

LESSON 5

Objective

Investigate slope and relate linear equations to proportional relationships.

Common Core State Standards

- **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

Expressions and Equations

Lines Through $(0, 0)$ and $(0, b)$

Students know that corresponding angles in similar triangles are congruent. They also know that corresponding side lengths in similar triangles are proportional. Using similar triangles, students can visualize why the slope between any two points on a line is the same as the slope between any other two points on the same line. Students also can use this idea to see that a line described by $y = mx$ is the same as a line described by $y = mx + b$ for a given m , except for where it crosses the y -axis.

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- **Ask:** *How are the lines different? How are they the same?*
- **Ask:** *What do the similar triangles show us about the slope of a line? Say:* They show us that the slope is the same between any two points on the line.
- **Ask:** *Which line represents a proportional relationship? Say:* The line for Caitlyn is a proportional relationship because it includes the origin.
- **Ask:** *What kind of relationship does the line for JT express? Say:* It is called a linear relationship. A proportion is just a specific type of linear relationship.
- Have students compare and contrast the equations they wrote— $y = \frac{3}{5}x$ and $y = \frac{3}{5}x + 6$. Elicit that the only difference is JT's free points.

Solve It

Reread the problem with the students. Have each student graph the lines on dot paper. Have them determine the unit rate for Caitlyn's line and the slope for JT's line. Ask students to write equations for the lines.

More Ideas

For another way to teach about slope and relating linear equations to proportional relationships—

- Use masking tape to make a coordinate grid on the floor. Have students stand in the locations of the points in the problem. Use string to graph the lines. Make similar triangles with string to demonstrate the slopes of the lines and find equations for the lines.

Formative Assessment

Have students try the following problem.

Lenny graphs the number of books he reads versus the time it takes him in weeks. He plots two points: $(3, 6)$ and $(6, 12)$. What is the equation of the line that passes through these two points?

Try It! 25 minutes | Groups of 3

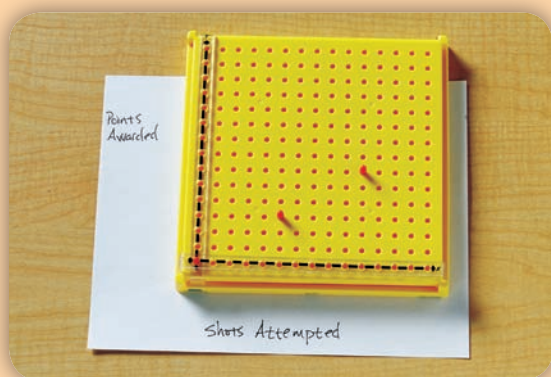
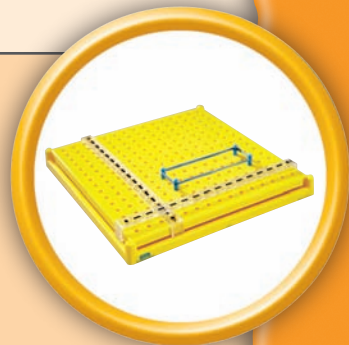
Here is a problem about slope and relating linear equations to proportional relationships.

Caitlyn and her younger sister JT are shooting a basketball to see who can earn more points in 10 shots. One point will be awarded for each shot made, and Caitlyn has spotted JT 6 free points. Caitlyn makes 3 shots in 5 attempts and 6 shots in 10 attempts. JT makes 3 shots in 5 attempts and 6 shots in 10 attempts. Make a graph of points awarded versus shots attempted for the girls. Write the equation of the line that represents each girl's results.

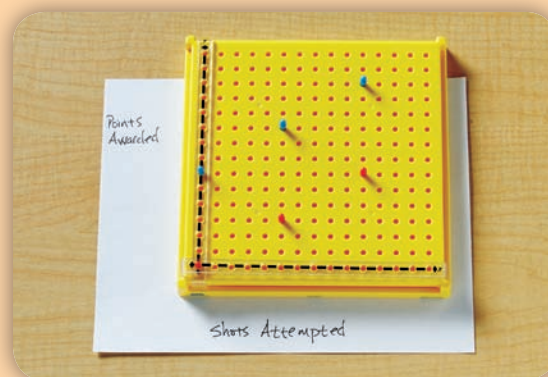
Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

Materials

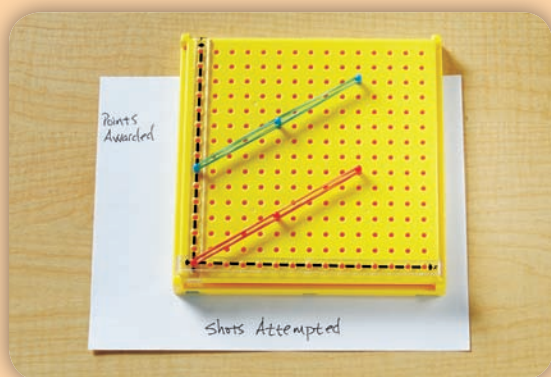
- XY Coordinate Pegboard
- Dot Paper (BLM 7; 1 per student)
- straightedge



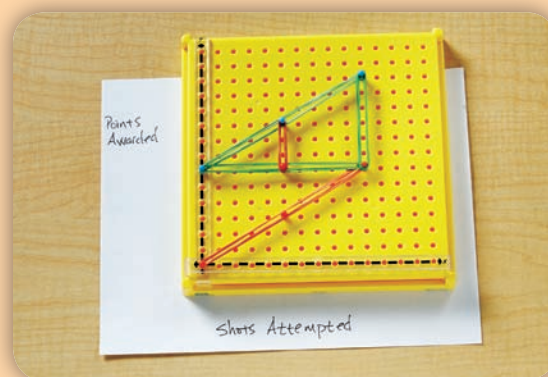
1. Have students prepare a pegboard for graphing in the first quadrant. Discuss what each axis will represent. **Ask:** *What points do we plot for Caitlyn?* Elicit from students that they should plot the points given in the problem—(5, 3) and (10, 6)—plus (0, 0), because Caitlyn did not have any points at the start.



2. **Ask:** *What do we plot for JT?* Elicit that the first point for JT is (0, 6). Elicit further that JT earned 3 more points in 5 shot attempts and 6 more points in 10 attempts, so the second point for JT is (5, 9) and the third point is (10, 12). Have students plot the points.



3. Have students stretch a rubber band around the pegs for each girl. Elicit that for each girl the pegs lie on a line. **Ask:** *Does either line represent a proportion? Which one?* **Say:** *Find the unit rate and write an equation for y versus x.*

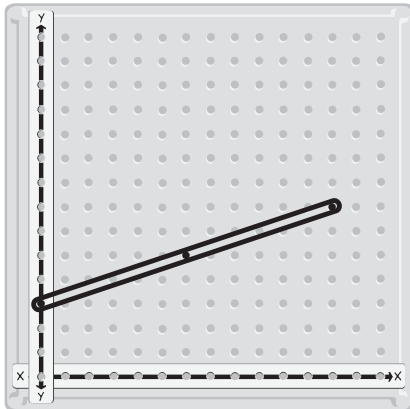


4. Have students add a peg at (5, 6) and make two triangles—one using (0, 6), (10, 6), and (10, 12), the other using (0, 6), (5, 6), and (5, 9). Have students see the triangles are similar, find the ratio of height to base length for each, and use the ratio to write an equation of the line for JT.

Use an XY Coordinate Pegboard. Build the model and use it to answer the questions.

(Check students' work.)

1.



Louis is collecting silver dollars. In 6 months, he had 5 silver dollars in his collection. In 12 months, he had 7 silver dollars. Make a graph of dollars versus months. Assume the relationship is linear.

Does the line go through the origin? no

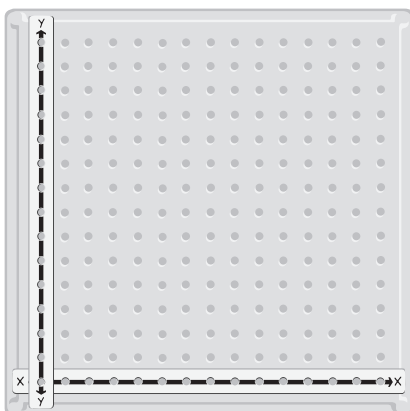
How many silver dollars did Louis start with? 3

Write the equation of the line. $y = \frac{1}{3}x + 3$

Using an XY Coordinate Pegboard, model the problem. Draw the model and use it to answer the questions.

(Check students' models.)

2.



Jo paid \$7 for 6 pounds of apples. She paid \$14 for 12 pounds of apples. Make a graph of cost versus pounds. Assume the relationship is linear.

Does the line go through the origin? yes

Is the relationship a proportion? yes

Write the equation of the line. $y = \frac{7}{6}x$

Use dot paper to graph the relationship. Answer the questions.

3. At 8:00 a.m. on a cold Saturday, Juan decided to track the temperature on his patio. At 11:00 a.m., the temperature was 8°F. At 2:00 p.m., the temperature was 14°F. Make a graph of temperature versus elapsed time. Assume the relationship is linear.

Is the relationship a proportion? no

What was the temperature at 8:00 a.m.? 2°F

Write the equation of the line. $y = 2x + 2$

Answer Key

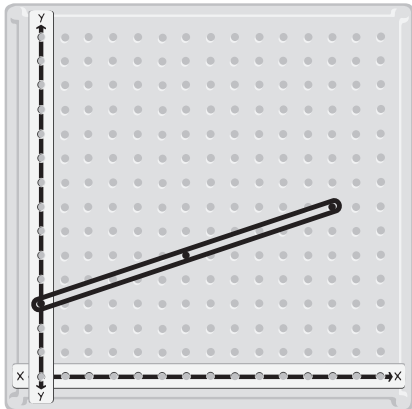
Challenge! How do you know whether the equation of a line will be of the form $y = mx$ or $y = mx + b$? What is b ?

Challenge: (Sample) If the line goes through the origin, it will be $y = mx$. If the line does not go through the origin, it will be $y = mx + b$. The constant b is where the line crosses the y -axis or how far up it is translated from $y = mx$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

Use an XY Coordinate Pegboard. Build the model and use it to answer the questions.

1.



Louis is collecting silver dollars. In 6 months, he had 5 silver dollars in his collection. In 12 months, he had 7 silver dollars. Make a graph of dollars versus months. Assume the relationship is linear.

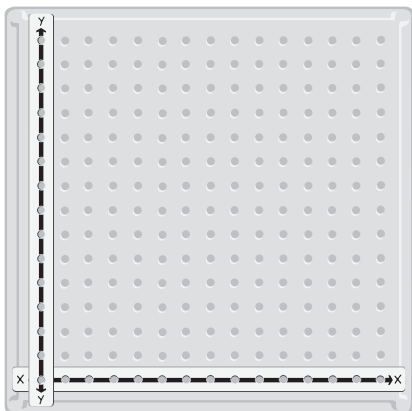
Does the line go through the origin? _____

How many silver dollars did Louis start with? _____

Write the equation of the line. _____

Using an XY Coordinate Pegboard, model the problem. Draw the model and use it to answer the questions.

2.



Jo paid \$7 for 6 pounds of apples. She paid \$14 for 12 pounds of apples. Make a graph of cost versus pounds. Assume the relationship is linear.

Does the line go through the origin? _____

Is the relationship a proportion? _____

Write the equation of the line. _____

Use dot paper to graph the relationship. Answer the questions.

3. At 8:00 a.m. on a cold Saturday, Juan decided to track the temperature on his patio. At 11:00 a.m., the temperature was 8°F. At 2:00 p.m., the temperature was 14°F. Make a graph of temperature versus elapsed time. Assume the relationship is linear.

Is the relationship a proportion? _____

What was the temperature at 8:00 a.m.? _____

Write the equation of the line. _____

Name _____

Challenge! How do you know whether the equation of a line will be of the form $y = mx$ or $y = mx + b$? What is b ?

[illegible]

Name _____

