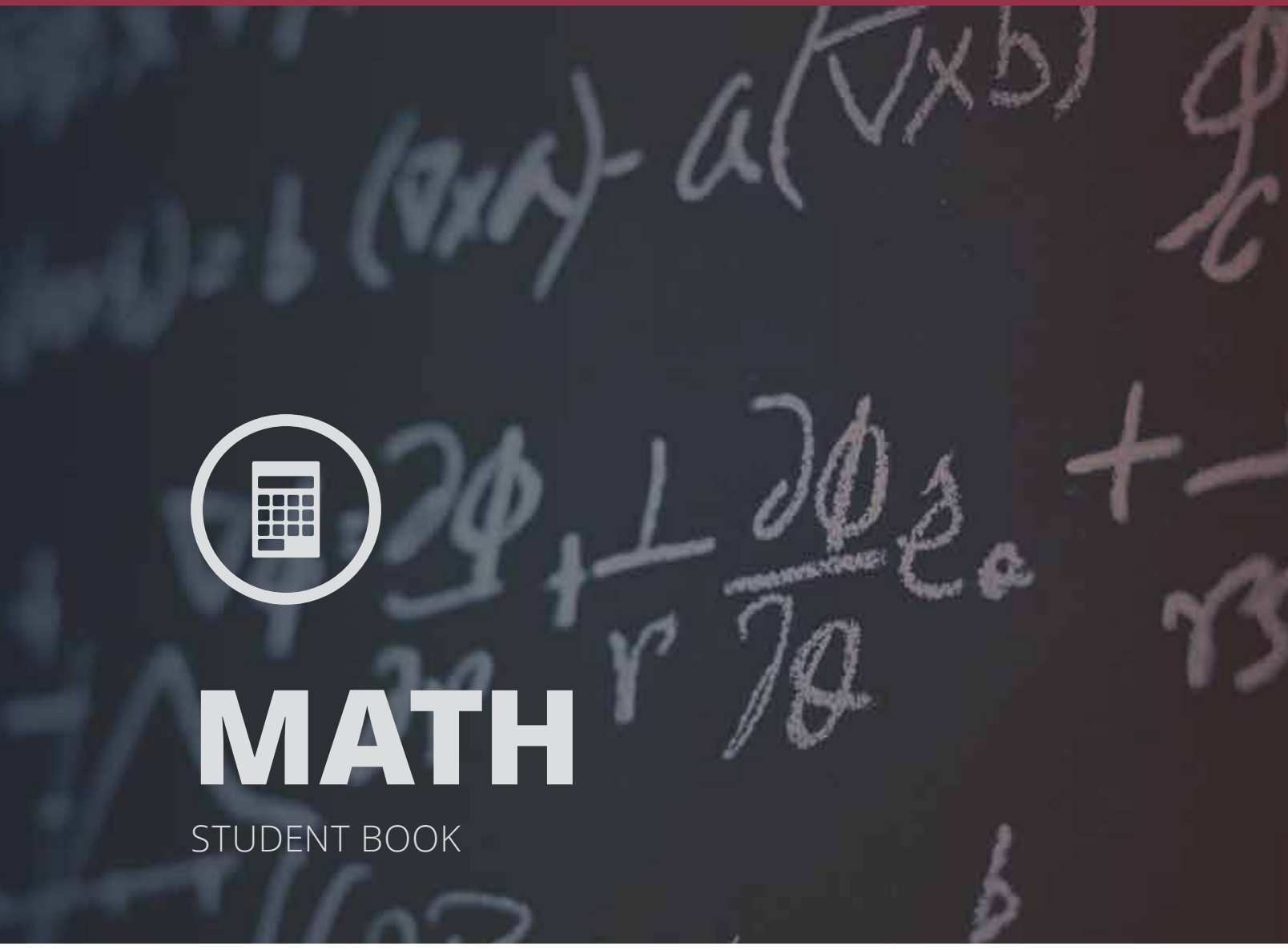
LIFEPAC®



MATH

STUDENT BOOK

7th Grade | Unit 1

Alpha Omega
PUBLICATIONS

Math 701
Integers

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Integers

Introduction

Mathematics 700 is designed to prepare junior-high students for Pre-algebra. This course focuses on strengthening needed skills in problem solving, number sense, and proportional reasoning. It also introduces students to integers, equations, and geometric concepts. Students will begin to see the "big picture" of mathematics and learn how numeric, algebraic, and geometric concepts are woven together to build a foundation for higher mathematical thinking.

By the end of the course, students will be expected to do the following:

- Gain an increased awareness of how math is a life skill.
- Understand how math gives us different ways to model or express the same thing.
- Explore concepts taught in previous math courses, but at higher levels, applying the concepts to real world situations.
- Use proportional reasoning in order to model and solve real world problems.
- Utilize new skills and concepts that will help them in future math courses.

In this unit, the student will be formally introduced to the set of integers. The number line will be used as a tool for students to locate and order integers, as well as find the absolute value of a number. It will also be used as a tool for adding, subtracting, multiplying, and dividing with integers. In addition, the real number properties, exponents, and the order of operations will be addressed and applied to integers.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Locate integers on the number line.
- Compare and order integers.
- Determine absolute value.
- Add, subtract, multiply, and divide integers.
- Use integers to solve word problems.
- Define the real number system and its properties.
- Use exponents.
- Use the order of operations to simplify expressions.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. Integers

Integers on the Number Line

Objectives

- Represent positive and negative values.
- Locate integers on the number line.

VOCABULARY

infinite—increasing or decreasing without end

integer—a number belonging to the set made up of the whole numbers and their opposites

natural number—a number belonging to the set made up of the counting numbers: 1, 2, 3, and so on

negative number—a number that is less than zero

number line—a line that graphically represents all numbers

point—a dot that marks a location on a graph

positive number—a number that is greater than zero

whole number—a number belonging to the set made up of zero and the natural numbers

What did Carlton mean when he said that Ondi has "less than no money"? Well, not only does she have zero dollars, but she actually owes a dollar. As soon as she earns a dollar, she'll have to pay it back to Carlton. So right now, Ondi actually has less than zero dollars

Connections! Can you think of any other situations in which you can have a value that is less than zero? Think about temperature. In many places in the world, the temperature gets below zero, or less than zero!

Special signs are used to show whether a number is positive or negative. Up until now, the numbers you have worked with had no sign in front of them. That means that they were *positive numbers*. Positive numbers can either have no sign or a positive sign (+) in front of the digit. But *negative numbers* must have a negative sign (-) in front of the digit. For example, +3 or 3 can be used to represent "positive three." "Negative three" is represented as -3.

Words can also be used to show if a number is positive or negative. Phrases like "above zero," "more than zero," or "greater than zero" indicate positive numbers. Phrases such as "below zero" and "less than zero" indicate negative numbers.

Think about it! What about the number zero? Is it positive or negative? Actually, it's neither. Zero is the only number that doesn't have a sign.

Example:

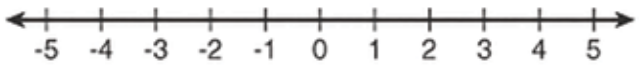
- What are some ways to represent the number 4?

Solution:

- A digit with no sign represents a positive number. So 4 can also be expressed as +4, positive four, four above zero, four more than zero, or four greater than zero.

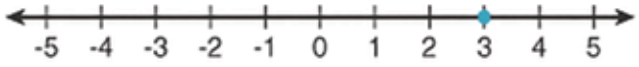
The different groups of positive and negative numbers have special names. The counting numbers, like 1, 2, 3, and so on, are called *natural numbers*. The *whole numbers* are exactly the same as the natural numbers except that the group also includes zero. You have probably worked a lot with these sets of numbers. Now you'll also be working with the *integers*. The integers include both the positive counting numbers, the negative numbers, and zero. So numbers like -8, 4, 0, and -2 are all considered integers.

Integers can be represented visually on a *number line*. A number line is just a graph that is used to represent numbers. Here's an example:



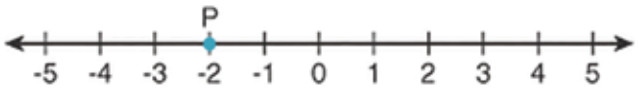
The arrows on each end of the number line mean that this line continues forever in both directions. That's because the set of integers continues forever. It is *infinite*. Also, notice that numbers to the right of zero are positive and numbers to the left of zero are negative.

Right now, the number line above is empty. No specific integers have been graphed. To graph a specific integer, use a point. A point is a dot on the line that represents the location of a specific number. For example, on the following number line, the point shown represents the number three.



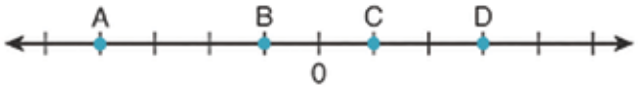
Key point! A number line with no points is like an empty game board. You place a game piece on the board to represent your specific location on the board. On a number line, you place a point to represent the location of a specific number.

Points on the number line are often labeled using a single letter. For example, in the following number line, point P is located at -2.



Example:

- Which point represents the number -1? (Note: Each tick represents 1.)



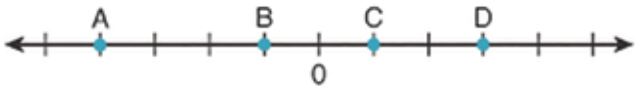
Solution:

- Negative numbers are to the left of zero. Negative one is one place to the left of zero, so B represents -1.

Look at one more example.

Example:

- Which point is represented by the point D? (Note: Each tick represents 1.)



Solution:

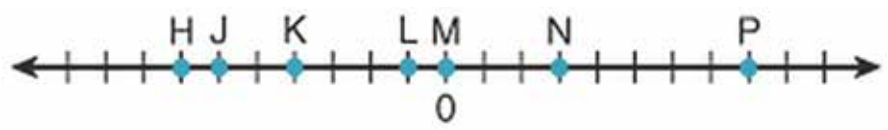
- Positive numbers are to the right of zero. Point D is three places to the right of zero, so 3 is represented by point D.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Positive numbers are greater than zero. Negative numbers are less than zero.
- The set of integers includes the positive counting numbers, the negative numbers, and zero.
- Integers can be graphed as points on a number line.

Use the number line to answer the following questions. Each tick represents 1.



1.1 Point H is located at -6.
- ○ True
- ○ False

1.2 Point N is three more than zero.
- ○ True
- ○ False

1.3 Point K is four less than zero.
- ○ True
- ○ False

1.4 Point L is located at 1.
- ○ True
- ○ False

1.5 Points H, J, K, L, and M are negative.
- ○ True
- ○ False

1.6 Points N and P are positive.
- ○ True
- ○ False

1.7 Which of the following *cannot* be used to express the number 8?
- ☐ eight more than zero
- ☐ eight below zero
- ☐ +8
- ☐ positive eight

1.8 Which of the following *cannot* be used to express the number -6?
- ☐ six below zero
- ☐ negative six
- ☐ six less than zero
- ☐ six above zero

1.9 Which of the following can be used to describe the location that is 9 units to the right of zero on the number line?
- ☐ negative nine
- ☐ nine less than zero
- ☐ positive nine

1.10 How would you graph a point at -5?
- ☐ Put a point 5 units to the right of zero.
- ☐ Put a point 5 units to the left of zero.
- ☐ Put a point at zero.

Complete the following activities.

1.11 Draw a number line and use point N to represent the number 2.

1.12 Draw a number line and use point P to represent the number -1.

Use the game board below like a number line to answer the following 3 questions.
The pawn represents zero on a number line. Each space represents 1.

1.13 Which space corresponds with -2 on the number line? _____

1.14 Which space corresponds with 3 on the number line? _____

1.15 What value is represented by space A on the game board? _____

Comparing and Ordering Integers

Suppose you're on a game show where you have to answer questions to win points, and the questions get harder as you go. The current category is "Name the Larger Number." The first couple questions are really easy. Check them out:

Question: Which number is larger: 13 or 7?

Answer: 13

Question: Which number is larger: 8 or 0?

Answer: 8

Now it's on to the third question. Do you know the answer?

Question: Which number is larger: 4 or -7?

Answer: ?

In this lesson, you'll be comparing integers and using special symbols to show how two numbers are related to each other.

Objectives

- Compare two integers using inequality symbols.
- Put a group of integers in order.

VOCABULARY

inequality—statement showing a relationship between numbers that are not necessarily equal; uses the symbols >, <, ≥, ≤, or ≠

How do you know which number is larger when one of them is a negative number? You can use money to help you figure it out. In terms of money, a positive number means you have that much money. A negative number means you owe that much money. So a positive 4 means that you have four dollars. A -7 means that you owe seven dollars. Which is greater: having $4 or owing $7? Of course having any money at all is always going to be more than owing money! Any positive number is larger than any negative number, no matter what the numerals are. So the answer to the last question is 4 because 4 is larger than -7.

What if the questions continue to get harder? How about comparing two negative numbers? Look at the following question.

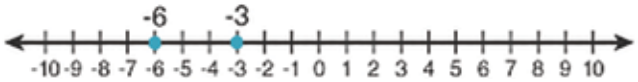
Question: Which number is larger: -6 or -3?

Answer: ?

Are you stumped by this one? Use the number line, rather than money, to help you this time.

First, look at the positive numbers. What happens as you move to the right on the number line? The positive numbers get larger, right? What happens as you move to the left? The positive numbers get smaller. The same is true for negative numbers. As you move to the right on the number line, the negative numbers get larger. As you move to the left, the negative numbers get smaller. That means that if you're comparing two numbers, the number that is farther to the right on the number line is the larger number.

Now go back to the previous question. Which number is farther to the right on the number line: -6 or -3? Negative three is, so it's the larger number.



It may seem odd that -3 is larger than -6. You've always been taught that larger numerals have a larger value. That's true, but only with positive numbers. Negative numbers are the opposite of positive numbers. With a negative number, the larger the numeral after the negative sign, the smaller the value. Think of it in terms of money again. The more money you owe (or the larger the numeral), the less money you have in terms of value.

Key point! The number line can be used to compare numbers. Numbers to the right are larger than numbers to the left.

Before you try an example, review what you've found so far:

- Positive numbers are larger than negative numbers.
- With negative numbers, the larger numeral has the smaller value.
- With positive numbers, the larger numeral has the larger value.
- Numbers to the right on the number line are larger than the numbers to the left.

What about zero? Where does it fit in? By definition, negative numbers are less than zero, and positive numbers are greater than zero.

Remember: Zero is greater than negative numbers and less than positive numbers.

Example:

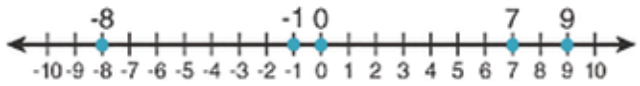
- Put the following numbers in order from smallest to largest.
- 7, -1, 0, -8, 9

Solution:

- This group of numbers has both negative and positive numbers. Using the rules, your list will start with the negative numbers, followed by zero, and then end with the positive numbers. Look at the negative numbers first. According to the rule, the larger numeral has the smaller value. So -8 is smaller than -1. The number 0 comes next in your list, followed by the positive numbers. With positive numbers, the larger numeral has the larger value. So 9 is larger than 7.

- The correct order is -8, -1, 0, 7, 9.

How can you check your answer to see if it's correct? Use the number line! Your list is in the correct order if the numbers move from left to right on the number line.



Besides making a list, there is another way you can show the relationship between numbers. An *inequality* is a math sentence that uses special symbols to show how two numbers are related. In this lesson, you'll use just two of those symbols—the "greater than" symbol (>) and the "less than" symbol (<). The rule to help you know which symbol to use is that the opening of the symbol always faces the larger number.

For example, earlier you compared the numbers -3 and -6 and found that -3 is larger than -6. There are two sentences you can create to show this relationship:

- -3 > -6 — "Negative three is greater than negative six."
- -6 < -3 — "Negative six is less than negative three."

Notice that in each sentence the opening of the symbol faces -3 because it is the larger number. Take a look at a few more examples.

Example:

- Complete the sentence with the correct inequality symbol.
- -5 ___ 0

Solution:

- -5 < 0

Example:

- Complete the sentence with the correct inequality symbol.
- -23 ___ -26

Solution:

- -23 > -26

Example:

- Complete the sentence with the correct inequality symbol.
- 18 ___ 16

Solution:

- 18 > 16

Example:

- Complete the sentence with the correct inequality symbol.
- 4 ___ -7

Solution:

- 4 > -7

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- All positive numbers are larger than negative numbers.
- On the number line, numbers to the right are larger than numbers to the left.
- Negative numbers are the opposite of positive numbers. With negative numbers, the larger numeral has the smaller value.

Complete the following activities.

1.16 -7 > -10
- ○ True
- ○ False

1.17 -9 < 9
- ○ True
- ○ False

1.18 -3 > 0
- ○ True
- ○ False

1.19 -4 > -1
- ○ True
- ○ False

1.20 -5 < -8
- ○ True
- ○ False

1.21 0 < 5
- ○ True
- ○ False

1.22 Match each number with its place in order from smallest (1st) to largest (6th).

_____ 1st
_____ 2nd
_____ 3rd
_____ 4th
_____ 5th
_____ 6th

-56
-80
59
-84
48
90

1.23 Which of the following lists is in the correct order from smallest to largest?
- ☐ -13, -19, -21, -24
- ☐ -7, -3, 6, 2
- ☐ 11, -12, 15, -19
- ☐ -19, -14, 5, 11

1.24 Which of the following lists is in the correct order from largest to smallest?
- ☐ 82, 80, 13, -2
- ☐ 35, 13, -7, -1
- ☐ -6, -3, 1, 2
- ☐ 18, 21, -8, -11

1.25 Which of the following sentences is not true?
- ☐ The symbol > means “greater than.”
- ☐ A negative number with a larger numeral is a larger number.
- ☐ Smaller numbers are farther left on the number line.
- ☐ All negative numbers are less than zero.

Complete the sentence with the correct inequality symbol.

1.26 0 ___ -8

1.27 -3 ___ -10

1.28 -2 ___ 1

Put each set of numbers in order from smallest to largest.

1.29 0, 1, -2, 3, -4, 5, -6, 7

1.30 -2, 5, 6, -8, 10, -11, -12, 15

Absolute Value

In this lesson, you'll learn how numbers can be opposite, too.

Objectives

- Find pairs of opposite numbers.
- Determine the absolute value of a number.

VOCABULARY

absolute value—the distance from zero on the number line

inequality—statement showing a relationship between numbers that are not necessarily equal; uses the symbols >, <, ≥, ≤, or ≠

opposite numbers—two numbers that are the same distance from zero on the number line but in opposite directions

Opposite Numbers and Absolute Value

You often match pairs of opposite words. For example, the opposite of *hot* is *cold*. And the opposite of *big* is *small*. Can you think of some opposites in math? What's the opposite of greater than? That's right—less than! What's the opposite of moving to the right on the number line? Moving to the left. What's the opposite of positive? Negative!

All of these math opposites have to do with *opposite numbers*. Opposite numbers are two numbers that lie the same distance from zero on the number line but in opposite directions. Look at the number line to find some pairs of opposite numbers.

What opposite numbers are two units from zero? They are 2 and -2. What opposite numbers are ten units from zero? They are 10 and -10. Do you see a pattern? Opposite numbers have the same numeral but different signs! One number is positive, and one number is negative.

Remember! Opposite numbers have opposite signs.

You know that opposite numbers lie the same distance from zero. That distance actually has a special name—*absolute value*. The absolute value of a number is its distance from zero on the number line. Look at the number line again to find the absolute value of some numbers.

The absolute value of +6 is 6, and the absolute value of -3 is 3. The absolute value of +2 is 2, and the absolute value of -2 is 2. What do you notice? No matter what the sign of the number is, the absolute value of the number is positive! That's because absolute value represents a distance, and distance cannot be negative.

Key point! The absolute value of a number is either zero or positive.

What else do you notice about the last two numbers: 2 and -2? This is a pair of opposite numbers. What is true about their absolute values? They're the same! This is true for every pair of opposite numbers.

Remember: Opposite numbers have the same absolute value.

Special symbols are used to represent absolute value. Two vertical bars surround the number that you are to find the absolute value of. For example, the absolute value of -7 can be represented as |-7|, and the absolute value of 3 can be represented as |3|. Once you've found the absolute value of the number, you can remove the vertical bars. Look at some examples.

Example:

- Find |-7|.

Solution:

- The number -7 is 7 units from 0, so |-7| = 7.

Example:

- Find |3|.

Solution:

- The number 3 is 3 units from 0, so |3| is 3.

Example:

- Find |0|.

Solution:

- The number 0 is at 0 on the number line, so |0| is 0.

Example:

- What two numbers have an absolute value equal to 4?

Solution:

- |-4| = 4 and |4| = 4

Complete the following activities.

1.31 The _____ of 2 is -2.

☐ absolute value ☐ sign

☐ opposite

1.32 The _____ of 2 is 2.

☐ absolute value ☐ sign

☐ opposite

1.33 The absolute value of a negative number is always _____.

☐ Negative ☐ Positive ☐ zero

1.34 The opposite of a positive number is always _____.

☐ Negative ☐ Positive ☐ zero

1.35 Match each number to its opposite.

_____ 11	8
_____ -8	-11
_____ 31	1
_____ -1	-31
_____ -17	17

1.36 Find |-14|.

☐ 14 ☐ -14

1.37 Find |27|.

☐ 27 ☐ -27

1.38 Find |-54|.

☐ 54 ☐ -54

1.39 What is the opposite of -12?

☐ 12 ☐ -12

1.40 What is the opposite of 80?

☐ 80 ☐ -80

Inequalities and Absolute Value

Recall that *inequality* symbols can be used to show how two numbers are related. The > symbol represents "greater than," and the < symbol represents "less than." There are more symbols that can be used as well.

A line under the > and < symbols means "or equal to." For example, ≥ means "greater than or equal to," and ≤ means "less than or equal to." Suppose you were comparing the two numbers -5 and 5. Since -5 is less than 5, you can write either of the following two inequalities:

- $-5 < 5$
- $5 > -5$

Keep in mind! The opening of the inequality symbol should always face the larger number. In this case, the opening must face 5.

Now you can write two more inequality statements using the two new symbols:

- $-5 \leq 5$— "Negative five is less than or equal to five."
- $5 \geq -5$— "Five is greater than or equal to negative five."

Because of the word or in the symbol meaning, only one part of the symbol has to be true in order for the entire statement to be true. For example, in the first statement above, -5 is less than 5 but not equal to 5. However, since one part of the statement is true, the entire statement is true. Look at a few examples.

Example:

- Determine if the following statement is true.
- $|-5| \geq 5$

Solution:

- This statement reads, "The absolute value of -5 is greater than or equal to 5." The expression $|-5|$ tells you that you need to find the absolute value of -5. The number -5 lies 5 units from zero, so $|-5| = 5$. The statement can now be rewritten as $5 \geq 5$. You know that 5 equals 5, so the entire statement is true.

Be Careful! In this example, the statement is true because 5 equals 5. If the symbol had not included the "or equal to" part, this statement would have been false. That's because 5 is not greater than 5. No number can be greater than or less than itself. It can only equal itself.

Example:

- Determine if the following statement is true.
- $|-7| \leq -9$

Solution:

- This statement reads, "The absolute value of -7 is less than or equal to -9." The absolute value of -7 is 7, so this statement can be rewritten as $7 \leq -9$. Since any negative number is less than any positive number, -9 is less than 7, not the other way around. And you know that 7 and -9 are not equal to each other. Since neither part of the statement is true, the entire statement is false.

Notice that in each example, the first step is to find the absolute value of any number that is surrounded by the absolute value symbols. Then check to see if the inequality is true or false.

There is one final sign that can be used in an inequality: ≠. The sign ≠ means "is not equal to." Here is an example that uses this sign.

Example:

- |-7| ≠ -7

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson:

- Opposite numbers have opposite signs.
- Opposite numbers have the same absolute value.
- The absolute value of both positive and negative numbers is always positive.
- A line under the "less than" or "greater than" signs indicates "or equal to."
- Only one part of the "or" statement in the inequality has to be true for the entire statement to be true.

Complete the following activities.

1.41 |1| = 1
- ○ True
- ○ False

1.42 |1| > -1
- ○ True
- ○ False

1.43 |-3| ≤ 0
- ○ True
- ○ False

1.44 |-2| ≥ |-3|
- ○ True
- ○ False

1.45 |-4| < |4|
- ○ True
- ○ False

1.46 Match each number with its place in order from smallest (1st) to largest (5th).

_____ 1st	|9|
_____ 2nd	11
_____ 3rd	1
_____ 4th	-2
_____ 5th	|-4|

Complete each number sentence with ≤ or ≥.

1.47 $|-6|$ ___ 3

1.48 $|-2|$ ___ $|-5|$

1.49 $|-8|$ ___ -1

1.50 $|-5|$ ___ -5

1.51 $|4|$ ___ $|-9|$

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

Self Test 1: Integers

Complete the following activities (4 points, each numbered activity).

1.01 Select all that apply. Which of the following phrases can be used to represent 7?

- ☐ the opposite of -7
- ☐ the absolute value of 7
- ☐ seven less than zero
- ☐ seven above zero
- ☐ the absolute value of -7
- ☐ positive seven

1.02 Which of the following phrases can be used to represent -11?

- ☐ positive eleven
- ☐ the opposite of -11
- ☐ eleven greater than zero
- ☐ eleven below zero

1.03 Select all that apply. Which symbols could be used to make the following statement true? -15 ___ -19

- ☐ >
- ☐ ≥
- ☐ =
- ☐ <
- ☐ ≤
- ☐ ≠

1.04 Select all that apply. Which symbols could be used to make the following statement true? |-8| ___ |8|

- ☐ >
- ☐ ≥
- ☐ =
- ☐ <
- ☐ ≤
- ☐ ≠

1.05 The absolute value of 9 is 9.

- ○ True
- ○ False

1.06 Positive numbers are located to the left of zero on the number line.

- ○ True
- ○ False

1.07 If a number is located farther right on the number line than another, then it is larger.

- ○ True
- ○ False

1.08 0 > -8

- ○ True
- ○ False

1.09 The opposite of 1 is -1.

- ○ True
- ○ False

1.010 -4 ≥ 1

- ○ True
- ○ False

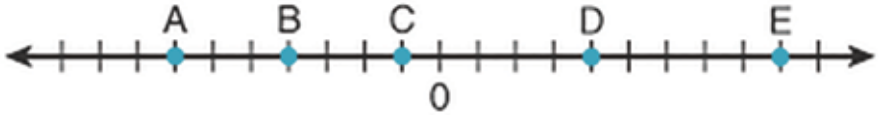
1.011 Negative numbers do not need the negative sign (-) in front of them.

- ○ True
- ○ False

1.012 Which of the following is in order from smallest to largest?

- ☐ -1, -11, 3, 9, 12
- ☐ -11, -1, 3, 9, 12
- ☐ -1, 3, 9, -11, -12
- ☐ 12, 9, 3, -11, -1

Use the number line to answer the questions. Each tick represents 1.



1.013 Which point is located at 4?

- ☐ B
- ☐ A
- ☐ D
- ☐ none of these

1.014 Which point is located at -4?

- ☐ B
- ☐ A
- ☐ D
- ☐ none of these

1.015 Find |-65|.

- ☐ 65
- ☐ -65
- ☐ 0

1.016 Which of the following is in order from largest to smallest?

- ☐ |-13|, 0, 4, |5|
- ☐ 0, 4, |5|, |-13|
- ☐ |5|, 4, 0, |-13|
- ☐ |-13|, |5|, 4, 0

1.017 Explain how you would graph point P at -3 on a number line.

Complete each number sentence with ≤ or ≥.

1.018 $|-3|$ ___ $|-10|$

1.020 $|5|$ ___ 1

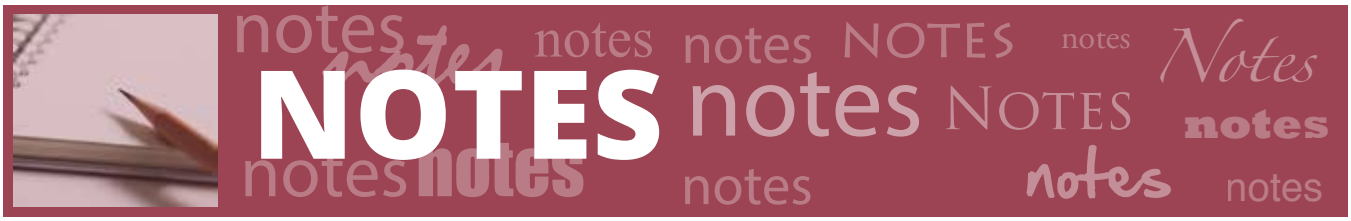
1.019 $|4|$ ___ $|2|$

Put each set of numbers in order from smallest to largest.

1.021 -4, 5, 6, $|-7|$, -8, $|9|$, 10, -11, -12, 13

1.022 $|-2|$, 3, -4, -6, $|10|$, -11, $|-14|$, 18

70/88 **SCORE** _____ **TEACHER** _____ _____
initials date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