

## PROBLEMS

1. **Problem 1**

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function satisfying the functional equation

$$f(x+y) = f(x) + f(y) \quad \text{for all } x, y \in \mathbb{R}.$$

Assume that  $f$  is continuous at  $x=0$ . Prove that  $f$  is linear, i.e., there exists a constant  $c \in \mathbb{R}$  such that  $f(x) = cx$  for all  $x \in \mathbb{R}$ .

2. **Problem 2**

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function satisfying the functional equation

$$f(x+y) = f(x)f(y) \quad \text{for all } x, y \in \mathbb{R}.$$

Assume that  $f$  is not identically zero and  $f(1) = 2$ . Find  $f(x)$  for all  $x \in \mathbb{R}$ .

3. **Problem 3**

Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function satisfying the functional equation

$$f(x+y) = f(x) + f(y) + xy \quad \text{for all } x, y \in \mathbb{R}.$$

Assume that  $f$  is continuous at  $x=0$ . Find  $f(x)$  for all  $x \in \mathbb{R}$ .