The Number System

In eighth grade, students develop greater understanding of the number system by exploring irrational numbers. **Irrational numbers** are numbers that cannot be expressed as ratios. They have endless, non-repeating digits to the right of the decimal point (e.g., $\pi \approx 3.14592$ and $\sqrt{2} \approx 1.414213$).

Previously, students learned that **rational numbers** are numbers that can be written as ratios. Rational numbers can be written as fractions and as decimals that either repeat (e.g., $\frac{4}{3} = 1.3333...$) or terminate (e.g., $\frac{5}{4} = 1.25$). Any whole number is a rational number because it can be written as a fraction (e.g., $8 = \frac{8}{1}$). Rational numbers also can be written in other forms that simplify to more familiar forms ($\sqrt{16} = 4$).

Students may be interested in knowing why the differences between rational and irrational numbers matter. It might help to explain that using numbers grew out of the practical need to count objects. In ancient times, people counted objects in units, such as 1, 2, 3 "apples." Over time, the need for additional numbers grew as the need to measure arose—there was a need to express numbers between 0 and 1, 1 and 2, 2 and 3, and so on. Precise measuring needed to account for all places on the ruler. Rational and irrational numbers allow for that to occur in that all rational and irrational numbers can be located or approximated on a number line.

The Grade 8 Common Core State Standards for The Number System specify that students should–

- Know that there are numbers that are not rational, and approximate them by rational numbers.

The following hands-on activities will help students understand the differences between rational and irrational numbers. They will equip students to apply the processes necessary to compare the sizes, estimate the values, and find the locations or approximate locations of rational and irrational numbers on the number line. The activities will give students concrete experiences that will help them reason abstractly and make sense of the number system and understand the real-world contexts of these numbers.

The Number System

Contents

LESSON

1

Objective

Estimate the irrational square root of given positive integers.

Common Core State Standards

- **8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

The Number System

Approximating Square Roots

By now, students have studied various types of rational and irrational numbers, including square roots. In previous grades, students learned how to find the principal square root of numbers that produce a positive rational number. In this lesson, students will estimate the square roots of numbers that produce positive irrational numbers.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Look at a 1 × 1 square. How many squares make up a 1 × 1 square?* Elicit 1. *What is the square root of 1?*
- **Ask:** *Look at a 2 × 2 square. How many squares make up a 2 × 2 square?* Elicit 4. *What is the square root of 4?* Continue similarly for another example or two.
- **Ask:** *What is the next square after 16?* (25) *What is the square root of 25?* Continue through 64 and its square root (8), if necessary.
- **Say:** *Use a calculator to find the square root of 55. Compare it to your estimate. Is it between 7 and 7.5?*

Solve It

Reread the problem with students. Have students build squares to determine between which integers the square of their target number falls. Have students roughly estimate the square root, using the relative distance between the squares of two integers.

More Ideas

For other ways to teach about squares and square roots—

- Have students create a table of all integers from 1 to 30 and estimate the square roots of the numbers using Color Tiles.
- Have students find the approximate location of $\sqrt{3}$ on a Folding Number Line.

Formative Assessment

Have students try the following problem.

Which of the following is the best estimate for the square root of 45?

A. between 4 and 5 but closer to 4

B. between 4 and 5 but closer to 5

C. between 6 and 7 but closer to 6

D. between 6 and 7 but closer to 7

Try It! 30 minutes | Pairs

Here is a problem about estimating square roots.

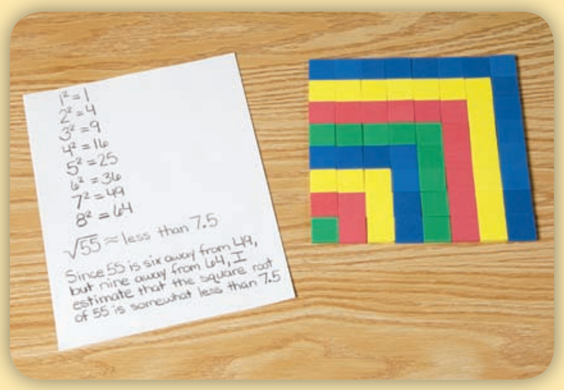
Mr. Malcom has enough 1′ by 1′ carpet tiles left over from a building project to cover an area of 55 square feet. He has offered to donate the tiles to a neighborhood preschool. The preschool wants to carpet a square play area with the tiles. What is the largest square they can make?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials
- Color Tiles (65 per pair)
- paper (1 sheet per pair)
- pencils (1 per pair)

1. Have students build up the following squares with their Color Tiles: 1 × 1, 2 × 2, 3 × 3, 4 × 4, 5 × 5, 6 × 6, 7 × 7, and 8 × 8. Have students determine the area of each of the squares as they work. (Note: Students may not have Color Tiles in the proportions shown here. The layering is done for clarity.)

2. Now have students determine which squares the number of carpet tiles falls between. **Ask:** *Between which squares does 55 fall?* Elicit from students that it falls between 7 (the square is 49) and 8 (the square is 64).



3. Ask: *Do you think the square root is greater or less than 7.5? Explain.* Elicit from students that the square root is less than 7.5 because 55 is closer to 49 than to 64.

⚠ Look Out!

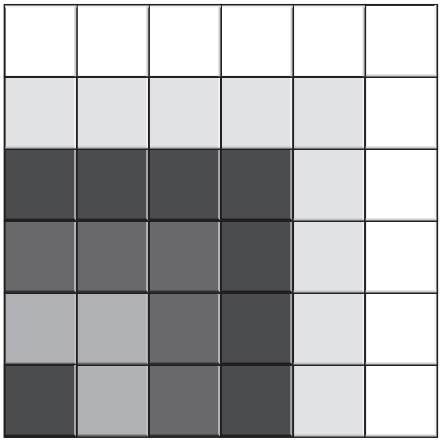
Some students may not be able to articulate how to estimate square roots without hearing others explain it first. Approximating square roots should improve with practice. Point out to students that the word *root* in mathematics means "the answer." For a given number of tiles, the square root of that number will give you the dimensions of the square that you can build with that number of tiles.

Answer Key

Use Color Tiles to estimate the square root of the given number. Fill in the blanks. Write a sentence about the estimate of the square root.

(Check students' work.)

1. $\sqrt{28}$



28 is between the square numbers of 25 and 36.

$\sqrt{28}$ is between 5 and 6.

It is closer to 5.

Using Color Tiles, model square numbers to help you estimate the given square root. Sketch the model. Write the estimate and justify it.

(Check students' models.)

2. $\sqrt{76}$

76 is between the square numbers of 64 and 81.

$\sqrt{76}$ is between 8 and 9.

It is closer to 9.

Estimate each square root. Write the two numbers the square root is between and circle the number it is closer to.

3. $\sqrt{15}$

between 3 and 4, but closer to 4

4. $\sqrt{45}$

between 6 and 7, but closer to 7

5. $\sqrt{33}$

between 5 and 6, but closer to 6

6. $\sqrt{65}$

between 8 and 9, but closer to 8

7. $\sqrt{20}$

between 4 and 5, but closer to 4

8. $\sqrt{50}$

between 7 and 8, but closer to 7

Answer Key

Challenge! Explain how you decide which two numbers the value of a square root is between.

Challenge: (Sample) Find the greatest square number that is less than the number under the radical symbol. Find the least square number that is greater than the number under the radical symbol. Decide which square number is closer to the number under the radical symbol.

LESSON 2

The Number System

Irrational Square Roots

The square root, r, of a number, x, is the number such that $r^2 = x$. This also can be written as $\sqrt{x} = r$. In previous work, students have found the principal square roots of numbers. Students need to understand that not all square roots are rational numbers. When a square root is a nonterminating decimal with no repeating pattern, it is irrational. Irrational square roots must be estimated.

Objective

Estimate irrational square roots.

Common Core State Standards

- **8.NS.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *What is the square root of 2? Why is $\sqrt{2}$ irrational? Why must $\sqrt{2}$ be estimated?*
- **Ask:** *How can $\sqrt{2}$ be estimated? Is $\sqrt{2}$ closer to 1 or 2? How did you make the estimate more accurate?*
- **Ask:** *What is the best range for the estimate of $\sqrt{2}$ on the number line?* Have students show the 1.414–1.415 interval. **Ask:** *Can you ever find the exact location?* Emphasize that the exact location is impossible to find because $\sqrt{2}$ is irrational.

Solve It

Reread the problem with students. Have students draw the square with the diagonal and label it $\sqrt{2}$. Then have them draw the section of the number line with the best range for the estimate indicated.

More Ideas

For other ways to teach about estimating irrational square roots—

- Have students make a table to find the integers between which the square root of 2 falls using tenths, hundredths, and thousandths increments.
- Have students use their calculators and the Folding Number Line to find the best range for the estimate of $\sqrt{3}$, $\sqrt{5}$, and $\sqrt{7}$.

Formative Assessment

Have students try the following problem.

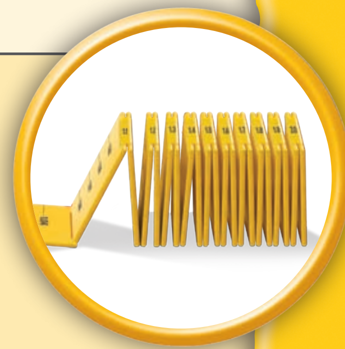
A picture frame has a square opening that measures 3 inches on each side. The diagonal length is $\sqrt{18}$ inches. Which is the best estimate of this length?

A. between 4 and 5, but closer to 5

B. between 4.1 and 4.2, but closer to 4.2

C. between 4.23 and 4.24, but closer to 4.24

D. between 4.24 and 4.25, but closer to 4.24



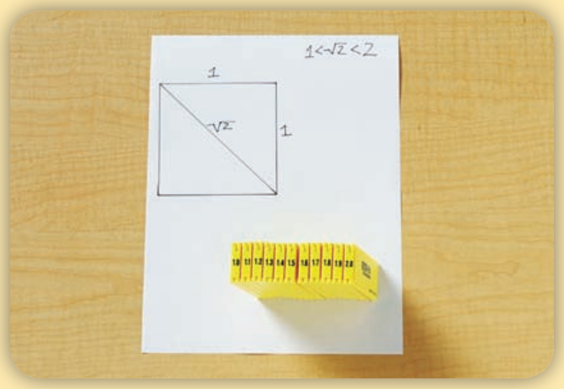
Try It! 15 minutes | Groups of 4

Here is a problem about estimating irrational square roots.

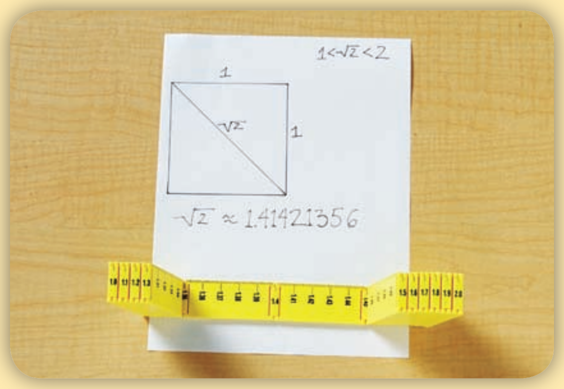
Carol drew a square with side lengths of 1 inch. Then she drew the diagonal and wanted to know its length. She calculated that it is $\sqrt{2}$. What is the approximate length of the diagonal?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

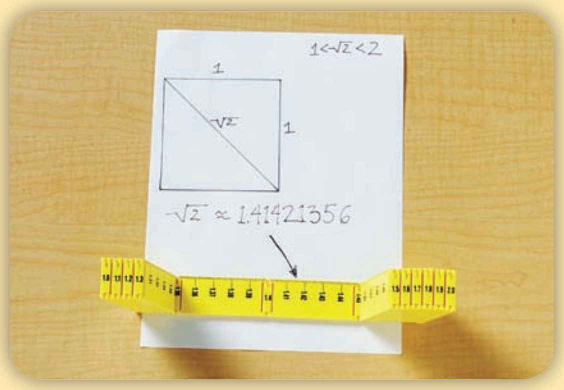
Materials
- Folding Number Line
- Calculator
- paper (1 sheet per group)
- pencils (1 per group)



1. Ask: *Between what two whole numbers is $\sqrt{2}$ found?* Note that 2 is between the two perfect squares 1 and 4 so $\sqrt{2}$ is between $\sqrt{1}$ and $\sqrt{4}$, or between 1 and 2. **Ask:** *Which number line should you use to locate $\sqrt{2}$?* Elicit that the 1–2 number line is the one they should use.



2. Have students use their calculators to find $\sqrt{2}$. Explain that a nonterminating number such as this, with no repeating pattern, is an irrational number. **Ask:** *How close to 1 is $\sqrt{2}$?* Have students look at the tenths place in the number to see where $\sqrt{2}$ would be located on the Folding Number Line. Have students find the 1.4–1.5 interval on the line.



3. Ask: *How can we make our estimate more accurate? Which hundredths interval should we locate?* Have students find the 1.41–1.42 interval on the line. **Ask:** *On which thousandths interval is $\sqrt{2}$ located?* Students should find the 1.414–1.415 interval.

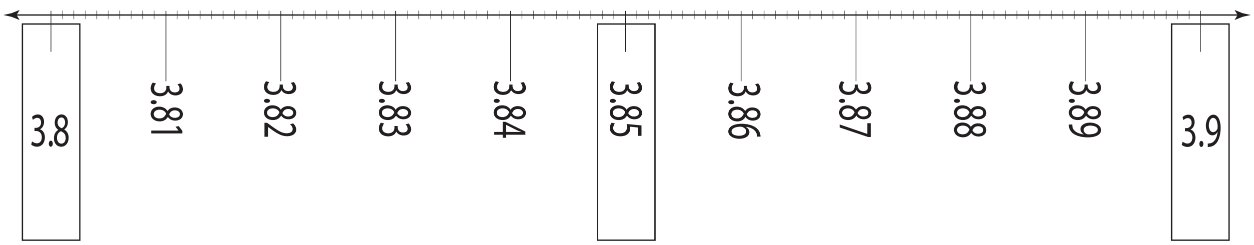
⚠ Look Out!

Students sometimes divide by 2 to find the square root. Reinforce the definition of square root, explaining that dividing by 2 is not the same.

Use the Folding Number Line to estimate the square root. Fill in the blanks.

(Check students' work.)

1. $\sqrt{15}$



$\sqrt{15}$ is between the whole numbers 3 and 4.

A better estimate is between 3.8 and 3.9.

A better estimate is between 3.87 and 3.88.

A better estimate is between 3.872 and 3.873.

Using the Folding Number Line, estimate the square root. Fill in the blanks.

2. $\sqrt{38}$

$\sqrt{38}$ is between the whole numbers 6 and 7.

A better estimate is between 6.1 and 6.2.

A better estimate is between 6.16 and 6.17.

A better estimate is between 6.164 and 6.165.

Give the tenths interval on which the irrational number falls.

3. $\sqrt{75}$ 8.6–8.7

4. $\sqrt{56}$ 7.4–7.5

5. $\sqrt{117}$ 10.8–10.9

6. $\sqrt{48}$ 6.9–7.0

Answer Key

Challenge! Using the Folding Number Line, show $\sqrt{17}$ and explain why increasing the number of decimal places in the endpoints of a range makes the estimate of an irrational square root more accurate.

Challenge: (Sample) $\sqrt{17}$ = 4.123. Increasing the number of decimal places for the endpoints of a range makes the range smaller. It limits the possibilities for the square root to that smaller range, which means the estimate is more accurate.