Number and Operations—Fractions

In fifth grade, students use their understanding of **fractions** to model and perform addition and subtraction of fractions with unlike denominators and solve related word problems. They use benchmark fractions and fraction number sense to estimate and assess the reasonableness of answers. They also use number sense to move between decimal and fraction equivalents.

Students in fifth grade apply and extend previous learning to multiply a fraction or a whole number by a fraction. They extend their understanding of multiplication beyond the concept of repeated addition. For example, they interpret multiplication as scaling, or resizing, to compare the size of a product to the size of one factor based on the size of the other factor.

Students learn that fractions represent the division of two quantities. They apply and extend previous understanding of division to divide unit fractions by whole numbers and whole numbers by unit fractions and solve related real-world problems. They understand and explain why the procedures for multiplying and dividing fractions make sense.

The Grade 5 Common Core State Standards for Number and Operations–Fractions specify that students should–

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

The following hands-on activities provide students opportunities to use concrete models and create visual representations that help them understand fractions. Students should be able to evaluate the utility of models and determine which are most useful in different problem solving contexts. A good concrete foundation helps students evaluate the results of problems and tell whether results make sense.

Number and Operations—Fractions

Contents

LESSON 1

Objective

Add fractions with unlike denominators.

Common Core State Standards

- **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
- **5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Number and Operations–Fractions

Add Fractions with Unlike Denominators

Students build on their knowledge of fractions as they use models to add fractions with unlike denominators. They may use different approaches, such as number sense or reasoning, to find the solution to a problem. In this activity, students follow the standard algorithm using their knowledge of equivalent fractions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Why do we need common denominators?*
- Have students explain how they know which fractions to rename when they find the sum.
- **Ask:** *How do you use your knowledge of equivalent fractions to add fractions with unlike denominators?*

Solve It

Reread the problem with students. Have students draw a possible map of Emilio's ride and label the distances between destinations. Then have them explain how they found the total distance Emilio rode to school that morning.

More Ideas

For other ways to teach adding fractions—

- Use Fraction Tower® Equivalency Cubes to model the problems.
- Use the Fraction Measurement Ring from a set of Rainbow Fraction Circle Rings along with the Deluxe Rainbow Fraction® Circles to model the problem. Students can place fraction pieces showing each addend inside the ring to find the sum. Make sure students understand why the denominator in the sum differs from the denominator in one or both of the addends.

Formative Assessment

Have students try the following problem.

Deon grows carrots in $\frac{1}{6}$ of his garden. He grows potatoes in another $\frac{1}{4}$ of the garden. The rest of the garden is planted with flowers. What fraction of Deon's garden is used to grow vegetables?

A. $\frac{1}{10}$ **B.** $\frac{2}{12}$ **C.** $\frac{2}{10}$ **D.** $\frac{5}{12}$

Try It! 15 minutes | Groups of 4

Here is a problem about adding fractions with unlike denominators.

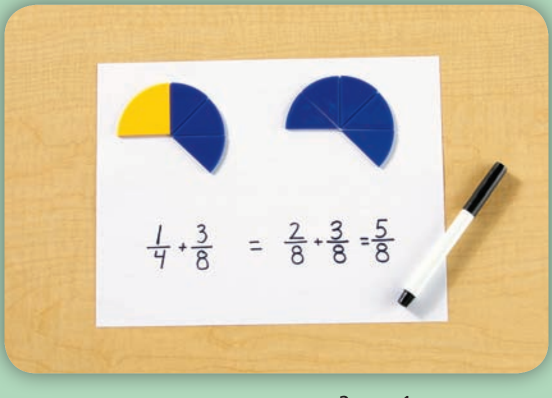
Emilio rides $\frac{1}{4}$ mile from his house to his friend Jake's house. Together they ride $\frac{3}{8}$ mile to school. How far does Emilio ride to school that day?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Circles, paper, and pencils to students.

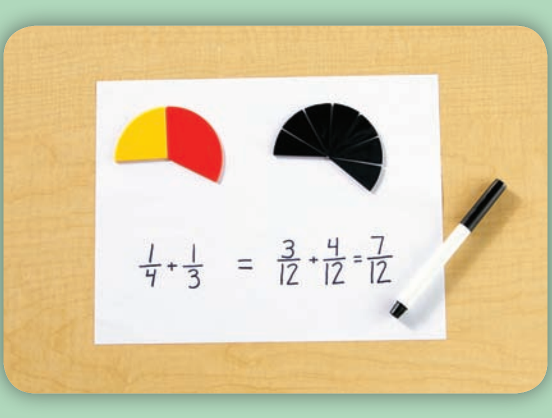
Materials

- Deluxe Rainbow Fraction® Circles (1 set per group)
- paper (1 sheet per group)
- pencils (1 per group)

1. Ask: *How do you know what operation to use to solve the problem?* Write $\frac{1}{4}+\frac{3}{8}$ on the board. Have students model each fraction. **Ask:** *How can you rename one fraction so that both have the same denominator?*



2. Have students substitute $\frac{2}{8}$ for $\frac{1}{4}$. **Ask:** *How do you find the sum of two fractions when the denominators are the same?* Have students complete the number sentence.



3. Ask: *How would your method change if Jake lived $\frac{1}{3}$ mile from school?* Write $\frac{1}{4}+\frac{1}{3}$ on the board. Have students model $\frac{1}{4}+\frac{1}{3}$ to see that both denominators must be changed before adding. Then have them write an equation for the model.

Look Out!

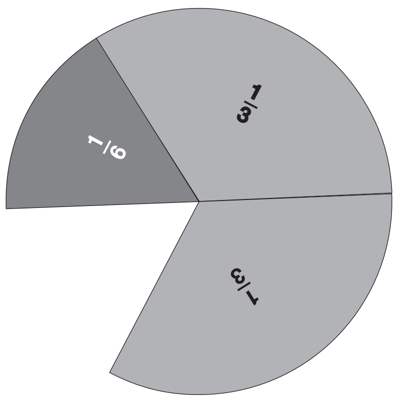
Some students may not find a common denominator before adding. These students may add the numerators and then add the denominators. Suggest that they check their solution by using fraction pieces to show the sum, and then place those pieces over the tops of the models for the addends to verify they are an exact match. If they are not an exact match, suggest that students use other same-size fraction pieces to discover a combination that does fit. Students also may need to fit pieces over the original model to find equivalent fractions.

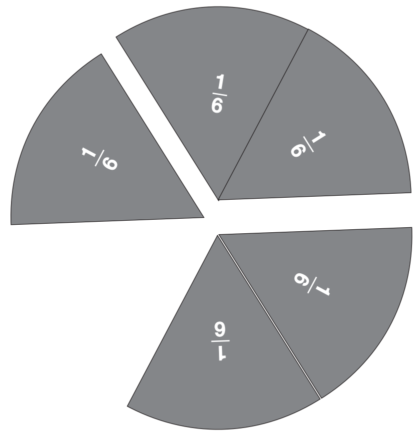
Answer Key

Use Fraction Circles to model the fractions shown. Write the addition sentences modeled.

(Check students' work.)

1.





$\frac{1}{6} + \frac{2}{3} = \frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

Using Fraction Circles, model the fractions to find the sum. Sketch the models. Write an addition sentence to show the sum.

(Check students' models.)

2. $\frac{2}{5} + \frac{3}{10}$

$\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

Find each sum. Simplify.

3. $\frac{1}{3} + \frac{5}{12} =$ $\frac{3}{4}$

4. $\frac{5}{6} + \frac{1}{3} =$ $\frac{7}{6} = 1\frac{1}{6}$

5. $\frac{3}{4} + \frac{1}{12} =$ $\frac{5}{6}$

6. $\frac{3}{5} + \frac{7}{10} =$ $\frac{13}{10} = 1\frac{3}{10}$

7. $\frac{2}{5} + \frac{7}{10} =$ $\frac{11}{10} = 1\frac{1}{10}$

8. $\frac{1}{4} + \frac{5}{8} =$ $\frac{7}{8}$

Answer Key

Challenge! Write instructions for how to find the sum in Problem 8.

Challenge: (Sample) Exchange the Fraction Circle for $\frac{1}{4}$ for two $\frac{1}{8}$ sections. That gives seven $\frac{1}{8}$ sections. So, the sum is $\frac{7}{8}$.

LESSON 2

Objective

Subtract fractions with unlike denominators.

Common Core State Standards

- **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
- **5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Number and Operations–Fractions

Subtract Fractions with Unlike Denominators

Students model the subtraction of fractions with unlike denominators as the first step toward using an algorithm to subtract. Many students will use prior experience with finding equivalent fractions to find common denominators before subtracting. Students also may use number sense or reasoning to find the solution to a problem.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *When finding the difference between two fractions, why can you subtract the numerators of two fractions with the same denominator, but cannot subtract the numerators of two fractions with unlike denominators?*
- **Ask:** *Which fraction or fractions would you rename to find $\frac{9}{10} - \frac{2}{5}$?*
- Have students compare addition and subtraction of fractions with unlike denominators. **Ask:** *How can you use addition to check your answer to a subtraction problem?*

Solve It

Reread the problem with students. Have students explain in writing why they needed to subtract to find the solution. Then have them describe how they found the quantity of milk needed from the second carton.

More Ideas

For other ways to teach subtracting fractions with unlike denominators—

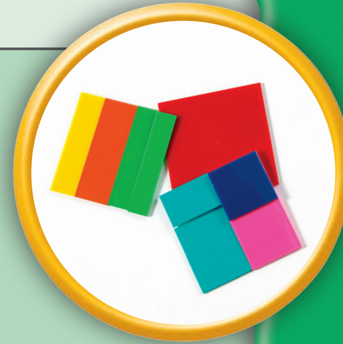
- Have students use Deluxe Rainbow Fraction® Circles or Fraction Tower® Equivalency Cubes to model subtraction problems.
- Have students use the hexagon, blue rhombus, trapezoid, and triangle from a set of Pattern Blocks to model subtraction. Let the hexagon represent one whole; the blue rhombus, $\frac{1}{3}$; the trapezoid, $\frac{1}{2}$; and the triangle, $\frac{1}{6}$. Have students write as many subtraction sentences as they can, using these fractions. Suggest that students fit pieces over larger pieces to help them find each difference.

Formative Assessment

Have students try the following problem.

Jordan and Mark are painting opposite sides of a fence. Mark has painted $\frac{7}{10}$ of his side. Jordan has painted $\frac{1}{2}$ of his side. How much more has Mark painted than Jordan?

A. $\frac{5}{8}$ B. $\frac{5}{10}$ C. $\frac{1}{5}$ D. $\frac{1}{10}$



Try It!

15 minutes | Groups of 4

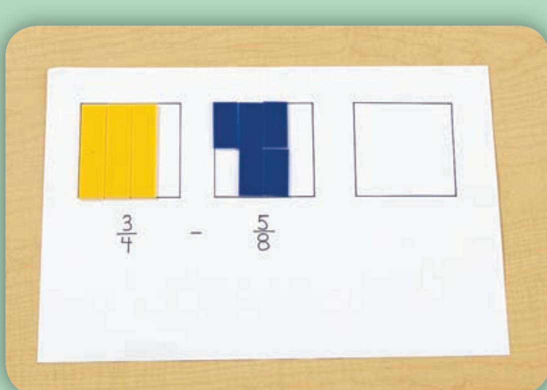
Here is a problem about subtracting fractions with unlike denominators.

A cornbread recipe calls for $\frac{3}{4}$ cup of milk. Rachel uses the last $\frac{5}{8}$ cup of milk in one carton. She opens another carton and pours the remaining amount needed. How much milk does Rachel use from the newly opened carton?

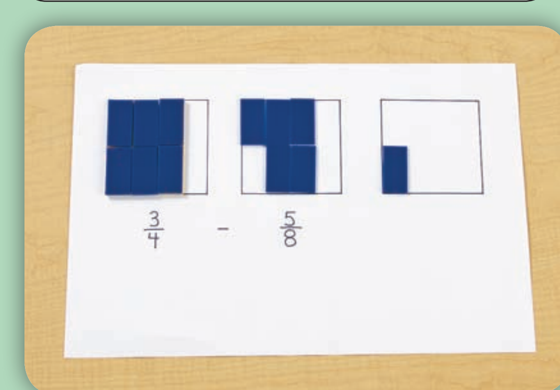
Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Squares, paper, and pencils to students. Have students trace 3 whole squares in a row on the paper.

Materials

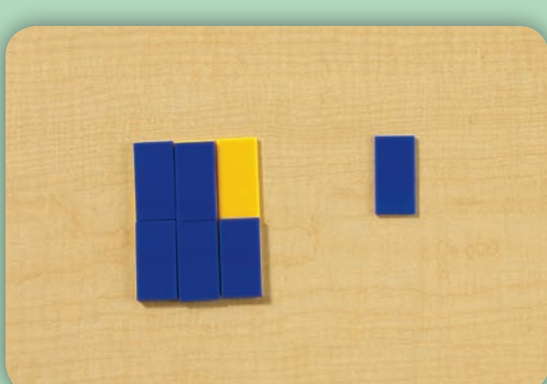
- Deluxe Rainbow Fraction® Squares (2 sets per group)
- paper (11" x 17"; 1 sheet per group)
- pencils (1 per group)



1. Have students model each fraction. **Ask:** *What expression can you write to show the situation?* Write $\frac{3}{4} - \frac{5}{8}$ on the board. **Ask:** *Using what you know about adding two fractions, what do you think you should do first to subtract these two fractions?*



2. Have students substitute $\frac{6}{8}$ for $\frac{3}{4}$. **Ask:** *Now that both fractions have the same denominator, how can you find the difference?* Have students write and model the subtraction equation for the situation.



3. Say: *The Fraction Squares can be used to check your answer.* Have students model each fraction again. This time have them place the five blue pieces on top of the three yellow pieces. **Ask:** *What piece is needed to completely cover the three yellow pieces? What fraction does this piece represent? Is this fraction the same as your answer?*

Look Out!

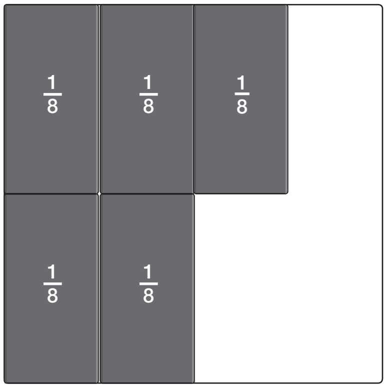
Some students may not measure the squares properly when finding equivalent fractions. Suggest that these students place the Fraction Squares over each other to ensure that the fractional values are exactly the same. Other students may not recognize the importance of finding common denominators, instead looking for a Fraction Square that will be the same size. Have these students explain why their solution works and how they can use it to find the differences in other situations.

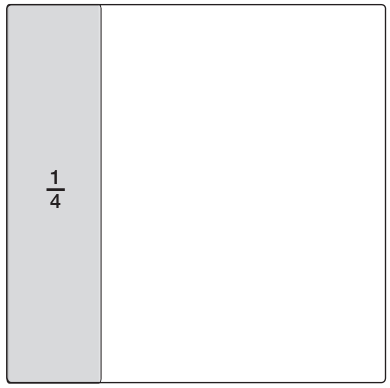
Answer Key

Use Fraction Squares to model the fractions shown. Use Fraction Squares to find fractions with the same denominators. Write the fractions and then find the difference.

(Check students' work.)

1.





$\frac{5}{8}$ – $\frac{1}{4}$ = $\frac{3}{8}$

Using Fraction Squares, model the subtraction problem. Sketch the model. Write the difference.

(Check students' models.)

2. $\frac{5}{6} - \frac{1}{3}$ $\frac{1}{2}$

3. $\frac{3}{4} - \frac{3}{8}$ $\frac{3}{8}$

Find each difference.

4. $\frac{3}{4} - \frac{5}{12} =$ $\frac{1}{3}$

5. $\frac{5}{6} - \frac{2}{3} =$ $\frac{1}{6}$

6. $\frac{3}{5} - \frac{1}{10} =$ $\frac{1}{2}$

7. $\frac{3}{4} - \frac{1}{12} =$ $\frac{2}{3}$

8. $\frac{5}{8} - \frac{1}{4} =$ $\frac{3}{8}$

9. $\frac{2}{5} - \frac{1}{10} =$ $\frac{3}{10}$

10. $\frac{2}{3} - \frac{5}{12} =$ $\frac{1}{4}$

11. $\frac{7}{12} - \frac{1}{4} =$ $\frac{1}{3}$

Answer Key

Challenge! Explain how you can use addition to check that you subtracted correctly. Draw a picture to help,

Challenge: (Sample) You can work backward by adding the answer to the number you subtracted to get the number you started with.

LESSON 3

Objective

Interpret a fraction as division of the numerator by the denominator.

Common Core State Standards

- **5.NF.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Number and Operations-Fractions

Fractions as Division

Division problems are frequently written as stacked fractions. This lesson introduces students to fractions as division problems and helps familiarize them with mixed-number answers. Fraction Circles and other concrete models will help students visualize these concepts.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *Will each camper get a whole liter of water? More than a liter? How many pieces does the fifth Fraction Circle need to be divided into?*
- **Ask:** *How can you write the division problem as a fraction? Which number is the numerator? Which number is the denominator? How do you know?*
- Have students write the problem as the improper fraction $\frac{5}{4}$.

Solve It

Reread the problem with students. Have students draw the problem and write it as a fraction equal to the correct mixed-number answer.

More Ideas

For other ways to teach about fractions as division—

- Have students multiply $\frac{3}{4}$ by 4, $\frac{4}{5}$ by 5, and $\frac{5}{6}$ by 6. Discuss that if $\frac{3}{4} \times 4 = 3$, then $3 \div 4 = \frac{3}{4}$. Have students model the multiplication problems with Deluxe Rainbow Fraction® Circles.
- Have students use Fraction Tower® Equivalency Cubes and Folding Number Lines (fractions side) to model the following problem: Maya wants to divide 40 pounds of oats evenly among her 6 horses. How many pounds of oats will each horse get? Have students identify between what two whole numbers the answer lies (6 and 7) and model dividing the remainder.

Formative Assessment

Have students try the following problem.

If 3 friends want to divide 5 oranges equally, how many oranges does each friend get?

A. $\frac{3}{5}$ **B.** $1\frac{2}{3}$ **C.** $2\frac{1}{3}$ **D.** $2\frac{1}{2}$

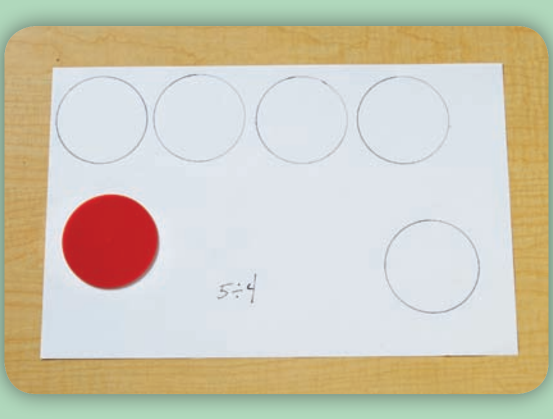
Try It! 20 minutes | Groups of 4

Here is a problem about using fractions as division.

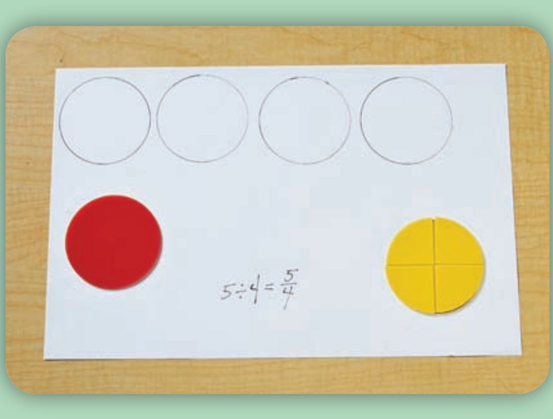
Four campers need to share 5 liters of water equally. How many liters of water does each camper get?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Circles, paper, and pencils to students.

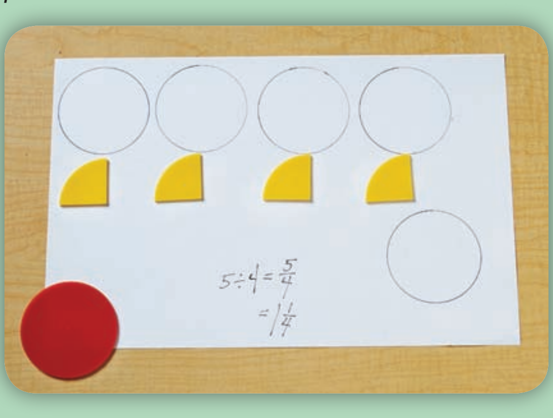
Materials
- Deluxe Rainbow Fraction® Circles (1 set per group)
- paper (11" × 17"; 1 sheet per group)
- pencils (1 per group)



1. Ask: *How many liters of water are there?* Have students show the number 5 with 5 whole Fraction Circles or by tracing a whole Fraction Circle 5 times. **Ask:** *How many campers are there? How would you write this as a division problem?* Have students write 5 ÷ 4.



2. Ask: *How can you divide those 5 circles evenly by 4? If each camper gets one whole liter, how will they divide the remaining liter? What fraction of the remaining liter of water will each camper get?*



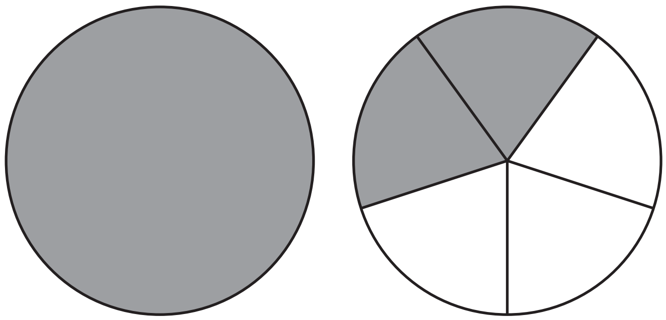
3. Say: *You can write this division problem as the fraction $\frac{5}{4}$.* **Ask:** *What mixed number does the fraction $\frac{5}{4}$ equal?*

⚠ Look Out!

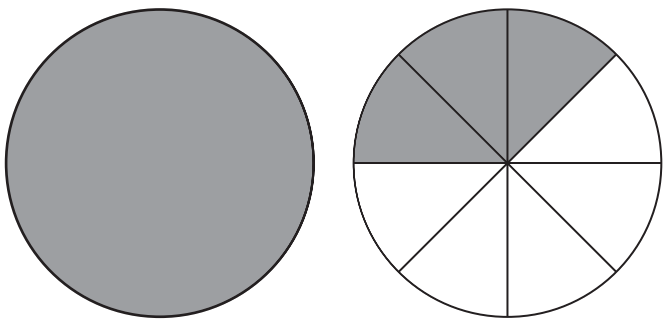
Students may have difficulty with Step 2. Remind them that each of the 4 campers will need an equal share of the remaining liter, so they would use fourths to model this step.

Use Fraction Circles to model each division problem. Write the problem as a fraction. Write the quotient as a mixed number. (Check students' work.)

1. $7 \div 5 = \underline{\frac{7}{5}} = \underline{1\frac{2}{5}}$



2. $11 \div 8 = \underline{\frac{11}{8}} = \underline{1\frac{3}{8}}$



Using Fraction Circles, model each division problem. Sketch the model. Write the problem as a fraction. Write the quotient as a mixed number. (Check students' models.)

3. $8 \div 6 = \underline{\frac{8}{6}} = \underline{1\frac{2}{6} = 1\frac{1}{3}}$

4. $9 \div 4 = \underline{\frac{9}{4}} = \underline{2\frac{1}{4}}$

Write each quotient as a mixed number.

5. $14 \div 6 = \underline{2\frac{2}{6}}$

6. $27 \div 5 = \underline{5\frac{2}{5}}$

7. $15 \div 4 = \underline{3\frac{3}{4}}$

8. $9 \div 7 = \underline{1\frac{2}{7}}$

Answer Key

Challenge! Explain how knowing $\frac{6}{5} \times 5 = 6$ can help you solve $6 \div 5 = \frac{6}{5}$.

Challenge: (Sample) I know that multiplication and division have an inverse relationship, so by knowing $\frac{6}{5} \times 5 = \frac{30}{5} = 6$, I can better understand $6 \div 5$.

LESSON 4

Objective

Multiply a fraction by a whole number.

Common Core State Standards

- **5.NF.4a** Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

Number and Operations–Fractions

Multiplying Fractions by Whole Numbers

Using concrete models helps students develop the fundamental understanding that multiplying a fraction by a whole number can be represented as repeated addition. Models also can help them interpret the multiplication in different ways for different situations. Real-world situations and mathematical problems will provide students opportunities to practice these concepts.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Write $\frac{2}{3} \times 4$ on the board. **Say:** *When we multiply a fraction by a whole number, we can multiply the numerator by the whole number and divide by the denominator. $\frac{2}{3} \times 4$ is the same as $2 \times 4 \div 3$.* Write $\frac{2}{3} \times 4 = 2 \times 4 \div 3 = \frac{8}{3}$.
- **Say:** *When a fraction has a numerator that is larger than the denominator, it is an improper fraction and represents more than one whole.* **Ask:** *How many whole circles do you have in your model? How many thirds are remaining?* Write $\frac{8}{3} = 2\frac{2}{3}$.

Solve It

- Reread the problem with students. Have them draw circles divided into thirds to answer the problem and write the multiplication sentence $\frac{2}{3} \times 4 = 2 \times 4 \div 3 = \frac{8}{3} = 2\frac{2}{3}$.

More Ideas

For other ways to teach multiplying a fraction by a whole number—

- Have students use Fraction Tower® Equivalency Cubes to solve similar problems. Have them trace copies of a whole tower as needed.
- Have students investigate different interpretations of the multiplication. For example, have them model $\frac{2}{3} \times 4 = 2 \times \frac{4}{3}$ by building the $\frac{4}{3}$ model first and doubling it.

Formative Assessment

Have students try the following problem.

Samira's cat spends about $\frac{5}{6}$ of each day napping. How many hours a day does Samira's cat sleep?

A. $\frac{5}{6} \times 24 = \frac{120}{144}$ **B.** $\frac{5}{6} \times 24 = \frac{96}{6}$ **C.** $\frac{5}{6} \times 24 = \frac{120}{6}$ **D.** $\frac{5}{6} \times 24 = \frac{144}{6}$

Try It! 20 minutes | Groups of 4

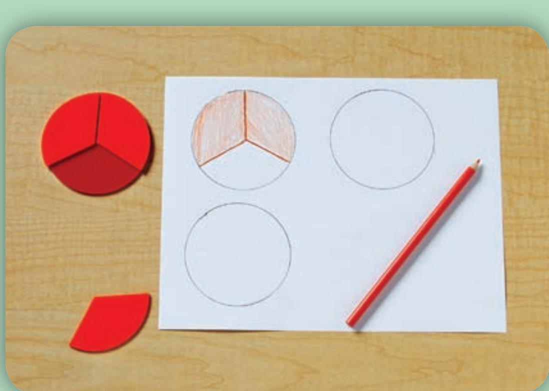
Here is a problem about multiplying fractions by whole numbers.

The fifth graders are making 4 batches of muffins for the school bake sale. They need $\frac{2}{3}$ cup of walnuts for each batch. How many cups of walnuts do they need?

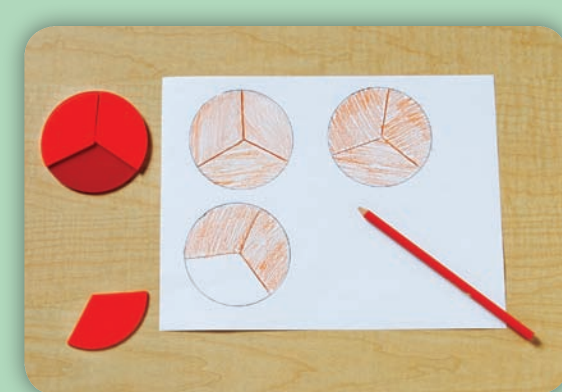
Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Circles, paper, and pencils to students.

Materials

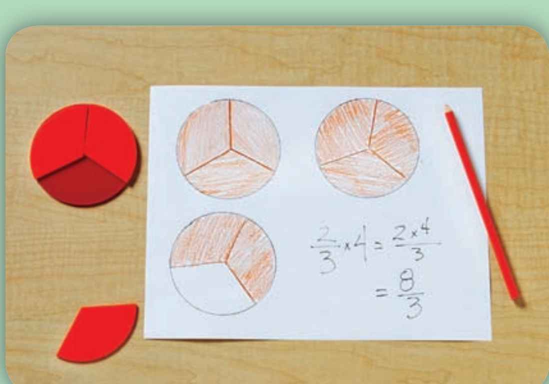
- Deluxe Rainbow® Fraction Circles (1 set per group)
- paper (1 per group)
- pencils (1 per group)
- colored pencils (1 per group)



1. Say: *Find the Fraction Circle that represents thirds. Show how many cups of walnuts are needed for one batch of muffins.* **Ask:** *How many times do we need this amount?* Explain that since $\frac{2}{3}$ cup is needed for each of 4 batches, students need to show $\frac{2}{3}$ four times. Have students trace 3 whole circles onto a piece of paper, divide them into thirds, and color $\frac{2}{3}$ of the first circle orange.



2. Say: *One way to multiply a fraction by a whole number is to multiply the numerator by the whole number and then divide by the denominator.* **Ask:** *What is our numerator? What is 2 times 4? How many more thirds do we need to add to the model?* Have students color 6 more thirds on their paper.



3. Ask: *How many equal parts do we have? What improper fraction does your model represent?* Have students write the number sentence, $\frac{2}{3} \times 4 = 2 \times 4 \div 3 = \frac{8}{3}$.

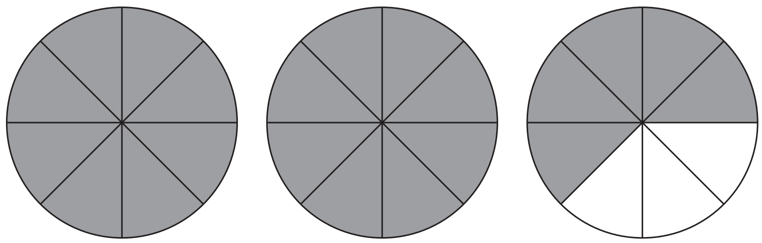
⚠ Look Out!

Watch for students who want to write the answer as 8 instead of $\frac{8}{3}$. Have them group their 2 thirds pieces into 4 groups and count them (one third, 2 thirds, etc.) to see they have 8 thirds. Then have them push 3 of the thirds pieces together two times to see that $\frac{8}{3} = 2\frac{2}{3}$.

Answer Key

Use the model to solve the problem. Write the answer as an improper fraction and as a mixed number. (Check students' work.)

1. Sarah ran 7 miles in one week. Her younger brother ran $\frac{3}{8}$ as far. How many miles did her brother run?

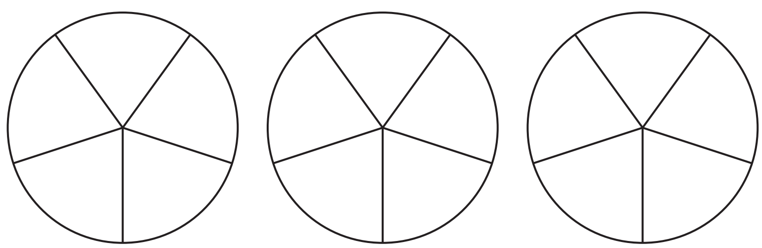


$\frac{3}{8} \times 7 = 3 \times 7 \div 8 = \underline{\frac{21}{8}} = \underline{2\frac{5}{8}}$ miles

Draw a model to solve the problem. Write the equation.

2. Jershom made 6 baskets in a basketball game. He got $\frac{2}{5}$ as many rebounds as baskets. How many rebounds did Jershom get?

(Check students' models.)



$\frac{2}{5} \times 6 = \underline{2 \times 6 \div 5} = \underline{\frac{12}{5}} = \underline{2\frac{2}{5}}$ rebounds

Multiply to complete the equations.

3. $\frac{2}{9} \times 6 = \underline{\frac{12}{9}} = \underline{1\frac{3}{9}}$

4. $\frac{3}{7} \times 21 = \underline{\frac{63}{7}} = \underline{9}$

5. $\frac{7}{8} \times 4 = \underline{\frac{28}{8}} = \underline{3\frac{4}{8}}$

6. $\frac{2}{5} \times 9 = \underline{\frac{18}{5}} = \underline{3\frac{3}{5}}$

7. $\frac{1}{7} \times 28 = \underline{\frac{28}{7}} = \underline{4}$

8. $\frac{2}{3} \times 27 = \underline{\frac{54}{3}} = \underline{18}$

Answer Key

Challenge! Create a story context for the expression $\frac{3}{8} \times 5$, and solve the problem.

Challenge: $\frac{3}{8} \times 5 = \frac{15}{8} = 1\frac{7}{8}$; stories will vary.

LESSON 5

Multiplying Two Fractions

Objective

Multiply a fraction by a fraction.

Common Core State Standards

- **5.NF.4a** Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
- **5.NF.4b** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Number and Operations–Fractions

Students will build upon their understanding of multiplying a fraction by a whole number to now multiply a fraction by a fraction. Using concrete models to build rectangles with fractional side lengths, students can show that the area is the same as it would be by multiplying side lengths. Visualizing fraction products as rectangular areas will increase understanding.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Write $\frac{3}{4} \times \frac{1}{2}$ on the board. **Ask:** *What fraction do the yellow pieces represent? What fraction does the pink piece represent? What fraction do the blue pieces represent?* Write $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$. **Ask:** *What is the area of the park?*
- **Say:** *To multiply fractions without using models, use the algorithm: Multiply the numerators and multiply the denominators.* Write $\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$.
- **Ask:** *Do you think you could show any fraction multiplication problem picture using this method when each of the two factors is less than 1?*

Solve It

Reread the problem with students. Have them draw or trace the Fraction Squares pieces on the BLM and shade the overlapping area to answer the problem and write the solution as an equation.

More Ideas

For other ways to teach about multiplying a fraction by a fraction—

- Have students use Color Tiles to represent the fractional side lengths of rectangles like denominators making sure they use the appropriate number of tiles to represent the fractional parts. Encourage them to use different colors to represent the length and the width.
- Have students trace Fraction Square pieces on Fraction Squares BLM 8 using different colored pencils, and shade the overlapping section to show the area of the fraction product.

Formative Assessment

Have students try the following problem.

Eva wants to plant pumpkins in a field that is $\frac{1}{3}$ mile by $\frac{1}{4}$ mile. What is the area of the field?

A. $\frac{1}{16}$ square mile **B.** $\frac{1}{12}$ square mile **C.** $\frac{1}{7}$ square mile **D.** $\frac{1}{2}$ square mile

Try It!

20 minutes | Groups of 4

Here is a problem about multiplying a fraction by a fraction.

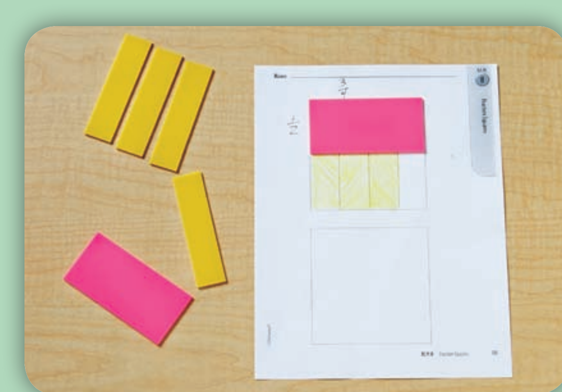
Zeke runs a lawn service. He needs to find the area of a rectangular park. The park measures $\frac{3}{4}$ mile by $\frac{1}{2}$ mile. What is the area of the park?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Squares, Fraction Squares sheets, and pencils to students.

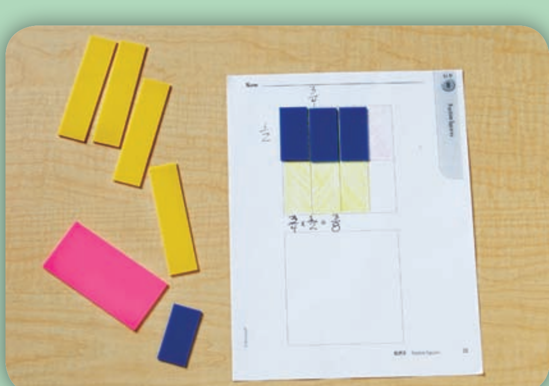
Materials

- Deluxe Rainbow Fraction® Squares (1 set per group)
- Fraction Squares (BLM 8; 1 per group)
- pencils (1 per group)

1. Say: *Find the Fraction Square that represents fourths and show $\frac{3}{4}$ on your paper.* Have students place three $\frac{1}{4}$ pieces vertically in the top square of the BLM. **Say:** *This fraction, $\frac{3}{4}$, represents the side of the park that is $\frac{3}{4}$ of a mile.* Have students mark the side as $\frac{3}{4}$.



2. Say: *Now find the Fraction Square that represents halves. Place a $\frac{1}{2}$ piece across the top half of your $\frac{3}{4}$ pieces to mark off the other side of the rectangle.* Have students place the $\frac{1}{2}$ piece horizontally across the top half of the other pieces and mark the side as $\frac{1}{2}$.



3. Say: *Find out what fraction pieces you can use to fill in the overlap area.* If necessary, help students determine that eighths pieces will work. **Ask:** *How many eighths fit in the overlap area?* Have students write the number sentence, $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$.

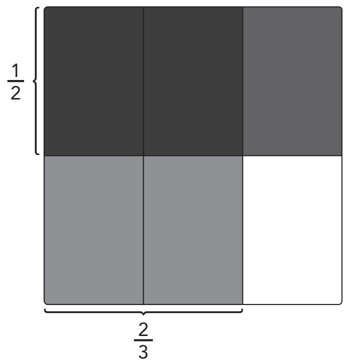
⚠ Look Out!

Watch for students who want to lay the $\frac{1}{2}$ piece alongside the $\frac{3}{4}$ pieces instead of across the top of them. Watch also for students who want to lay four $\frac{1}{8}$ pieces across the top of the $\frac{1}{2}$ piece. Point out that there is no yellow piece under the last fourth of the $\frac{1}{2}$ piece.

Answer Key

Use Fraction Squares to model the problem. Write the solution. (Check students' work.)

1. Alex wants to cover a bulletin board with cloth. The board measures $\frac{2}{3}$ yard by $\frac{1}{2}$ yard. What is the area of the bulletin board?



$\frac{2}{3} \times \frac{1}{2} =$ $\frac{2}{6}$ or $\frac{1}{3}$ square yard

Using Fraction Squares, model the problem. Sketch the model. Write the multiplication sentence that shows the solution.

2. A frame measures $\frac{5}{6}$ foot by $\frac{3}{4}$ foot. What is the area of the frame? (Check students' models.)

$\frac{5}{6}$ × $\frac{3}{4}$ = $\frac{15}{24}$ or $\frac{5}{8}$ square foot

Find each product.

3. $\frac{1}{2} \times \frac{4}{5} =$ $\frac{4}{10}$ or $\frac{2}{5}$

4. $\frac{1}{3} \times \frac{3}{4} =$ $\frac{3}{12}$ or $\frac{1}{4}$

5. $\frac{2}{5} \times \frac{3}{5} =$ $\frac{6}{25}$

6. $\frac{1}{3} \times \frac{9}{10} =$ $\frac{9}{30}$ or $\frac{3}{10}$

7. $\frac{3}{10} \times \frac{5}{8} =$ $\frac{15}{80}$ or $\frac{3}{16}$

8. $\frac{2}{3} \times \frac{3}{8} =$ $\frac{6}{24}$ or $\frac{1}{4}$

Answer Key

Challenge! Create a story context for the expression $\frac{2}{3} \times \frac{4}{5}$, and solve the problem.

Challenge: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$; stories will vary.

LESSON 6

Objective

Multiply with fractions, including a fraction with a whole number and a fraction with a fraction.

Common Core State Standards

- **5.NF.5a** Interpret multiplication as scaling (resizing), by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Number and Operations–Fractions

Multiply with Fractions

As students learn how to multiply with fractions, they continue to use the idea of multiplication as repeated addition. The use of models helps them discover why multiplying with fractions results in a product that is less than one or both factors. This is surprising to students because it is different from multiplying whole numbers where the product is greater than the factors (except when a factor is 0 or 1).

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *How can you use repeated addition in this problem?*
- **Ask:** *Why is the product $\frac{1}{2} \times \frac{1}{3}$ less than $\frac{1}{3}$? Explain the answer in terms of scaling. Elicit that multiplying $\frac{1}{2}$ by any number means you are making the number half as large.*

Solve It

Reread the problem with students. Point out that the "of" in $\frac{1}{2}$ *of* $\frac{1}{3}$ means to multiply, so it can be replaced by the multiplication symbol.

More Ideas

For other ways to teach multiplying with fractions—

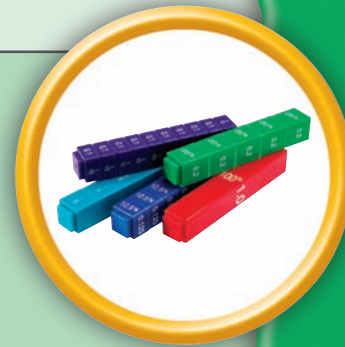
- Extend the lesson by using Fraction Tower® Equivalency Cubes to solve this problem: *2 friends share $\frac{3}{4}$ quart of lemonade. What fraction of the lemonade does each friend drink, if they share it equally?* Remind students that $\frac{3}{4}$ is $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, so half of $\frac{3}{4}$ is half of $\frac{1}{4}$ taken 3 times. Have students use two $\frac{1}{8}$ pieces to build a tower equal to $\frac{1}{4}$. Have them show that $\frac{1}{8}$ is $\frac{1}{2}$ of $\frac{1}{4}$ and that $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ is $\frac{3}{8}$, so $\frac{1}{2} \times \frac{3}{4}$ is $\frac{3}{8}$.
- Provide different scenarios to give students practice in multiplying fractions and whole numbers, such as $\frac{2}{5} \times 4$ and $\frac{3}{8} \times 3$. Have students use Deluxe Rainbow Fraction® Squares to model and solve the problems.

Formative Assessment

Have students try the following problem.

Kelly wants to decrease a recipe by $\frac{2}{3}$. How much less milk should she use if the recipe calls for 6 cups of milk?

A. $\frac{1}{6}$ cup　**B.** $\frac{1}{3}$ cup　**C.** 2 cups　**D.** 4 cups



Try It! 20 minutes | Groups of 4

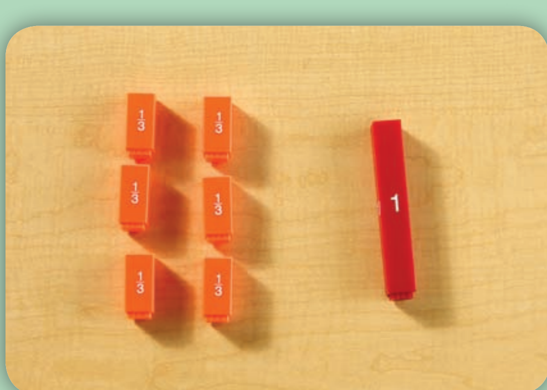
Here is a problem about multiplying with fractions.

Helen has two hot dogs. She cuts each hot dog into thirds to feed to her son, Matt. Matt eats all but $\frac{1}{2}$ of the last piece. How much of a hot dog is left over?

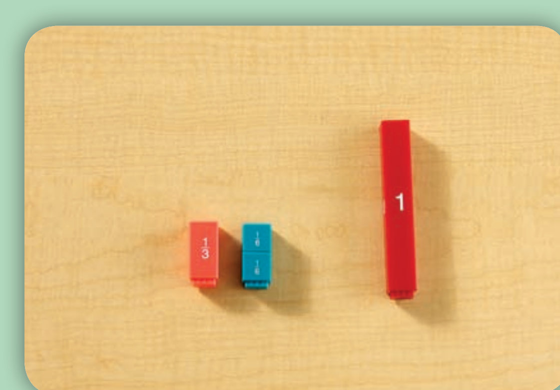
Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Equivalency Cubes, paper, and pencils to students.

Materials

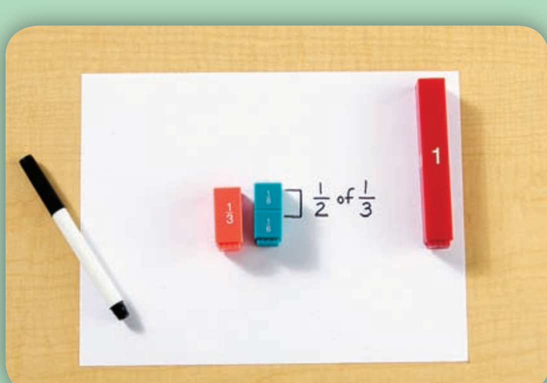
- Fraction Tower® Equivalency Cubes (2 sets per group)
- paper (1 sheet per group)
- pencils (1 per group)



1. Say: *You can use repeated addition to solve the problem. Since there are 6 hot dog pieces, you can add $\frac{1}{3}$ six times to represent the total.* Have students use six $\frac{1}{3}$ pieces to model the problem. Have students group the pieces and compare them to a whole.



2. Say: *Matt eats all but $\frac{1}{2}$ of the last piece. Find the fraction that represents the amount of a hot dog that Matt did not eat.* Have students place a $\frac{1}{3}$ piece on the table and then build an equivalent tower using two pieces of the same size. **Ask:** *Why does the equivalent tower need to have two pieces?* Be sure students realize that the tower must be equivalent to the $\frac{1}{3}$ piece.



3. Ask: *How can you use the equivalent tower to find $\frac{1}{2}$ of $\frac{1}{3}$?* Have students use a grouping symbol to show that each piece in the tower represents $\frac{1}{2}$ of $\frac{1}{3}$. **Ask:** *What is $\frac{1}{2}$ of $\frac{1}{3}$?*

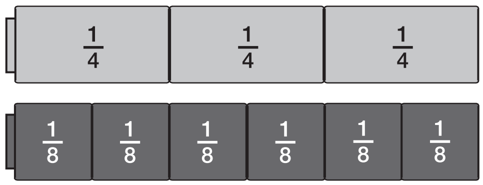
⚠ Look Out!

Watch for students who compare the $\frac{1}{2}$ piece to the $\frac{1}{3}$ piece when trying to find $\frac{1}{2}$ of $\frac{1}{3}$. Explain that the equivalent tower needs to show $\frac{1}{3}$ divided into halves. To emphasize this connection, have them trace around the $\frac{1}{3}$ piece and then use the divisions in the $\frac{2}{6}$ tower to divide the diagram into halves. Have them shade $\frac{1}{2}$ of the diagram and then compare it to the $\frac{2}{6}$ tower.

Use Fraction Towers to model the equivalent fractions. Complete the number sentence.

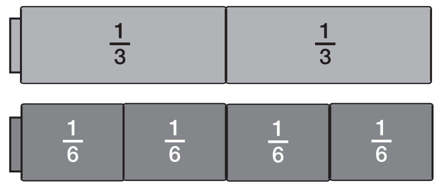
(Check students' work.)

1. What is $\frac{1}{2}$ of $\frac{3}{4}$?



$\frac{1}{2} \times \frac{3}{4} =$ $\frac{3}{8}$

2. What is $\frac{1}{4}$ of $\frac{2}{3}$?



$\frac{1}{4} \times \frac{2}{3} =$ $\frac{2}{12} = \frac{1}{6}$

Using Fraction Towers, model each product. Sketch the model. Write a number sentence for each product.

(Check students' models.)

3. What is $\frac{1}{2}$ of $\frac{5}{6}$?

$\frac{5}{12}$

4. What is $\frac{1}{3}$ of $\frac{3}{4}$?

$\frac{3}{12} = \frac{1}{4}$

Find each product.

5. $\frac{1}{2} \times \frac{1}{6}$

$\frac{1}{12}$

6. $\frac{1}{2} \times \frac{2}{5}$

$\frac{2}{10} = \frac{1}{5}$

7. $\frac{1}{5} \times \frac{5}{6}$

$\frac{5}{30} = \frac{1}{6}$

8. $\frac{1}{2} \times \frac{3}{5}$

$\frac{3}{10}$

9. $\frac{1}{3} \times \frac{6}{8}$

$\frac{6}{24} = \frac{1}{4}$

10. $\frac{1}{2} \times \frac{3}{10}$

$\frac{3}{20}$

Answer Key

Challenge! Explain why $\frac{1}{2} \times \frac{2}{3}$ is less than 1. Draw a picture to help.

Challenge: (Sample) I am finding one-half of something that is already smaller than 1.

LESSON 7

Objective

Divide a unit fraction by a whole number.

Common Core State Standards

- **5.NF.7a** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*
- **5.NF.7c** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Number and Operations–Fractions

Divide a Unit Fraction by a Whole Number

Students begin their exploration of fractions by dividing shapes into equal-size sections. They learn how to write a unit fraction to represent one section. Just as one whole can be divided into equal-size parts, one of those parts can be further divided. In this lesson, students apply this idea to divide a unit fraction by a whole number.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Ask:** *If Bonnie has 2 children, how much land will each child get? How much land will the 2 children get together?*
- **Ask:** *If Bonnie has 3 children, how much land will each child get? How much land will the 3 children get together?*
- **Say:** *Continue, and describe the patterns in this problem.*

Solve It

Reread the problem with students. In every case, the children will get half of the grandfather's land. As that half is divided among more and more children, the half is divided into more and more pieces. Have students write number sentences to express the division for 2, 3, 4, 5, and 6 children.

More Ideas

For other ways to teach about dividing a unit fraction by a whole number—

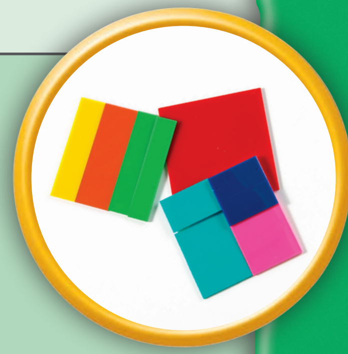
- Have students do this problem again using Deluxe Rainbow Fraction® Circles. They can stack the equal areas for $\frac{1}{2}$ and spread the colors out to see the increase in the number of pieces for 2, 3, 4, 5, and 6 children. Ask students what the fraction would be for 10 children.
- Use Deluxe Rainbow Fraction® Circles and Squares to divide different unit fractions by whole numbers.

Formative Assessment

Have students try the following problem.

What is $\frac{1}{3} \div 5$?

A. $\frac{1}{15}$ **B.** $\frac{1}{8}$ **C.** $\frac{3}{5}$ **D.** $\frac{5}{3}$



Try It!

15 minutes | Groups of 5

Here is a problem about dividing a unit fraction by a whole number.

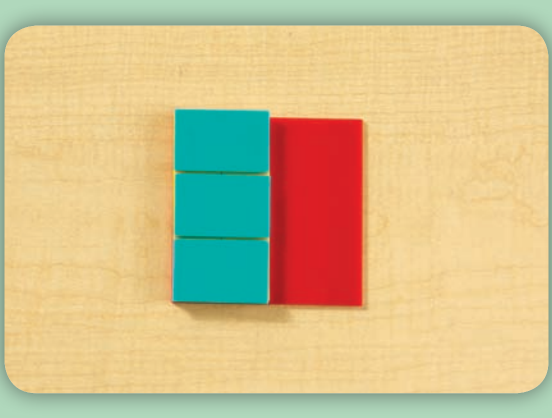
Bonnie bought half of her father's land and will divide it evenly among her own children. What fraction describes the portion of their grandfather's land that each child will receive if Bonnie has 2 children? 3 children? 4 children? 5 children? 6 children?

Introduce the problem. Then have students do the activity to solve the problem. Each student in the group can answer one question. Distribute the Fraction Squares, paper, and pencils to students.

Materials

- Deluxe Rainbow Fraction® Squares (1 set per group)
- paper (2 sheets per group)
- pencils (1 per group)

1. Say: *Use the red square to represent Bonnie's father's land. Place a pink rectangle on the square to represent Bonnie's land, which is half of her father's land. Now show how Bonnie's land can be divided evenly for 2 children.* Students place 2 yellow rectangles on top of the pink rectangle to divide it into two equal halves.



2. Ask: *How can you show Bonnie's land divided into 3 equal parts for 3 children?* Students place 3 aqua rectangles on top of the 2 yellow rectangles to divide the area into three equal thirds.

3. Say: *Continue the problem for 4, 5, and 6 children.* Students continue with blue, purple, and then black rectangles. Have students undo the stack, line up the colors on paper, and write a fraction for each color.

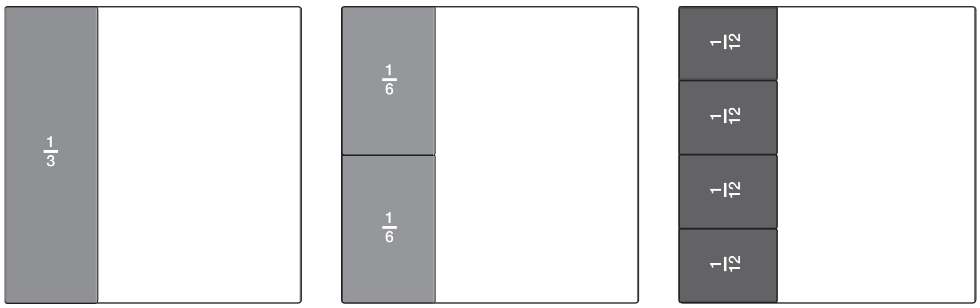
4. Elicit from students that the models they have lined up represent $\frac{1}{2}$ divided into different numbers of equal parts. Write $\frac{1}{2} \div 2 = ?$ on the board, and have students use the yellow model to determine the quotient. Continue for the aqua model, the blue model, and so on.

Answer Key

Use Fraction Squares to model the fraction compared to 1. Complete each equation.

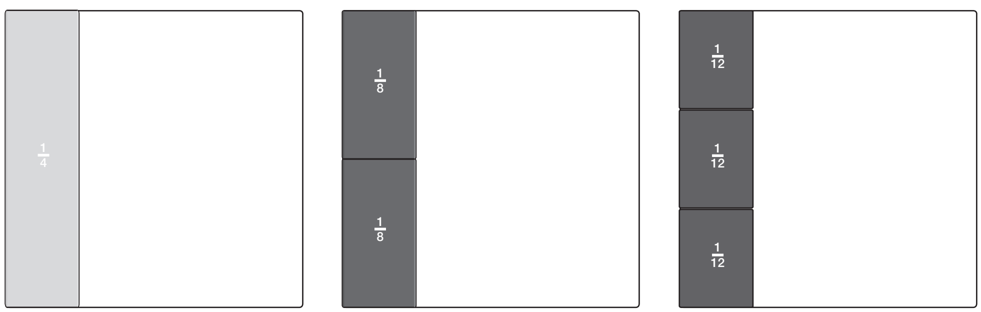
(Check students' work.)

1.



$\frac{1}{3} \div 2 = \frac{1}{6}$ $\frac{1}{3} \div 4 = \frac{1}{12}$

2.



$\frac{1}{4} \div 2 = \frac{1}{8}$ $\frac{1}{4} \div 3 = \frac{1}{12}$

Using Fraction Squares, model the division. Sketch the models. Write the quotient.

(Check students' models.)

3. $\frac{1}{5} \div 2 = \frac{1}{10}$

Find the quotient

4. $\frac{1}{2} \div 7 = \frac{1}{14}$

5. $\frac{1}{3} \div 3 = \frac{1}{9}$

6. $\frac{1}{4} \div 4 = \frac{1}{16}$

7. $\frac{1}{5} \div 4 = \frac{1}{20}$

8. $\frac{1}{6} \div 2 = \frac{1}{12}$

9. $\frac{1}{8} \div 2 = \frac{1}{16}$

Answer Key

Challenge! What method do you use to divide a unit fraction by a whole number when you don't use the Fraction Squares? Why does it work?

Challenge: (Sample) Multiply the denominator of the unit fraction by the whole number. It works because dividing by a whole number is the same as multiplying by its reciprocal.

LESSON

8

Objective

Divide a whole number by a unit fraction.

Common Core State Standards

- **5.NF.7b** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

Number and Operations–Fractions

Dividing a Whole Number by a Fraction

Students will build upon their understanding of multiplying a fraction by a whole number to now divide a whole number by a fraction. Using concrete models and real-world situations will help students visualize the relationship between multiplying and dividing whole numbers by fractions.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- Write $3 \div \frac{1}{4}$ on the board. **Ask:** *What do the three whole towers represent? Why did you divide the three towers into fourths? What does each fourth represent?*
- **Say:** *You know that multiplication and division are inversely related.* Write, *If* $12 \times \frac{1}{4} = 3$, *then* $3 \div \frac{1}{4} = 12$. **Ask:** *How can you use this to help you divide a whole number by a fraction?*

Solve It

Reread the problem with students. Have them draw the tower pieces on the Fraction Towers BLM and number them 1–12 to answer the problem and write the equation.

More Ideas

For other ways to teach dividing a whole number by a unit fraction—

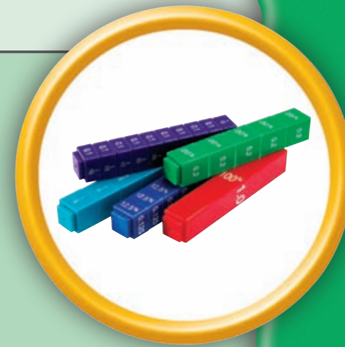
- Have students use Fraction Tower® Equivalency Cubes to divide whole numbers by unit fractions. Encourage students to color each fractional segment a different color. Have them write the equation as a division problem and then as a multiplication problem.
- For additional problems involving whole numbers divided by unit fractions, have students use Deluxe Rainbow Fraction® Circles. Encourage students to write the equation as a division problem and then as a multiplication problem.

Formative Assessment

Have students try the following problem.

Wanda has 12 feet of ribbon with which to tie party bags. If she cuts $\frac{1}{3}$ *foot of ribbon for each bag, how many pieces of ribbon can she cut?*

A. 4 **B.** 24 **C.** 15 **D.** 36



Try It!

15 minutes | Groups of 4

Here is a problem about dividing a whole number by a fraction.

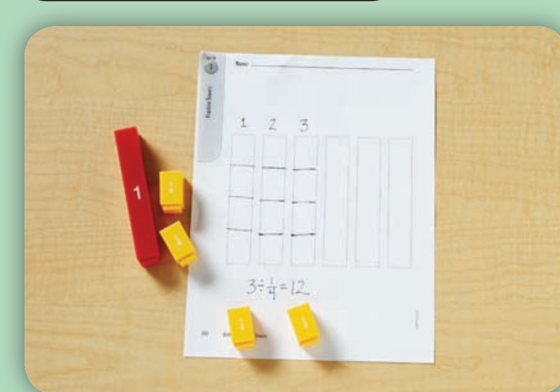
Jin has 3 cups of potting soil, small peat pots, and a package of seeds. If he puts $\frac{1}{4}$ cup of soil and a seed in each peat pot, how many peat pots can he fill with the soil?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Tower Equivalency Cubes, Fraction Tower sheets, and pencils to students.

Materials

- Fraction Tower® Equivalency Cubes (1 set per group)
- Fraction Towers (BLM 7)
- pencils

1. Ask: *How many cups of soil does Jin have? How much does he want to put in each pot?* Explain to students that they will use three Fraction Tower outlines to represent the three cups of soil. Have them find the tower that represents fourths and use it to mark off $\frac{1}{4}$ sections on three outlines. Have students write $3 \div \frac{1}{4}$.



2. Ask: *How many fourths are there in one whole tower? Three whole towers? How many peat pots can Jin fill with $\frac{1}{4}$ cup of soil in each?* Have students write $3 \div \frac{1}{4} = 12$.

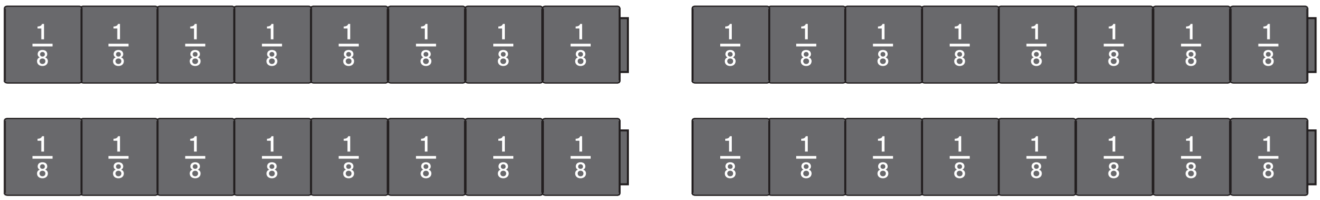
3. Say: *Our towers show $3 \div \frac{1}{4} = 12$. You also know that multiplication and division have an inverse relationship. You can use this to think of the problem as $12 \times \frac{1}{4} = 3$.* Have students write the number sentence, $12 \times \frac{1}{4} = 3$.

⚠ Look Out!

Watch for students who draw four lines on the towers instead of three and end up dividing the towers into fifths instead of fourths.

Use Fraction Towers to model the problem. Write the quotient. (Check students' work.)

1. Hugo has 4 sheets of green card stock he wants to cut into Earth Day bookmarks. How many bookmarks can he make if each one is $\frac{1}{8}$ of a sheet?



$4 \div \frac{1}{8} =$ 32

Using Fraction Towers, model the problem. Sketch the model. Write the equation that shows the solution. (Check students' models.)

2. How many $\frac{2}{3}$ foot pieces of fabric can be cut from 4 yards of fabric?

12 $\div$ $\frac{2}{3}$ $=$ 18

Divide.

3. $4 \div \frac{1}{5} =$ 20

4. $8 \div \frac{1}{4} =$ 32

5. $6 \div \frac{3}{8} =$ 16

6. $15 \div \frac{3}{10} =$ 50

7. $9 \div \frac{1}{3} =$ 27

8. $9 \div \frac{3}{8} =$ 24

Answer Key

Challenge! Create a story that requires the expression $8 \div \frac{2}{5}$, and solve the problem.

Challenge : $8 \div \frac{2}{5} = 20$; stories will vary.