LESSON 6

Objective

Determine the probability for a random drawing without replacement.

Common Core State Standards

- **7.SP.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- **7.SP.8a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- **7.SP.8b** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Finding Probability Without Replacement

Students have learned how to find the probability of a single event. In this lesson, students will determine the probability of having the same outcome occur more than once in a row.

Try It! *Perform the Try It! activity on the next page.*

Talk About It

Discuss the Try It! activity.

- **Say:** *The probability of picking a blue sock can be found by taking the number of blue socks and dividing that number by the total number of socks in the drawer.* **Ask:** *What is the probability of picking a blue sock?* Elicit that the probability is $\frac{2}{6}$ or $\frac{1}{3}$.
- **Say:** *Suppose that Bill picks a blue sock and doesn't return it to the drawer.* **Ask:** *If he picks again, what are his chances of picking another blue sock?* Elicit that because there are now just five socks in the drawer and only one of them is blue, you divide one by five; the answer is $\frac{1}{5}$.
- **Ask:** *How could you use your data from the first step to estimate the probability of drawing two blue socks?* Have students obtain an estimate and compare it with the theoretical probability.

Solve It

Reread the problem with students. Ask them to write to Bill explaining how to determine the probability of picking two blue socks.

More Ideas

For another way to teach about probability for a random drawing—

- Extend the activity by having students determine the probability of picking two blue socks with replacement. Have students re-enact the scenario, but this time they should replace the first sock drawn. Compare the probabilities (both theoretical and experimental) with and without replacement.

Formative Assessment

Have students try the following problem.

Each of the numbers 1–6 is written on a card and put into a hat. Two cards will be drawn, one at a time, without replacement. What is the probability of drawing two odd numbers?

A. $\frac{7}{10}$ **B.** $\frac{1}{3}$ **C.** $\frac{1}{5}$ **D.** $\frac{1}{10}$

Try It! 40 minutes | Pairs

Here is a problem about a random drawing without replacement.

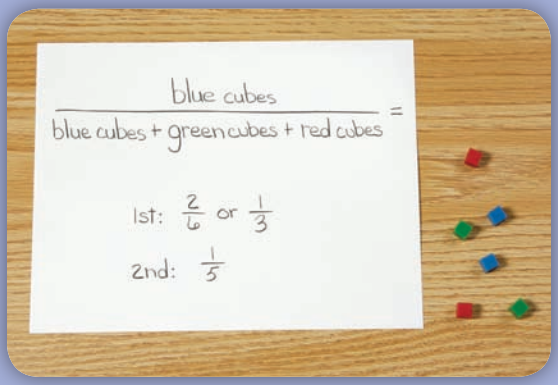
Bill's sock drawer contains 2 blue socks, 2 green socks, and 2 red socks. If he picks one sock at random, what is the probability it will be blue? If he then picks another sock (without returning the first sock), what is the probability the second sock will be blue? What is the probability that both socks will be blue?

Introduce the problem. Then have students do the activity to solve the problem. Distribute the materials.

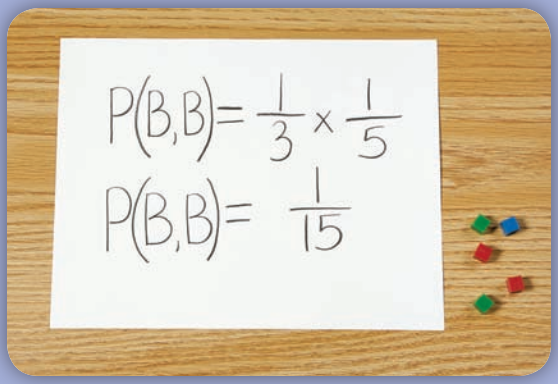
Materials

- Centimeter Cubes (2 blue, 2 green, and 2 red)
- BLM 12
- paper bag

1. Have students place 2 blue, 2 green, and 2 red cubes in a paper bag. One student should pick a cube at random. The other student should record its color on the recording chart. Without returning the cube to the bag, the first student should select another cube at random. Its color also should be recorded on the chart. Students should return the cubes to the bag and repeat the procedure 30 times.



2. Have students determine the probability of randomly choosing a blue sock from the sock drawer. Then have them determine the probability that the second sock drawn will be blue if the first sock drawn is blue and is not replaced.



3. Have students use the formula to determine the probability that both socks Bill draws will be blue. Remind students of the Conditional Probability formula, $P(A \text{ and } B) = P(A) \times P(B \text{ given } A)$.

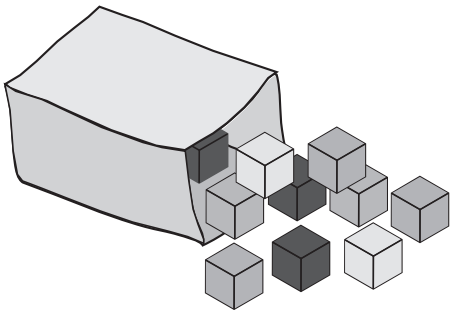
⚠ Look Out!

Watch for students who do not decrease the number of socks available when Bill draws the second sock. Have students re-enact the scenario with cubes to confirm that the number of socks in the drawer is now five and not six.

Answer Key

Use Centimeter Cubes to model the probability of each event, without replacement. Make a bag like the one shown. Answer the questions. (Check students' work.)

1. The bag has 2 yellow cubes, 5 green cubes, and 3 red cubes.



What is the probability of selecting a yellow cube at random? $\frac{1}{5}$

Without replacing the yellow cube, what is the probability of selecting a red cube at random? $\frac{1}{3}$

What is *P*(yellow, red)? $\frac{1}{15}$

What is *P*(yellow, yellow)? $\frac{1}{45}$

Using Centimeter Cubes, model each bag described. Find each probability without replacement.

2. A bag with 5 black cubes, 3 pink cubes, and 2 blue cubes

What is *P*(blue, black)? $\frac{1}{9}$

What is *P*(pink, blue)? $\frac{1}{15}$

What is *P*(black, black)? $\frac{2}{9}$

3. A bag with 6 orange cubes, 6 red cubes, and 6 brown cubes

What is *P*(orange, red)? $\frac{2}{17}$

What is *P*(red, red)? $\frac{5}{51}$

What is *P*(brown, red)? $\frac{2}{17}$

Find each probability without replacement.

4. A bag with 5 black marbles, 2 white marbles, and 8 yellow marbles

What is *P*(yellow, white)? $\frac{8}{105}$

What is *P*(white, black)? $\frac{1}{21}$

What is *P*(black, black)? $\frac{2}{21}$

What is *P*(black, yellow)? $\frac{4}{21}$

5. A bag with 3 solid ribbons, 4 striped ribbons, and 10 checkered ribbons

What is *P*(solid, solid)? $\frac{3}{136}$

What is *P*(checkered, striped)? $\frac{5}{34}$

What is *P*(striped, solid)? $\frac{3}{68}$

What is *P*(solid, checkered)? $\frac{15}{136}$

6. A bag with 12 red tiles, 10 black tiles, and 20 white tiles

What is *P*(red, white)? $\frac{40}{287}$

What is *P*(white, black)? $\frac{100}{861}$

What is *P*(red, black)? $\frac{20}{287}$

What is *P*(black, black)? $\frac{15}{287}$

7. A bag with 10 green marbles, 2 clear marbles, and 8 blue marbles

What is *P*(clear, clear)? $\frac{1}{190}$

What is *P*(green, clear)? $\frac{1}{19}$

What is *P*(blue, green)? $\frac{4}{19}$

What is *P*(green, green)? $\frac{9}{38}$

Answer Key

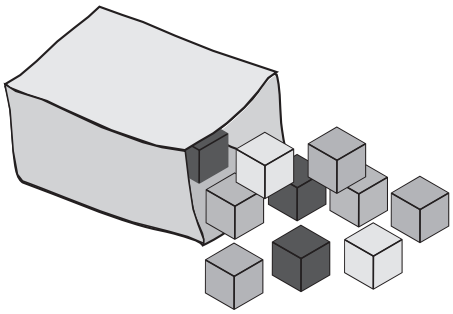
Challenge! Describe the numbers you multiply in the denominator when you find the probability of two events without replacement. When does the probability in simplest form have a denominator that differs from the product of the numbers you just described?

Challenge: (Sample) The denominator is the product of the number of items and one less than the number of items. When a numerator and a denominator have a common factor and you divide each by the common factor, the denominator of the probability in simplest form is not the product of the number of items times one less than the number of items.

Name _____

Use Centimeter Cubes to model the probability of each event, without replacement. Make a bag like the one shown. Answer the questions.

1. The bag has 2 yellow cubes, 5 green cubes, and 3 red cubes.



What is the probability of selecting a yellow cube at random? _____

Without replacing the yellow cube, what is the probability of selecting a red cube at random?

_____

What is *P*(yellow, red)? _____

What is *P*(yellow, yellow)? _____

Using Centimeter Cubes, model each bag described. Find each probability without replacement.

2. A bag with 5 black cubes, 3 pink cubes, and 2 blue cubes

What is *P*(blue, black)? _____

What is *P*(pink, blue)? _____

What is *P*(black, black)? _____

3. A bag with 6 orange cubes, 6 red cubes, and 6 brown cubes

What is *P*(orange, red)? _____

What is *P*(red, red)? _____

What is *P*(brown, red)? _____

Find each probability without replacement.

4. A bag with 5 black marbles, 2 white marbles, and 8 yellow marbles

What is *P*(yellow, white)? _____

What is *P*(white, black)? _____

What is *P*(black, black)? _____

What is *P*(black, yellow)? _____

5. A bag with 3 solid ribbons, 4 striped ribbons, and 10 checkered ribbons

What is *P*(solid, solid)? _____

What is *P*(checkered, striped)? _____

What is *P*(striped, solid)? _____

What is *P*(solid, checkered)? _____

6. A bag with 12 red tiles, 10 black tiles, and 20 white tiles

What is *P*(red, white)? _____

What is *P*(white, black)? _____

What is *P*(red, black)? _____

What is *P*(black, black)? _____

7. A bag with 10 green marbles, 2 clear marbles, and 8 blue marbles

What is *P*(clear, clear)? _____

What is *P*(green, clear)? _____

What is *P*(blue, green)? _____

What is *P*(green, green)? _____

Name _____

Challenge! Describe the numbers you multiply in the denominator when you find the probability of two events without replacement. When does the probability in simplest form have a denominator that differs from the product of the numbers you just described?

30-Trial Recording Chart

Name _____

Trial	First Cube	Second Cube