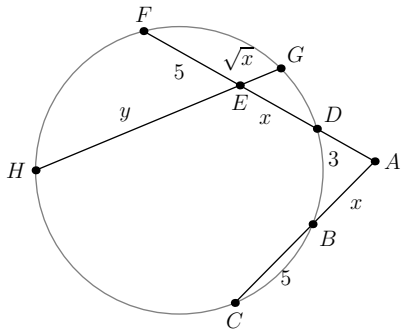
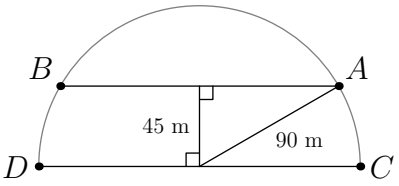
Do You Know? *Introduction to Geometry*, by R. Rusczyk

If you can solve nearly all of the following problems with little difficulty, then the text **Introduction to Geometry** would only serve as a review for you.

1. Prove the Pythagorean Theorem.
2. Find y in the diagram below.

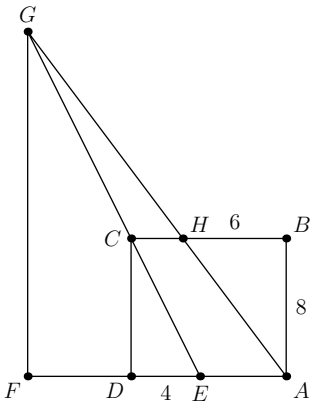


3. Marcia could walk from A to B along arc AB on the semicircular path, or she can walk along chord AB. Diameter CD has length $180m$. How much farther is it to walk along the arc as opposed to the chord?

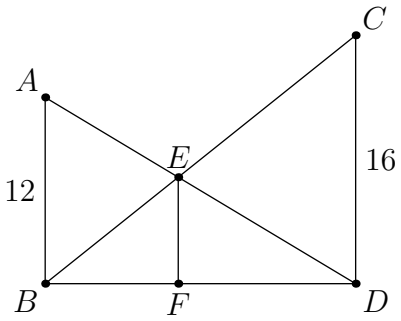


4. An ant starts at one vertex of a unit cube and walks to the opposite vertex along the surface of the cube. What is the minimum distance the ant can walk?
5. Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside the doghouse that Spot can reach?

6. In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on BC with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G, and F is on line AD with $GF \perp AF$. Find the length GF.



7. There are two flagpoles, one of height 12 and one of height 16. A rope is connected from the top of each flagpole to the bottom of the other. The ropes intersect at a point x units above the ground. Find x. In the accompanying diagram, this is equivalent to finding the length of EF.



8. Three spheres are tangent to a plane at the vertices of a triangle and are tangent to each other. Find the radii of these spheres if the sides of the triangle are 6, 8, and 10.

9. Derive a general formula for the volume of the frustum of a cone with bases of radius R and r and height h.

Don't look at the next page until you've attempted all the problems!

The answers to Do You Know **Introduction to Geometry** are below.

1. (Note that there are many acceptable proofs.) In right triangle ABC with right angle at A we wish to prove $AC^2 + AB^2 = BC^2$. Drop altitude AD to hypotenuse BC. $\triangle ABC \sim \triangle DAC \sim \triangle DBA$ giving us $\frac{DC}{AC} = \frac{AC}{BC}$ and $\frac{DB}{AB} = \frac{AB}{BC}$. Now $AC^2 = BC \cdot DC$ and $AB^2 = BC \cdot DB$, so $AC^2 + AB^2 = BC(DC + DB) = BC^2$.
2. 10
3. $60\pi - 90\sqrt{3}$
4. $\sqrt{5}$
5. 3π
6. 20
7. $\frac{48}{7}$
8. $r_1 = \frac{12}{5}, r_2 = \frac{15}{4}, r_3 = \frac{20}{3}$
9. $V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$