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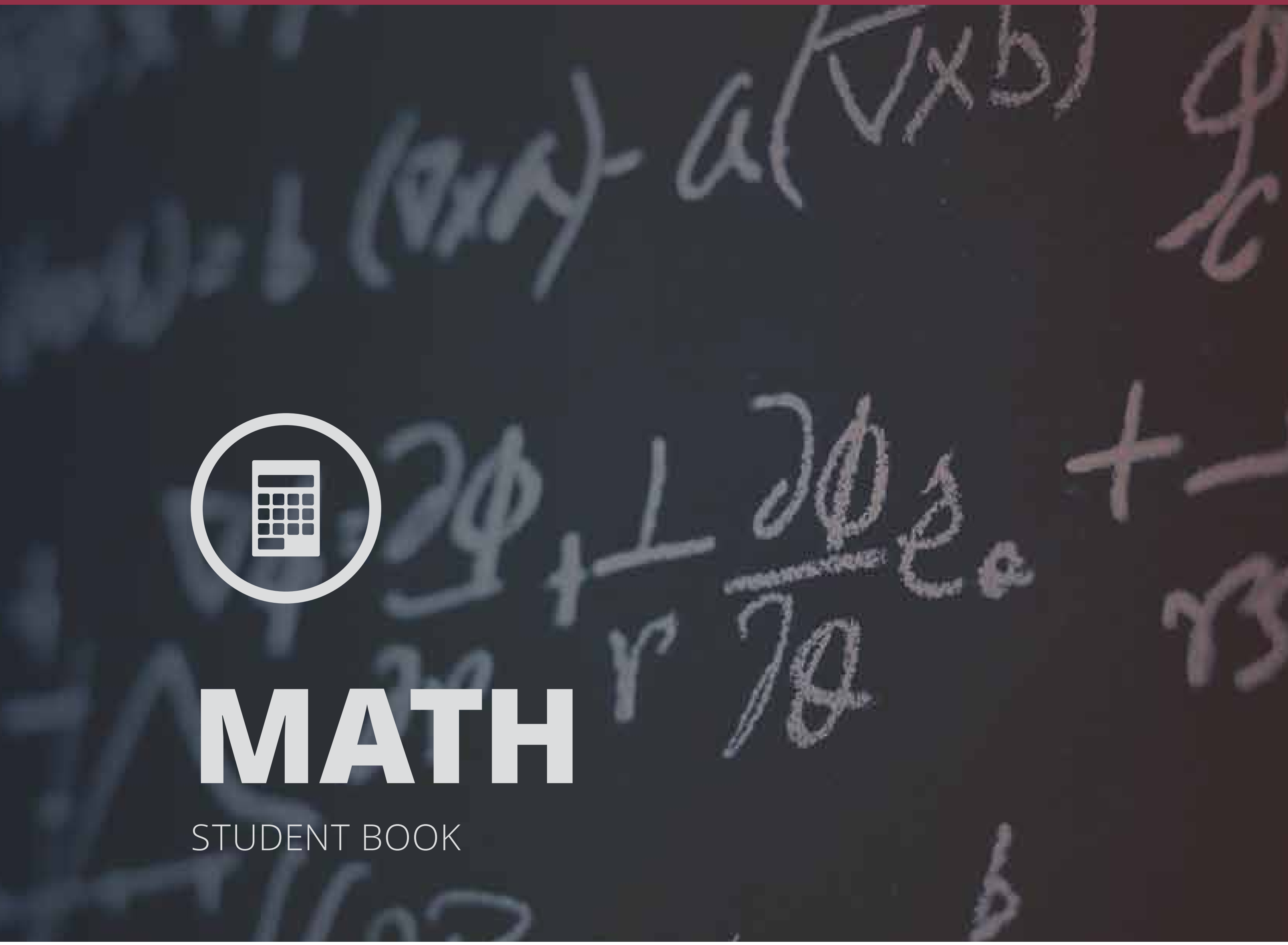

MATH

STUDENT BOOK

8th Grade | Unit 1

Alpha Omega PUBLICATIONS

Math 801
The Real Number System

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Alpha Omega PUBLICATIONS

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The Real Number System

Introduction

Pre-algebra is an introductory algebra course designed to prepare junior-high school students for Algebra I. The course focuses on strengthening needed skills in problem solving, integers, equations, and graphing. Students will begin to see the “big picture” of mathematics and learn how numeric, algebraic, and geometric concepts are woven together to build a foundation for higher mathematical thinking.

By the end of the course, students will be expected to do the following:

- Gain an increased awareness of how math is a life skill.
- Understand how math is like a language, with a set of conventions.
- Explore concepts taught in previous math courses at higher levels and in real world applications.
- Practice algebraic thinking in order to model and solve real world problems.
- Utilize new skills and concepts that will help them in future math courses.
- Introduce variable expressions and equations (single and multiple variable).
- Introduce linear functions, relationship between dependent and independent variables and coordinate graphing.

In this unit, the student is formally introduced to the subsets of the real number system, including irrationals. Venn diagrams are used to compare and contrast the subsets. The number line is used to discuss distance, midpoint, and absolute value, as well as to compare and order integers.

The properties of the real number system are reviewed. Exponents and order of operations are used to allow the student to apply properties of the real number system. Lastly, scientific notation is explained.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

- Classify numbers.
- Evaluate expressions that contain variables.
- Compare and order numbers.
- Determine absolute value.
- Apply the properties of real numbers.
- Use exponents.
- Write numbers in scientific notation.
- Write numbers with square roots.
- Use the order of operations to simplify expressions.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. Relationships

Subsets of the Real Number System

Numbers are concepts that are represented by symbols called numerals. Numerals are used to communicate the idea of "how many." Look at some numerals that have been used in the past to communicate the number ten:

Early Greek	Roman	Babylonian
Δ	X	

Even though the symbols are different, they each represent the same "idea." But, what if everyone had their own way to represent numbers? Can you imagine the chaos? Trying to communicate ideas, like what time it is or how much something costs, would be almost impossible!

The sets of numbers we use today exist because there came a time when a universal way to represent numbers was needed.

Objectives

- Classify numbers.
- Identify irrational numbers.

Vocabulary

infinite—increasing or decreasing without end

integer—a number belonging to the set made up of the whole numbers and their opposites

irrational number—a number which, when in decimal form, does not terminate or repeat

natural number—a number belonging to the set made up of the numbers that are used to count: 1, 2, 3, and so on

rational number—a number which can be written as a ratio in fraction form

real number—a number which can be written as an infinite decimal

whole number—a number belonging to the set made up of zero and the natural numbers

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Real Numbers

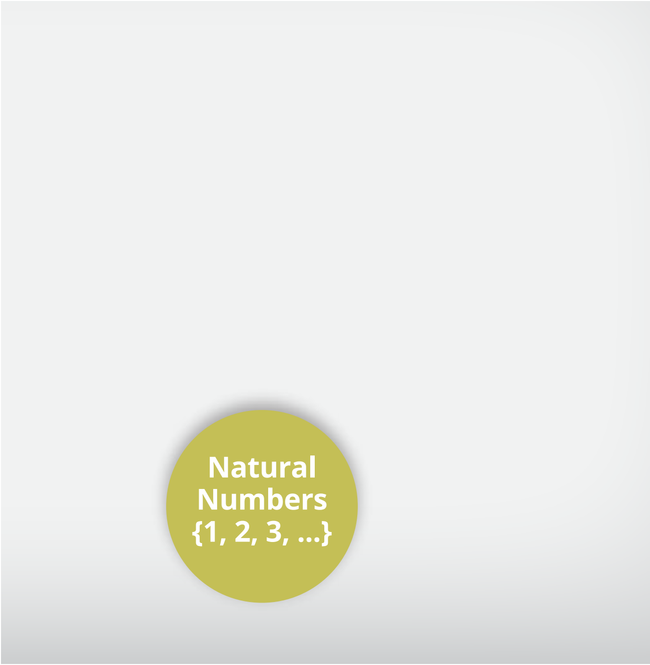
All of the numbers that you have worked with so far are called *real numbers*. You might wonder why we call them "real" numbers. Are there "unreal" numbers? Actually, yes, there are! The set of "unreal" numbers are called imaginary or complex numbers, and you will learn about those later on in math. In pre-algebra, we will focus on the real number system.

Within the system of real numbers, there are several sets or groups of numbers, called subsets. We will use a diagram to help us remember what each group of numbers is and how it fits within the real number system.

Vocabulary! Remember that a subset is a set in which all its members belong to another set. For example, a set listing several types of dogs is a subset of the set of all animals, because every dog is an animal.

People in ancient times began using numbers so they could record or talk about how many of something they had. Zero and negative numbers did not exist as we know them today because people had no need to communicate those concepts. Numbers really were only used for counting, so this set of numbers is called the "counting" numbers or *natural numbers*.

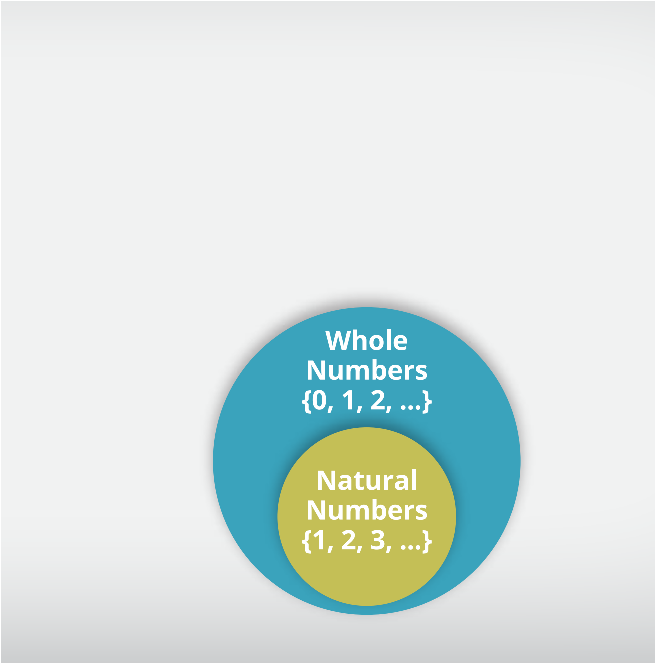
Natural Numbers



It is difficult to find the history of how zero evolved. The idea of zero was hard for people to accept. Math had always been used to solve "real" problems. People saw no need to represent zero. It did, however, start to get used as a place holder in numbers, just as we do today. For example, the number 306 would be 36 if we didn't use the zero to show that there is nothing in the tens place. Eventually, zero was used to represent the result when a number is subtracted from itself. Adding zero to the set of natural numbers gives us the *whole numbers*.

Keep in mind! The three dots, called an ellipsis, means that the set of numbers is infinite, or continues forever in the same pattern.

Whole Numbers

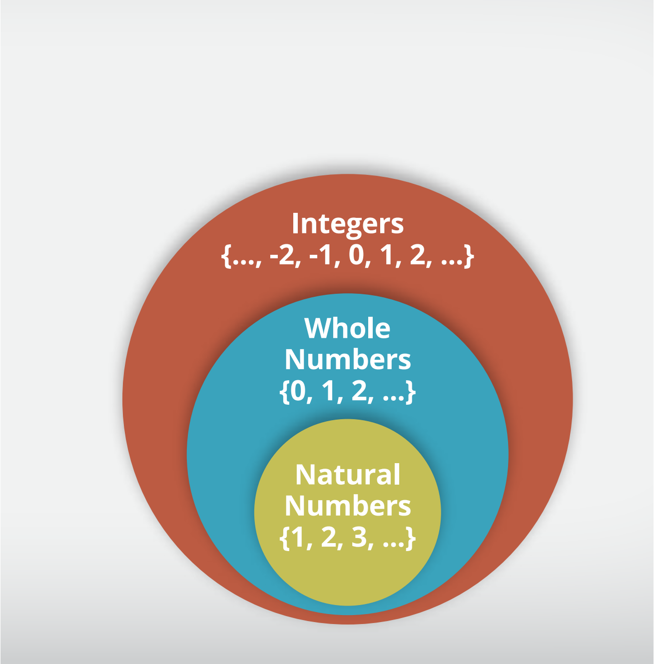


The negative numbers were ultimately the biggest challenge for mathematicians. As they discovered more about math, mathematicians began to solve problems that had negative answers. At first, people could not agree. Many said that negatives were not real numbers, so there could be no answer to the problems. Over time, however, negative numbers have come to be accepted. Today, we easily talk about negative temperatures or negative amounts of money (debt).

This might help! One way to remember the whole numbers is to think of zero as a "hole." The words are not spelled the same but sound the same. Try to connect "hole" with the set that starts with zero.

Take the counting numbers, make them negative, and combine them with the whole numbers. Now you have the *integers*.

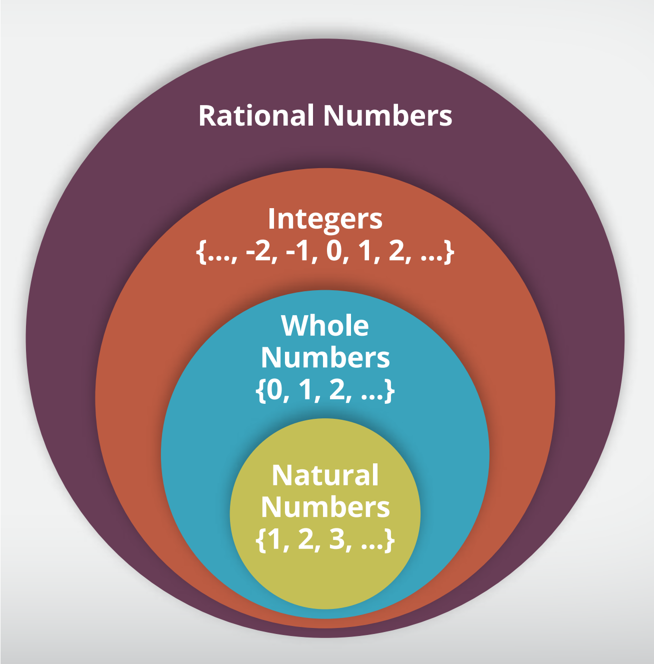
Integers



People were actually using fractions before zero or negative numbers. It made sense to people that they could have part of something. Just as with whole numbers, there may have been different symbols, but the concepts were the same. Everyone could "see" what one-half meant. The set of numbers containing all fractions is called the *rational numbers*. Look at the picture relating the sets of numbers:

Key point! On the diagram, a circle inside another circle shows that the smaller circle is a part of the larger circle. So, all natural numbers are also whole numbers and integers. And all whole numbers are also integers. It doesn't work the other way around though. Not all integers are whole numbers or natural numbers (like −1), and not all whole numbers are natural numbers (the number 0).

Rational Numbers



Rational numbers are numbers that can be written as fractions. The natural numbers, whole numbers, and integers all can be expressed as fractions by placing them over 1. So, they are all rational numbers. Notice in the rational numbers diagram that these three sets are all inside the set of rational numbers. This shows us that they are all rational numbers.

Think about it! The word rational contains the root word ratio. Ratios are often written in fraction form.

- Natural numbers can be written as fractions:

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \ldots$$

- The only whole number that is not a natural number is zero, but it can also be written as a fraction: $\frac{0}{1}$
- Negative integers can be written as fractions:

$$\frac{-1}{1}, \frac{-2}{1}, \ldots$$

- Terminating decimals can be written as fractions:

$$0.3 = \frac{3}{10}$$

$$2.07 = 2\frac{7}{100} = \frac{207}{100}$$

- [Remember... To change a mixed number to a fraction, multiply the whole number by the denominator and add the numerator. This number becomes your new numerator.]

$$-1.9 = -1\frac{9}{10} = -\frac{19}{10}$$

Vocabulary! The word terminating means to end. So terminating decimals are decimal numbers that have an end.

Most of the numbers that you have used in math so far are rational numbers.

Use the rational numbers diagram to help you classify numbers. We classify numbers by determining which sets they belong to. Remember that numbers may belong to more than one set. Here are some examples:

- Five is a natural number, whole number, integer, and rational number. We can see this in the diagram because the circle of natural numbers is inside the circles of the whole numbers, integers, and rational numbers.
- $\frac{2}{3}$ is a rational number only.
- −74 is an integer and rational number.
- $-\frac{6}{3}$ is an integer and rational number

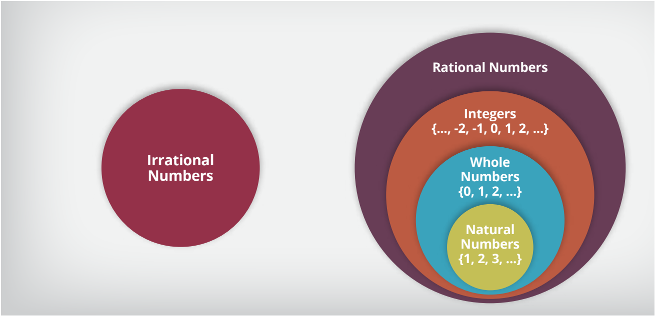
Keep in mind! In the last example, the fraction $-\frac{6}{3}$ doesn't look like an integer, but it does simplify to −2. Make sure to simplify the number, if possible, before determining which sets it belongs to.

Do you believe that even with all of these numbers, the set of real numbers is still not complete? The study of geometry introduced even more numbers that had not been necessary before. These numbers were decimal numbers that never terminated or repeated. Remember the number pi? We use the symbol π to represent it. Computers have written it out millions of digits and still there is no pattern to way the digits are written. Here is a little tiny bit of π: 3.14159265358979323846243 38327...

Notice that there is no repeating pattern in π and that the number is *infinite* (never ends)! These special decimals that don't repeat or terminate are called *irrational numbers*. Let's look at how the irrational numbers are related to the other sets of numbers.

Keep in mind! The prefix "ir-" means "not." For example, if something is irresistible, you are not able to resist it. Irrational numbers are not rational.

Real Numbers: Irrational or Rational



Numbers are either rational or irrational. They cannot be both! Notice in the diagram that the circles representing rational and irrational numbers do not overlap or touch each other in any way.

Make note! It is a good idea to copy this diagram into your notes, since it is a nice summary of the sets of numbers and how they are related. You'll want to have your notes handy when you are working on the problems and studying for your quiz and test.

There are three types of decimal numbers. Use the following examples to help you remember whether a decimal number is rational or irrational.

The decimal number is:	Examples	Type
terminating	0.25, 13.3457	rational
repeating	$0.\overline{25}, 1.\overline{3}, -74.\overline{125}$	rational
NOT terminating or repeating	713.6925419927...	irrational

The Real Numbers: Irrational or Rational diagram shows the entire real number system as we know it today. The real number system includes any number that can be written as an infinite decimal and represents all of the numbers that you are familiar with. This number system evolved as people needed to express number concepts in consistent ways.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- The real number system can be divided into sets of numbers.
- A number may belong to more than one set of numbers.
- A real number is either rational or irrational, but not both.

Complete the following.

1.1 Which of the following is an irrational number?

☐ π ☐ $-\frac{16}{3}$ ☐ $2.\overline{6}$ ☐ 18

1.2 The display of a student's calculator shows: 1.2731568 The student is most likely looking at _____.

☐ a natural number ☐ a whole number

☐ an integer ☐ an irrational number

1.3 The number −5 is all of the following *except* _____.

☐ an integer ☐ a whole number

☐ a rational number ☐ a real number

1.4 If a number is a whole number, then it *cannot* be _____.

☐ an irrational number ☐ an integer

☐ a natural number ☐ a rational number

1.5 Luis starts to do a division problem and notices that there is a pattern in the digits to the right of the decimal point. This number is _____.

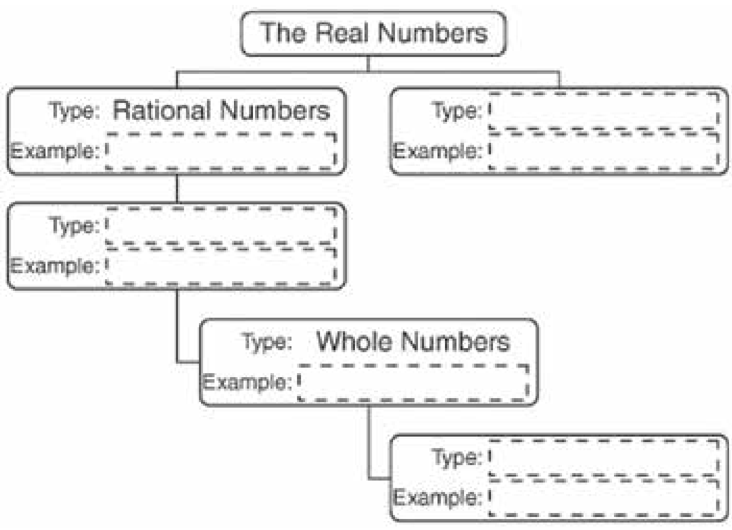
☐ irrational ☐ rational ☐ an integer ☐ natural

1.6 All of the following are rational *except* _____.

☐ 0 ☐ $-\frac{2}{5}$ ☐ $0.\overline{125}$ ☐ 3.14159...

1.7 Use the choices to complete the Real Numbers chart.

1/3
Irrational Numbers
1.175136981
Integers
-5
0
Natural Numbers
56



Using Variables

What if someone sent you a message that looked like this: Δ@\ 1/> #@>/> ~>##>/7 &$ 51#@?

How would you know what it says? What must you have in order to figure out the meaning of the message? A key! You would need to know what each number or symbol stands for. Otherwise, the message is meaningless to you. Here is the key to this code:

Decode the message to reveal a question. You can find the answer to the question in your lesson!

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	<	+	2	>	!	3	@	&	4	=	~	5	$	%	6	*	/	7	#	÷	8	Δ	ø	\	9

Objectives

- Identify a variable, term, or expression.
- Use substitution to simplify expressions and formulas.

Vocabulary

constant—a number; a term containing no variables
expression—one term or multiple terms connected by an addition or subtraction sign
formula—an expression that uses variables to state a rule
term—a number or a variable, or the product of a number and variable(s)
variable—a letter or symbol used to represent an unknown number

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Using Letters to Represent Numbers

Did you decode the message from the beginning of the lesson? It asked, "Why are there letters in math?" Let's find out why.

Algebra is kind of like a secret code that you have to work out. And, as you learn more and more parts of the key, you'll be able to solve harder and harder codes! But first, you'll need to learn a few of the words that you'll be seeing a lot from now on.

Just like the given code used symbols to represent letters, algebra uses letters to represent numbers. These letters stand for numbers that are unknown and are called *variables*, because they can change.

Numbers always represent themselves, so they are called *constants*.

Vocabulary! The word *variable* describes something that is able to change. The word *constant* describes something that never changes or always stays the same.

We can also combine variables and constants. When we combine them using multiplication or division, the result is called a *term*. Here are some examples of terms:

$$7x,\ abc,\ \frac{x}{2},\ \frac{3y}{5},\ r,\ 8$$

A term can be just a number, just a variable, or any combination of numbers and variables that uses multiplication or division.

In looking at the examples of a term, however, you may be wondering why there are no multiplication symbols. As you start using variables in math, it becomes very easy to confuse the letter (x) with the multiplication symbol (×). So, as a convention in math, we use a raised dot (•), parentheses, or write variables side-by-side to indicate multiplication between two numbers. For example, 7*x* means "7 times *x*," and *abc* means "*a* times *b* times *c*."

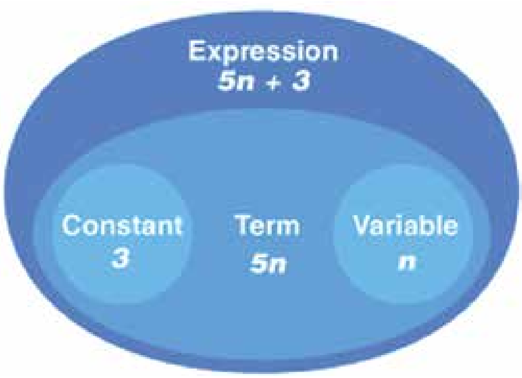
Key point! Letters written side-by-side, or a number and letters written side-by-side, indicate multiplication.

An *expression* is one term or a combination of multiple terms using an addition or subtraction sign. Here are some examples of expressions:

$$3x + 5,\ n - 11,\ \frac{1}{2}ab + 8 - y,\ \frac{a+2}{5},\ s,\ 9,\ -2mn$$

Notice that even a single term is considered an expression, not just multiple terms.

Did you notice how the vocabulary words we just looked at seemed to build on each other? Starting with constants and variables, we add operations to show relationships and to form terms and expressions. The Expression diagram will help you visualize this.



This might help! The diagram shows how the vocabulary words are related. Constants and variables are types of terms. And, you can combine constants, variables, and terms to make expressions.

You may be wondering, "What is an expression good for?" Well, expressions show a relationship between ideas. For example, the expression, 5n + 3, could represent the following scenario:

- *To park in a downtown parking garage, it costs a flat fee of $3, plus an additional $5 per hour.*

If you know the number of hours a person parked, you could evaluate what their total cost would be.

Vocabulary! Evaluating an expression means to find the numerical answer for an expression.

For now, you will be given expressions and asked to evaluate them. Later on, you'll be able to determine the relationships and write the expressions yourself!

Evaluating an expression is very simple. The first step is to replace the appropriate variables with any known values. After that, you just need to perform the computation. Let's look at a couple of examples.

Example:

- Evaluate the expression: $m \div 4$, if $m = 68$.

Solution:

$m \div 4$	Original equation.
$68 \div 4$	Replacing the variable with a known value, substitute $m = 68$.
17	Perform the division.

Example:

- Evaluate the expression: *abc*, if $a = 3$, $b = 4$, and $c = 6$.

Solution:

abc	Original equation.
$3 \cdot 4 \cdot 6$	Replace the variables with their known values.
72	Perform the mulitiplication.

One special type of expression is a *formula*. A formula uses variables to state a commonly known or frequently used rule. For example, to find the area of a rectangle, the rule is to multiply the length of the rectangle by the width of the rectangle. So, the formula is $A = lw$. Formulas use logical variables to stand for the different parts, such as the first letter of what they represent. In this case, *A* stands for Area, *l* for length, and *w* for width. Here are some other common formulas:

Distance	$d = rt$	d = distance; r = rate; t = time
Simple Interest	$i = prt$	i = interest; p = principal; r = rate; t = time
Area of a Triangle	$A = \frac{1}{2}bh$	A = area; b = base; h = height
Volume	$V = Bh$	V = volume; B = area of the base; h = height
Circumference of a Circle	$C = 2\pi r$	C = circumference; r = radius

Evaluating formulas is done the same way as evaluating other expressions. Simply substitute any given values in for the correct variables and complete the computation!

Example:

- Find the distance a man traveled, if his rate (r) was 50 miles per hour, and his time traveled (t) was 3 hours.
- Use the formula $d = rt$.

Solution:

$d = rt$	Formula for distance traveled.
$d = (50)(3)$	Replace variables with known values.
$d = 150$	Perform the mulitiplication.

- Answer: The man traveled 150 miles.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Constants, variables, and operations are used to form terms and expressions.
- Formulas use variables to state a commonly known rule.
- Expressions and formulas can be evaluated using substitution.

Complete the following

1.8 Six + *x* is an example of _____.

- ☐ a variable
- ☐ a constant
- ☐ an expression
- ☐ a formula

1.9 Seven is *not* an example of _____.

- ☐ a variable
- ☐ a constant
- ☐ an expression
- ☐ a term

1.10 *K* is an example of _____.

- ☐ a variable
- ☐ a constant
- ☐ an equation
- ☐ a formula

1.11 *I* = *prt* is an example of _____.

- ☐ a variable
- ☐ a constant
- ☐ an expression
- ☐ a formula

1.12 Evaluate each expression and match with the possible answers if $a = 6$, $b = 3$, and $c = 2$.

________	ab	6
________	$c + 4$	8
________	$\frac{a}{b}$	2
________	$a - b$	5
________	2c	1
________	$b \div 3$	3
________	$a - 1$	4
________	$6 + c$	18

1.13 The area of a triangle is found using the following formula: $A = \frac{1}{2}bh$
If $b = 3$ in. and $h = 8$ in., what is the area of the triangle?

- ☐ 5.5 in.^{2}
- ☐ 12 in.^{2}
- ☐ 24 in.^{2}
- ☐ 48 in.^{2}

1.14 If $m = 8$ and $n = 4$, all of the following represent the same value *except* _____.

- ☐ $m - n$
- ☐ $\frac{n}{1}$
- ☐ $\frac{1}{2}m$
- ☐ mn

1.15 The formula commonly used to find simple interest is $i = prt$. What is the simple interest if $p = 1000$, $r = 0.05$, and $t = 1$?

☐ 1001.05 ☐ 1050 ☐ 500 ☐ 50

1.16 All of the following are equivalent *except* _____.

☐ $4 \cdot y$ ☐ $4 + y$ ☐ $4y$ ☐ $(4)(y)$

1.17 Using the formula $d = rt$, which of the following statements is *not* true?

☐ If $r = 3$ and $t = 4$, then d = 12.
☐ If $r = 4$ and $t = 2$, then d = 8.
☐ If $r = 3$ and $t = 5$, then d = 8.
☐ If $r = 5$ and $t = 3$, then d = 15.

The Number Line

Your friends call and tell you that they are only three blocks from your house. They want you to hop on your bike and ride over to meet them. You start down the driveway and you suddenly come to a stop. Why?

Objectives

- Locate numbers on the number line.
- Find the distance between two points on the number line.
- Evaluate numerical expressions containing absolute value symbols.
- Find the opposite of a number.

Vocabulary

absolute value—the distance from zero on the number line

distance—the number of units between two numbers or points, regardless of direction

opposite numbers—two numbers that are the same distance from zero on the number line but in opposite directions

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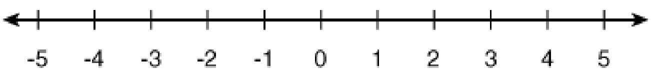
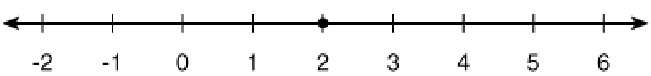
The Number Line

The number line, is a graph that lets us represent the real numbers.



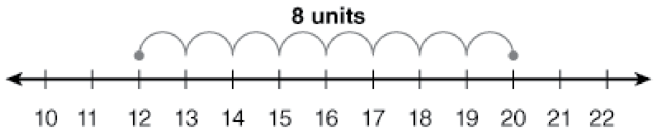
To the left of zero are the negative numbers, and to the right are positive numbers. To graph a number, simply put a point at the location of that number. The following graph is of the number two.

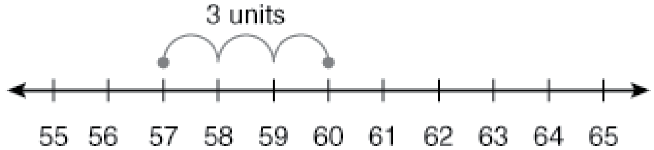


Recall that lines go on forever in both directions. The arrows on the graph are important because they show that the numbers go on forever. The term used to describe this is infinity. Numbers continue infinitely in both the positive and negative directions.

Distance on the number line means how many units there are between numbers. The distance between 1 and 6 is 5 units.

The following graphs show the distance between numbers. Do you see a connection to another math concept? You can find the distance between two numbers simply by subtracting the smaller number from the larger number!





Use the following number line to find the distance between −4 and 3.

Notice that distance is the number of units and has nothing to do with movement. Think back to your friends who are 3 blocks away. You stopped at the end of the driveway because you didn't know which direction to go. Just as we need the terms north, south, east, and west to describe direction on a map, we also need terms to describe direction on a number line. On the number line, we use the terms positive and negative. Movement to the right is in the positive direction and to the left is in the negative direction.

In addition to distance, the number line can also be used to find opposites. *Opposite numbers* are the same distance from zero on the number line but in opposite directions. Every real number has an opposite. The opposite of 2 is −2, and the opposite of −7 is 7.

S-t-r-e-t-c-h! The words opposite and negative can be used interchangeably. For example, we can say "the opposite of 2" instead of "negative 2" for −2. This is especially helpful when we want to simplify something, like −(−5). The first negative symbol can be read as "opposite." So, "the opposite of negative 5" is 5.

When you graph the opposite numbers 2 and −2, two different points are graphed. If you move 2 units to the right of zero, you are at 2. If you move 2 units to the left of zero, you are at −2. These numbers do represent different quantities. If you find the distance from zero to each of these numbers, however, it is the same. This distance is called the *absolute value* of a number. The absolute value symbol is a shorthand way of asking you to find the distance from zero to a number on the number line. It looks like this: | |.

Let's say that you want to find the distance from zero to 7 on the number line. Rather than writing that out, you can just write |7|. This is read as "the absolute value of 7," and it means "find the distance from zero to 7." Since −7 is also 7 units away from zero, then |−7| is also 7.

Look at the following examples:

Evaluate:

- $|38| = 38$
- $|-15| = 15$
- $\left|\frac{1}{2}\right| = \frac{1}{2}$
- $|-5.7| = 5.7$

Notice that the absolute value is positive because it represents a distance. It means "how far," not "what direction."

Key point! Distance is *always* positive because it only refers to the number of units between two numbers, or the length between two objects. It does not refer to direction.

Don't fall into the trap, though, of thinking that, if you see an absolute value symbol, the answer will be positive. Look carefully at the following problem and think about what it says to do: $-|14|$. You are told to take the opposite of something, but what? $|14| = 14$, so you must take the opposite of 14. Therefore, $-|14| = -14$.

Keep in mind! The absolute value symbol is considered a grouping symbol, like parentheses. When following the order of operations, make sure to first work inside all grouping symbols, including the absolute value symbol.

In the preceding problem, the absolute value was a positive number. But, then you were asked to take the opposite, which gives a negative number.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- The number line is a way to represent numbers graphically and to show relationships between numbers.
- The terms positive and negative are used to represent direction on the number line.
- Distance is always positive.
- Every number has an opposite, and opposite numbers have the same absolute value.

Complete the following.

1.18 $|-12| = 12$
- ○ True
- ○ False

1.19 $-|-37| = 37$
- ○ True
- ○ False

1.20 $-(-49) = -49$
- ○ True
- ○ False

1.21 $-|9| = -9$
- ○ True
- ○ False

1.22 The distance from −6 to 1 is 7.
- ○ True
- ○ False

1.23 Choose all of the expressions that are equal to −9.

- ☐ |−9|
- ☐ −|9|
- ☐ −(−9)
- ☐ the distance from zero to nine
- ☐ −|−9|
- ☐ the opposite of nine

1.24 Choose all of the expressions that are equal to 61.

- ☐ |−61|
- ☐ −|61|
- ☐ the opposite of −61
- ☐ −|−61|
- ☐ the opposite of 61
- ☐ −(−61)
- ☐ the distance from zero to −61

1.25 Which of the following is equal to the opposite of −45?

- ☐ −(−45)
- ☐ −45
- ☐ −|−45|
- ☐ −|45|

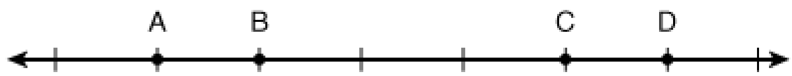
1.26 Which of the following statements *does not* represent the same value as the others?

- ☐ the distance between 0 and −9
- ☐ the opposite of 9
- ☐ the absolute value of negative 9
- ☐ the absolute value of 9

1.27 All of the following are true statements *except* _____.

- ☐ |93| = 93
- ☐ |−93| = 93
- ☐ −|93| = −93
- ☐ |−93| = −93

1.28 Answer this question based on the number line shown.



The distance from a point to point C is 1, and the distance from that same point to point B is 4. The point must be _____.

- ☐ point A
- ☐ between C and A
- ☐ point D
- ☐ between D and A

1.29 If you move from zero to 15 on the number line, you are representing all of the following *except* _____.

- ☐ the opposite of 15
- ☐ the absolute value of 15
- ☐ the distance between zero and 15
- ☐ the opposite of −15

1.30 All of the following expressions are equal to N *except* _____.

- ☐ $|-N|$
- ☐ $-(-N)$
- ☐ $|N|$
- ☐ $-N$

Comparing Rational Numbers

Objectives

- Place rational numbers on the number line.
- Use the correct inequality symbol to compare rational numbers.

Vocabulary

inequality—sentence showing a relationship between numbers that are not necessarily equal; uses the symbols >, <, ≥, ≤, or ≠

Comparing Numbers

Throughout history, symbols have been used to communicate mathematical concepts. Numerals are symbols that represent numbers. You are also familiar with the symbols that represent adding, subtracting, multiplying, and dividing. In fact, all these symbols are so commonly used that you probably read them without really thinking about their meaning. Let's look at a few more symbols that are commonly used to show the relationships between numbers.

We use the equality symbol (=) when we want to show that two quantities are equal. *Inequality* symbols are also used when we want to compare numbers. Following is a chart of the inequality symbols, and an overview of how they are read and used.

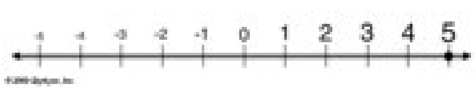
Inequality Symbol	Meaning	Example
$<$	less than	$2 < 5$
$>$	greater than	$5 > 2$
$\le$	less than or equal to	$2 \le 5$
$\ge$	greater than or equal to	$5 \ge 5$
$\ne$	not equal to	$2 \ne 5$

It is important to look at the examples with the ≤ and ≥ symbols. Notice that each of these symbols has an "or equal to" part (represented by the line underneath the < or > symbol). This means that the numbers you are comparing could possibly be equal. As long as one part of the statement is true, the whole thing is true. In the ≤ example, two is not equal to five. But two is less than five, so the entire sentence is true. In the ≥ example, five is not greater than five. But five does equal five, so this sentence is also true.

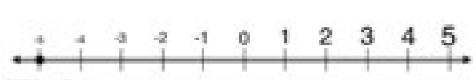
This might help! The opening of the symbol, or the *larger* side, goes with the *larger* number. So when you use >, the larger number is on the left side of the symbol. If you use <, the larger number is on the right side of the symbol.

Ever since you were very young, you have been comparing quantities. You understand what it means to be more or less than something when working with whole numbers. However, the comparison is not always as easy when dealing with all of the real numbers. Let's look at the number line and see how it can help us compare numbers when the relationship is not so obvious.

Start at -5 on the number line and move to the right.



The numbers are getting larger as you move to the right. What happens as you start to move to the left?

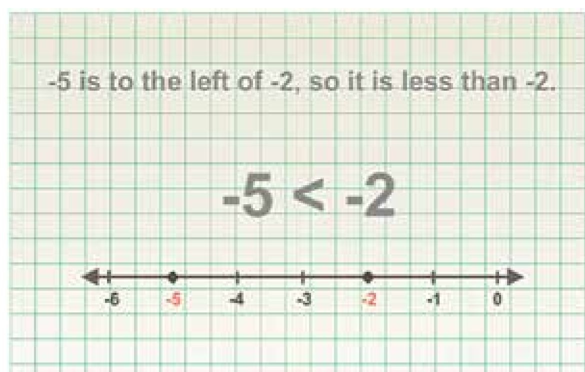


The numbers are getting smaller. This pattern continues as we move left into the negative numbers. As we look at the number line, if we are at a point "to the left of" a number, then we are "less than" that number. If we are "to the right of" a number, then we are "greater than" that number. Negative numbers are to the left of, and will always be less than, positive numbers.

This might help! Turn the number line on end like a thermometer—as you go down, the numbers get smaller. Or, you can remember that on the number line, numbers to the left are less.

Let's look at comparing some negative numbers.

Compare the following pair of numbers using an inequality symbol:



So, $-5 < -2$. Are there any other inequality symbols that would make this sentence true? Yes! $-5 \le -2$, and $-5 \ne -2$. Often, multiple symbols can describe the relationship between two numbers.

Example:

- Which of the following statements are *not* true?
  1. $27 \le 52$
  2. $-16 > -10$
  3. $5 > -24$
  4. $-2 \ne 2$
  5. $12 \ge 12$
  6. $0 < -7$
  7. $-3 \le 8$
  8. $-2 < -5$

Solution:

- The false statements are:
  2. −16 is to the left of −10 on the number line, so it must be less than −10, not greater than −10.
  6. 0 is to the right of −7 on the number line, so it must be greater than −7, not less than −7.
  8. −2 is to the right of −5 on the number line, so it must be greater than −5, not less than −5.

Comparing Fractions and Decimals

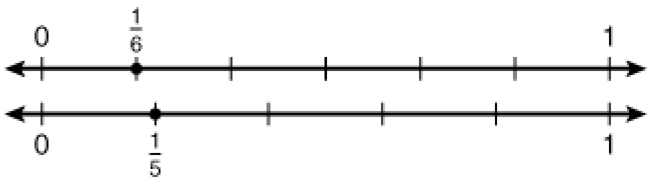
In addition to comparing integers, we also can compare numbers that have parts of whole units. Let's review how to make comparisons between some of the positive fractions and decimals:

Fractions with the same numerator:

- Compare the denominators: the larger the denominator, the smaller the fraction.

Example:

- $\frac{1}{6} < \frac{1}{5}$, because 6 > 5.



Think about it! Suppose you have two pies, and you cut one into 6 pieces and one into 5 pieces. The servings from the pie that was only cut into 5 pieces will be larger!

Fractions with the same denominator:

- Compare the numerators.

Example:

- $\frac{5}{7} > \frac{3}{7}$, because 5 > 3.

Mixed numbers:

- Compare the whole number part.

Example:

- $3\frac{2}{5} > 1\frac{7}{8}$, because 3 > 1.
  - If the whole numbers are the same, then compare the fractions.

Example:

- $7\frac{2}{3} < 7\frac{5}{6}$, because $\frac{2}{3} < \frac{5}{6}$.

Step-by-Step!

1) $7\frac{2}{3} < 7\frac{5}{6}$
2) $\frac{2}{3}$ is equivalent to (or the same as) $\frac{4}{6}$.
3) $\frac{4}{6} < \frac{5}{6}$
4) $7\frac{4}{6} < 7\frac{5}{6}$, so $7\frac{2}{3} < 7\frac{5}{6}$.

Decimals:

- Compare the whole number part (the numbers to the left of the decimal).

Example:

- 16.028 > 13.115, because 16 > 13.

- If the whole numbers are the same, and there are the same number of digits to the right of the decimal point, just compare the decimal portion.

Example:

- 1.25 < 1.27, because they both end in the hundredths place, and 25 < 27.

Compare! This is just like comparing two fractions that have the same denominator.

- If there are fewer digits to the right of the decimal point in one of the numbers, add zeros to the end of the number with fewer digits until have the same place value to compare.

Example:

- 35.02 < 35.9

Solution:

- Add a zero to 35.9, so that each number has the same number of digits. Then, 35.02 < 35.90, because they both end in the hundredths place, and 2 < 90.

Fractions and Decimals:

- Convert all of the numbers to either fractions or decimals.

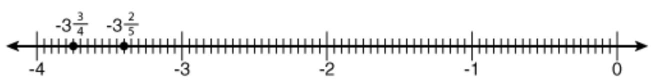
Example:

- $\frac{1}{2} > 0.3$

Solution:

- *Method 1*: Convert to fractions.
  - Using place value, $0.3 = \frac{3}{10}$.
  - $\frac{1}{2}$ is equal to $\frac{5}{10}$, which is greater than $\frac{3}{10}$.
- *Method 2*: Convert to decimals.
  - Using division, $\frac{1}{2}$ is equal to 0.5, which is greater than 0.3.

When comparing two negative numbers, remember that the one furthest to the left on the number line is the smallest. It may help to use the absolute value, or distance from zero. Let's say that you want to compare $-3\frac{2}{5}$ and $-3\frac{3}{4}$. Since the whole number parts are the same, compare the fractions. $\frac{3}{4} > \frac{2}{5}$, so $\frac{3}{4}$ is further from zero on the left side. This gives us that $-3\frac{3}{4} < -3\frac{2}{5}$.



Comparing numbers is something people do every day without even realizing it. At the store, a shopper compares prices. Scientists make comparisons when determining the greater amount of a substance in an experiment. They often work with very small numbers and need to understand how to compare fractional values. Golfers use negative numbers in scoring when they are under par. The smallest number wins. As you think about it, I'm sure that you can come up with many more examples of when people need to compare numbers.

Let's Review

Before going on to the practice problems, make sure you understand the main points of this lesson.

- Inequality symbols are used to communicate relationships between numbers.
- The number line is used to help to determine those relationships.

Complete the following activities.

1.31 If $r = 12.5$ and $s = |-13|$, which of the following inequalities is *not* true?

☐ $r \neq s$ ☐ $r < s$ ☐ $r \geq s$ ☐ $r \leq s$

1.32 Which of the following lists is ordered from least to greatest?

☐ $-1.5, -2, 0, \frac{2}{5}, \frac{3}{4}$ ☐ $-2, -1.5, 0, \frac{2}{5}, \frac{3}{4}$

☐ $-2, -1.5, 0, \frac{3}{4}, \frac{2}{5}$ ☐ $|-1.5|, -2, 0, \frac{2}{5}, \frac{3}{4}$

1.33 Which of the following numbers is between 0.62 and 1.05?

☐ 1.01 ☐ 0.065 ☐ 1.1 ☐ 0.6

1.34 $-\frac{1}{7}$ is greater than all of the following *except* _____.

☐ $-\frac{1}{3}$ ☐ −1 ☐ −2 ☐ −0.1

1.35 Which of the following statements is false?

☐ $2 \geq 8$ ☐ $2 \leq 8$ ☐ $8 \leq 8$ ☐ $2 < 8$

1.36 The following chart shows temperatures of cities on a winter morning.

City	Temperature
Buffalo	−8
Grand Rapids	−3
Denver	10
Chicago	−1

Which city had the coldest temperature?

☐ Buffalo ☐ Grand Rapids ☐ Denver ☐ Chicago

1.37 Tanisha and her friends were playing a card game where the smallest number wins. Look at their scorecard. Who won the game?

Tanisha	−15
Tyrell	−18
Carlos	−12
Samantha	−9

☐ Tanisha ☐ Carlos

☐ Tyrell ☐ Samantha

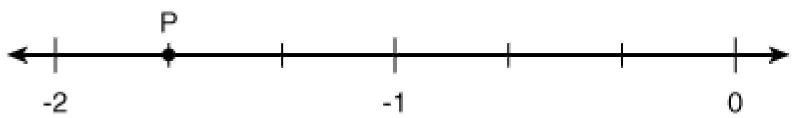
1.38 In which of these pairs of numbers is 1.75 greater than the first number but smaller than the second number?

☐ 0 and 1 ☐ 1 and $\frac{3}{2}$ ☐ $\frac{3}{2}$ and $1\frac{9}{10}$ ☐ $1\frac{9}{10}$ and 2

1.39 What is the best estimate of the number corresponding to A?

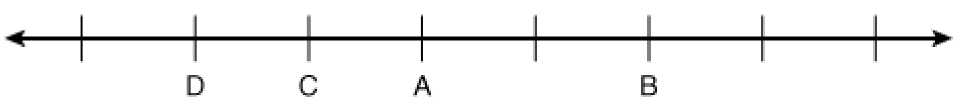
☐ 2.2 ☐ 2.3 ☐ 2.4 ☐ 2.6

1.40 Which of the following statements could be used to describe the location of point P, shown on the number line?



☐ P = −2.5 ☐ P < −1 ☐ P > −1 ☐ P < −2

1.41 Which of the following inequalities is true according to the number line?



☐ A > D ☐ B ≤ D ☐ C ≥ B ☐ D > C

Review the material in this section in preparation for the Self Test. The Self Test will check your mastery of this particular section. The items missed on this Self Test will indicate specific areas where restudy is needed for mastery.

SELF TEST 1: Relationships

Complete the following activities (6 points, each numbered activity).

1.01 Select all of the symbols that would make the comparison true. −7 ___ −4

☐ $<$ ☐ $\le$ ☐ $=$

☐ $>$ ☐ $\ge$ ☐ $\neq$

1.02 Select all of the symbols that would make the comparison true. 2.5 ___ 2.05

☐ $<$ ☐ $\le$ ☐ $=$

☐ $>$ ☐ $\ge$ ☐ $\neq$

1.03 Select all of the symbols that would make the comparison true. $-|-9|$ ___ −9

☐ $<$ ☐ $\le$ ☐ $=$

☐ $>$ ☐ $\ge$ ☐ $\neq$

1.04 All of the following are rational numbers *except* _____.

☐ $\frac{2}{3}$ ☐ $1.\overline{33}$ ☐ −16 ☐ 3.14159...

1.05 Which of the following is a true statement?

☐ The opposite of −45 is equal to the absolute value of −45.

☐ The opposite of 45 is equal to the absolute value of −45.

☐ The opposite of 45 is equal to the absolute value of 45.

☐ The opposite of −45 is not equal to the absolute value of 45.

1.06 The following chart shows the times of runners in the 100 meter sprint.

Rahn	11.33 sec
Miguel	11.5 sec
Tyrone	11.09 sec
George	11.28 sec

Who won the race?

☐ Rahn ☐ Tyrone

☐ Miguel ☐ George

1.07 Which of the following statements is false?

☐ If a number is a natural number, then it is rational.

☐ If a number is a fraction, then it is rational.

☐ If a number is a whole number, then it is rational.

☐ If a number is an integer, then it is irrational.

1.08 Which of the following numbers is between $-\frac{3}{4}$ and $\frac{5}{8}$?

☐ -1 ☐ $-\frac{1}{2}$ ☐ $\frac{7}{8}$ ☐ 0.9

1.09 Which of the following statements is false?

☐ -|-5| = -5 ☐ -(-5) = 5 ☐ -|5| = -5 ☐ |-5| = -5

1.010 Given the statement -12 ≤ -15, which of the following is correct?

☐ It is a true statement, because -12 is less than -15.
☐ It is a false statement, because -15 is less than -12.
☐ It is a false statement, because -12 is not equal to -15.
☐ It is true, because 12 is less than 15.

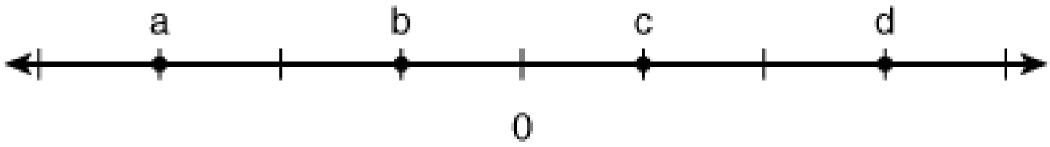
1.011 The distance between -7 and 2 on the number line is _____.

☐ 5 ☐ -5 ☐ 9 ☐ 10

1.012 π is an example of _____.

☐ a rational number
☐ an integer
☐ an irrational number
☐ a natural number

1.013 Which of the following statements is *not* true based on the given graph?



☐ $a \leq b$ ☐ $c > 0$ ☐ $|b| = c$ ☐ $-a = c$

1.014 Which of the following lists is ordered from least to greatest?

☐ -5, 0, 0.8, 1, $1\frac{1}{2}$
☐ -5, 0, 0.8, $1\frac{1}{2}$, 1
☐ -3, -5, 0, 0.8, 1
☐ 1, 0.8, $\frac{1}{2}$, 0, -5

1.015 If $h = 12$ and $g = 4$, which of the following has a value of 3?

☐ $h - g$ ☐ $\frac{h}{g}$ ☐ $h \div 3$ ☐ $g + 1$

1.016 *X* is all of the following *except* _____.

☐ a constant ☐ a term

☐ a variable ☐ an expression

1.017 Evaluate the formula $V = Bh$, if $B = 24$ and $h = 6$.

☐ $V = 4$ ☐ $V = 12$ ☐ $V = 30$ ☐ $V = 144$

82/102 **SCORE** ________ **TEACHER** ________ ________

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