

PROBLEMS

Problem 1 (10 points)

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation

$$f(x+y) = f(x) + f(y) \quad \text{for all } x, y \in \mathbb{R}.$$

Suppose that f is continuous at $x=0$. Prove that f is linear, i.e., there exists a constant $c \in \mathbb{R}$ such that $f(x) = cx$ for all $x \in \mathbb{R}$.

Problem 2 (10 points)

Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.$$

Problem 3 (10 points)

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying the functional equation

$$f(x+y) = f(x)f(y) \quad \text{for all } x, y \in \mathbb{R}.$$