

Are You Ready?

Precalculus, by R. Rusczyk

If you've mastered using the Pythagorean Theorem, solving basic equations and systems of linear equations, the basics of logarithms and polynomials, graphing simple functions on the Cartesian coordinate plane, determining angles and arcs in circles, and the arithmetic of complex numbers then you are ready for the Art of Problem Solving textbook, **Precalculus**. (Answers to these problems are on the following page.)

1. Pythagorean Theorem

- (a) Find the hypotenuse of a right triangle with legs of length 10 and 24.
- (b) Find the hypotenuse of a right triangle with legs of length 14 and 17.
- (c) Find the length of the shorter leg of a right triangle with hypotenuse of length 82 and longer leg of length 80.
- (d) If the hypotenuse of a right triangle has length 60 and one leg has length 45, find the length of the other leg.
- 2. Basics of Complex Numbers Find the value of each of the following
 - (a) The sum of 3 + 4i and -9 + 2i.
 - **(b)** (4+2i)(7-6i)
 - (c) $(\sqrt{3}+i)^3$
 - (d) |3+4i|

3. Algebra

- (a) Find all values of x that satisfy $2x^2 4x + 1 = 0$.
- (b) Solve for *a* and *b* where 3a 2b = -5 and -2a + 3b = 15.
- (c) Find the value of 10x + 3y where 2x + 5y = 13 and 6x 7y = 11.
- (d) Find all values of x such that $\sqrt{x+3} + \frac{3}{\sqrt{x+3}} = 4$.

4. Coordinates and Graphing

- (a) Point A is located at (3,5) on the Cartesian plane. What is the result of reflecting A across the x-axis?
- (b) Find the points of intersection of the graphs of $y = 3x^2 + x 7$ and $y = 2x^2 + 6x 13$.
- (c) What is the equation of a circle centered at (2,2) with radius 5?
- (d) In a coordinate plane, a circle with center (5,2) passes through the point (-7,7). What is the circumference of the circle?
- 5. Sequences and Series Evaluate each of the following.

(a)
$$\sum_{k=1}^{10} 6k - 2.$$

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6. Circles

- (a) What is the measure in degrees of an angle inscribed in a circle that subtends an arc of 80°?
- (b) Points *A*, *B*, and *C* lie on a circle in that order so that the measure of arc $AB = 110^{\circ}$ and the measure of arc $BC = 130^{\circ}$. What is the measure in degrees of $\angle ABC$?
- (c) What is the measure of the angle formed by the minute and hour hands of a clock at 1:30?

7. Functions

- (a) Find f(6) if $f(x) = 5 2x^2$.
- (b) Find the range of f(x) = 6 + 2|x|.
- (c) Find the domain of $f(x) = 3 \sqrt{2 x^2}$.

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The answers to Are You Ready for **Precalculus** are below. (The answers to problem sets and challenges given in the class will include full detailed solutions as opposed to the mere answers provided below.)

- 1. (a) 26 (b) $\sqrt{485}$ (c) 18 (d) $15\sqrt{7}$ 2. (a) -6+6i**(b)** 40 - 10*i* (c) 8*i* (d) 5 3. (a) $1 \pm \frac{\sqrt{2}}{2}$ (b) a = 3 and b = 7(c) 37 (d) x = -2 or 64. (a) (3, -5)(b) (2,7) and (3,23)(c) $(x-2)^2 + (y-2)^2 = 25$ **(d)** 26π 5. (a) 310 (b) $\frac{1}{780}$ **6**. (a) 40° **(b)** 60° (c) 135° 7. (a) −67
 - (b) All real numbers greater than or equal to 6. (Alternatively, $[6, +\infty)$.)
 - (c) All real numbers from $-\sqrt{2}$ to $\sqrt{2}$. (Alternatively, $[-\sqrt{2},\sqrt{2}]$.)

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