or her age or grade would suggest. However, the extra time or effort spent in building the basic foundations will be extremely beneficial to the student.

With the educator's or parent's observation of the student's current performance, a subjective decision may be reached. Toward this goal, we offer the recommendations listed below, grade by grade. To reach a more objective decision, the placement tests built into the program are used.

Kindergarten

This is the beginning of the program, and it is logical that the student begins with Level 1, where the number concepts are developed.

First Grade

If the student has developed the concepts of the numbers one through nine and knows how to read and write the symbols 1, 2, 3,..., 9, he or she may begin with Level 2. Otherwise, he or she should begin with Level l.

Second Grade

The student may have done some work with addition and subtraction, but because of the great importance of these foundations, we have found that most students must relearn them or at least review them. We strongly suggest that the student begin with Level 2. If he or she has already mastered the concepts and facts of addition and subtraction with the numbers one through nine, a review of these foundations will be of great benefit to the student's future learning of mathematics. Please do not regret the extra time spent on such basic knowledge.

Third Grade

As with second-graders, we have found that most students lack the quick recall of the basic facts of addition and subtraction. If the student cannot respond immediately to simple addition and subtraction problems or needs to count on his or her fingers, this is an indication of a critical need for remediation. The student must complete Levels 2 and 3. Otherwise he or she should begin with Level 4.

Fourth Grade

If the student has mastered all the facts of addition and subtraction, can perform addition and subtraction computations with two-place numbers quickly and accurately, and can solve verbal problems that require these skills, he or she is ready to be placed on Level 8.

If the student has difficulties with the simple facts of addition, the remediation of these facts is urgently needed, and Levels 2, 3 and 4 should be completed.

If there is difficulty with the computations and verbal problems that involve addition or subtraction, remediation of these very basic skills is needed. Levels 5, 6 and 7 should be the first assignment.

Fifth Grade

If the student has mastered the addition and subtraction computations and can apply them to verbal problems, and he or she has mastered the concepts and facts of multiplication and division, he or she will be ready to start Level 10.

If the student has difficulty with the addition and subtraction computations and applications, remediation of these skills is needed, and Levels 5, 6, and 7 must be completed.

If the student has difficulty with multiplication and division and their applications in verbal problems, then remediation is required, and Levels 8 and 9 must be completed. These two levels are devoted to the concepts and facts of multiplication and division, respectively.

Sixth Grade

If the student has mastered the four operations with whole numbers and can apply them successfully to verbal problems, he or she may start with Level 12. Otherwise, remediation for these skills is required, and Levels 10 and 11 should be assigned.

Seventh and Eighth Grades

Since these students are very close to the end of elementary mathematics and the beginning of algebra, they should have a solid foundation in fractions, decimals, and ratio proportion, percent, etc. For such students, we suggest that they start with Level 13 and continue through Level 16.

HOW TO MONITOR THE STUDENT'S LEARNING

Although the program is self-teaching, the educator or parent has a major role in implementing it successfully. After placing the student on the program, the following are the main tasks to be carried out by the educator or parent:

1. The student may need the educator's or parent's help in understanding a sentence, an example or a piece of work. Here the educator or parent should take the student through the text in the workbook and help him or her to understand it without changing the methodology or the way of thinking adopted in the book.

2. As the student works, the educator or parent should monitor his or her achievement, checking the work page by page, to locate any mistakes. It is important that not too much work be completed without checking, otherwise a pattern of mistakes may develop and become more difficult to correct later. There is room in the Student's Progress Chart for the parent to record his or her observations.

3. When the student completes a Unit in a Level, he or she should take the Unit Test. Then the educator or parent corrects it in order to determine whether the student has mastered that unit and is ready to move on to the next. The answers expected are given in the Instruction Guide or the Solution Manual.

4. When the student completes a level, he or she should take the Diagnostic Test for the level. The educator or parent then examines the student's performance. Any deficiencies revealed by the test must be remedied before the student begins the next level. The answers expected to be given by the student are provided in the Instruction Guide or the Solution Manual.

EDUCATIONAL BACKGROUND

Please complete the following student profile, including the applicable personal information and educational background. We use the data to include you on our mailing list and to provide you with our package for placing the student at the appropriate level within our curriculum.

First name _____

Last name _____

Street address _____

Apartment number _____

City or town _____

State and country _____

Zip or postal code _____

Telephone number _____

Facsimile number _____

E-mail address _____

Student's age _____

Student's birthday _____

Student's gender _____

- ☐ Male
- ☐ Female

Interests and activities:

- ☐ Crafts
- ☐ Music
- ☐ Sports
- ☐ Other _____

Additional information and comments _____

_____

_____

_____

PERSONAL INFORMATION

Student's grade _____

Teacher's name _____

School's name _____

Street address _____

City or town _____

State and country _____

Zip or postal code _____

Learning environment:

- ☐ Home school
- ☐ Private school
- ☐ Public school

Learning ability:

- ☐ Remedial
- ☐ Moderate
- ☐ Advanced

Do you have a computer for student use?

☐ Yes ☐ No

If yes, what brand, model and configuration? _____

_____

If yes, do you have access to the Internet?

☐ Yes ☐ No

If yes, are you interested in our *Mathematics Placement and Scoring System (MPASS)*?

☐ Yes ☐ No

Special needs? _____

_____

_____

_____

_____

After completing questionnaire, please remove along perforation and return to:

MATHEMATICS PROGRAMS ASSOCIATES

P.O. Box 2118 • Halesite, NY 11743

Tel: 516.643.9300 • Fax: 516.643.9301

Mathematics Programs Associates (MPA), a Long Island-based family enterprise providing educational products and consulting services, exists today primarily due to the vision and determination of its founder, Dr. L. George Saad. During the early 1950s, Dr. Saad taught mathematics education at the University of Ain-shams in Cairo, Egypt. In 1954, with an innovative idea for self-teaching, he enrolled as a doctoral candidate at the University of Birmingham in England. During the following three years, Dr. Saad devoted his research to the elementary and secondary students' understanding of basic mathematics, and he developed the methodology for a self-teaching mathematics program. In 1957, Dr. Saad received the Ph.D. in mathematics education. He then returned to Cairo and began the development of a government-sponsored mathematics curriculum for use throughout the country's elementary school system. In 1959, samples of Dr. Saad's materials were tested in the Cairo schools and, a few years later, his curriculum was being used throughout the country and in other Middle Eastern nations. Due to his popularity in the Middle East, in 1969, Dr. Saad was invited to the United States as a visiting professor at the State University of New York, and in the same year, accepted a professorship at Long Island University. In 1970, with an inspiration to repeat his success, Dr. Saad immigrated his family to the United States and began working on the rudiments of a self-teaching mathematics workbook series. In 1974, he incorporated MPA in New York to design, develop and distribute his work. Today, educators and students in the United States, and many other nations throughout the world, are benefiting from Dr. Saad's lifelong achievement,

Developmental Mathematics

A Self-Teaching Program

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MPA

Teaching From The Right Angle

P. O. Box 2118 • Halesite, NY 11743