



# Daily Math Fluency

K-5 Sample Lessons

Build math fluency  
through Math Talks  
& Number Strings



# Fill the fluency gap in today's math curriculum

**Daily Math Fluency** is a year-long supplemental program that provides everything educators need to teach and reinforce multiple strategies that build number sense in 10 minutes a day. The powerful combination of 60 Math Talks and 120 Number Strings improves a student's ability to think about numbers flexibly, efficiently, and accurately. Created by teachers for teachers, Daily Math Fluency helps students build a strong foundation of mathematical reasoning for future math success.

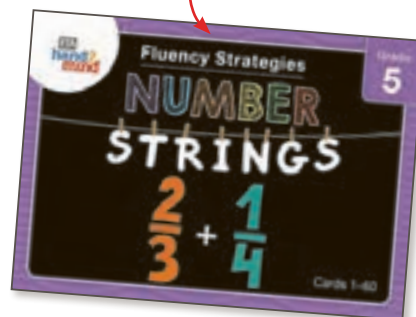
- **Teaches specific strategies** with targeted sets of related problems
- **Manipulatives and visual models** enable teacher demonstration
- **Easy to use** for quick classroom implementation



Daily Math Fluency, Grade 5

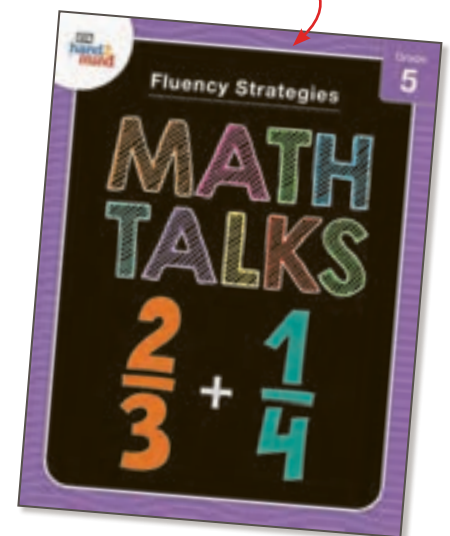
*Teacher Guide features implementation tips, strategies & big ideas*

*120 Numbers Strings cards with 5 problems per card*



Daily Math Fluency Number Strings, Grade 5

*60 Math Talks include possible strategies, teacher notes & facilitating questions*



Daily Math Fluency Math Talks, Grade 5



## Meet the lead author and co-creator

Brittany Goerig is the lead author and co-creator of Daily Math Fluency. Brittany spent 17 years in the classroom as an elementary math teacher and has created and conducted professional development for teachers for the past 15 years. She is passionate about helping children and teachers construct their own mathematical knowledge based on understanding the relationships in mathematics.



# Daily Math Fluency encourages students to think, not to memorize



## Creating a Generation of Independent Thinkers

by Brittany Goerig

### Valuing Productive Struggle

While it's important for students to be able to recall basic math, educators should persist in teaching the deep relationship between

numbers. **Instead of just memorizing facts, we want students to be able to visualize models and construct strategies.** A surface-level understanding of numbers isn't going to cultivate the confidence students need to independently and creatively solve math problems.

Math classrooms are changing by letting students productively struggle. **Rather than encouraging them to get the correct answer quickly, educators are giving students the time they need to reason with mathematics.** Most of the time, the teacher isn't teaching strategy directly. The strategy is a natural outcome of the product. After students experience a few number strings, they are able to pick up on the patterns. From there, they can construct the path to reason about the numbers.

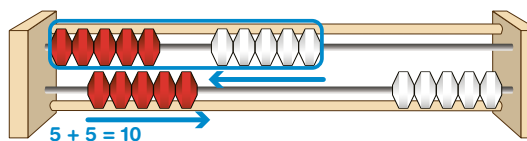
We don't know what future job markets look like, so **we have to make sure the future generation is full of thinkers.** We can help them prepare by equipping them with the ability to solve problems independently, without someone telling them what to do every step of the way.

—excerpted from *The Edvocate*; March 27, 2019.

$$5 + 5$$

Show 5 on top, 5 on the bottom for 3 seconds

**Teacher:** How many? How did you see it? **Goal:** 10; I saw 5 red on top and 5 red on the bottom. That is the same as 5 red and 5 white on top.

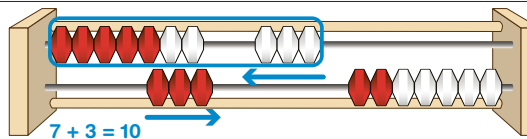


$$5 + 5 = 10$$

$$7 + 3$$

Show 7 on top, 3 on the bottom for 3 seconds

**Teacher:** How many? How did you see it? **Goal:** 10; I saw 7 on top and 3 on the bottom. I moved the 3 from the bottom to the 7 on top to get 10.

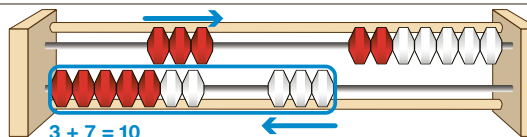


$$7 + 3 = 10$$

$$3 + 7$$

Show 3 on top, 7 on the bottom for 3 seconds

**Teacher:** How many? How did you see it? **Goal:** 10; It is switched around. I moved the 3 beads down this time to the 7 on the bottom to get 10.

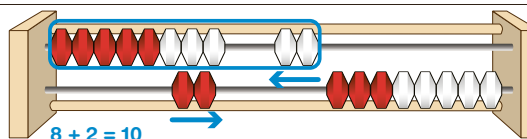


$$3 + 7 = 10$$

$$8 + 2$$

Show 8 on top, 2 on the bottom for 3 seconds

**Teacher:** How many? How did you see it? **Goal:** 10; I saw 8 on top and 2 on the bottom. I moved the 2 from the bottom to the 8 on top to get 10.

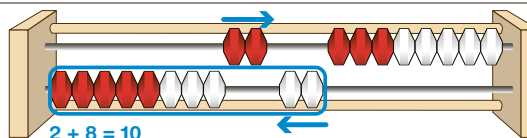


$$8 + 2 = 10$$

$$2 + 8$$

Show 2 on top, 8 on the bottom for 3 seconds

**Teacher:** How many? How did you see it? **Goal:** 10; It is switched around. I moved the 2 beads down this time to the 8 on the bottom to get 10.



$$2 + 8 = 10$$

## Strategies Taught

- Subitizing
- Counting On and Counting Back
- Use Five/Use Ten
- Use Doubles
- Get to Ten

## Manipulatives included

- Demonstration Rekenrek
- Number Path Pocket Chart
- Picture/Dot Subitizing Cards
- Five/Ten/Double Ten-Frame Cards

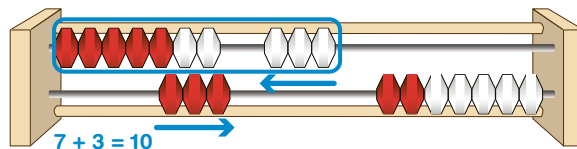


# Math Talks

$$7 + 3$$

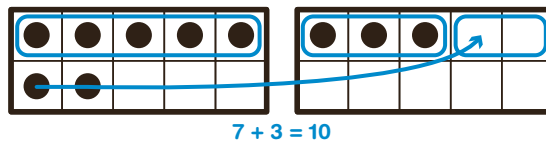
## Get to Ten

10; I take 3 and put it with the 7 to get 10.



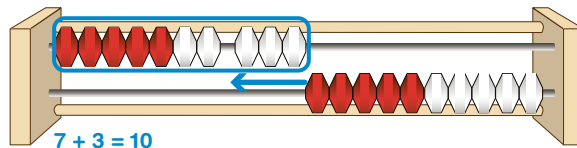
## Use the Five/Ten Structure

10; I think about 7 as 5 and 2. I know that 2 and 3 is 5. I add the 5 and 5 together to get 10.



## Use the Five/Ten Structure

10; I know 7 is 3 less than 10, so  $7 + 3$  is 10.



## Counting On

10; I start with 7 and count on 3 more, so 7, 8, 9, 10.

## Teacher Notes

Start the Math Talk by writing  $7 + 3$  and put it in context: Grace had 7 toys. She got 3 more for her birthday. How many toys does Grace have? Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear

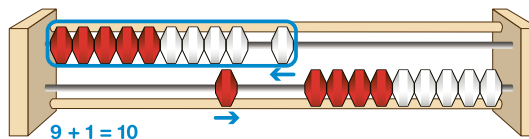
for all students to access. Write any equations that represent the strategy.

**Facilitating Questions:** **1.** Can you find two strategies that are similar? How are they the same? **2.** Are there any strategies that are more efficient than another? Why? **3.** After observing other strategies, did you revise your thinking? How?

$$9 + 1$$

Show 9 on top, 1 on the bottom for 3 seconds

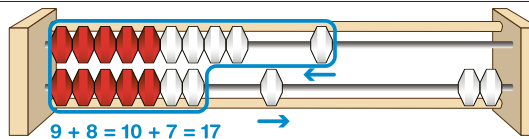
**Teacher:** How many beads? How did you see them? **Goal:** 10; I noticed there were 9 beads on top and 1 bead on the bottom. If the one bead on the bottom goes to the top it makes 10.



$$9 + 8$$

Show 9 on top, 8 on the bottom for 3 seconds

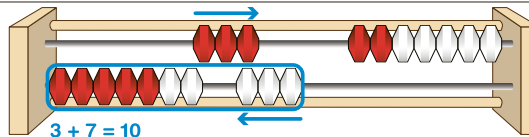
**Teacher:** How many beads? How did you see them? **Goal:** 17; There are 9 beads on top and 8 on the bottom. I moved one of the 8 beads to the top to make 10, so  $10 + 7 = 17$ .



$$3 + 7$$

Show 3 on top, 7 on the bottom for 3 seconds

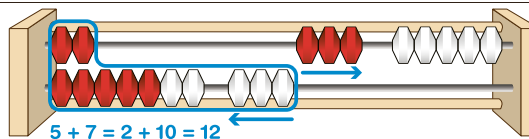
**Teacher:** How many beads? How did you see them? **Goal:** 10; I noticed 3 beads on top and 7 beads on the bottom. I moved the 3 beads on top to the bottom to make 10.



$$5 + 7$$

Show 5 on top, 7 on the bottom for 3 seconds

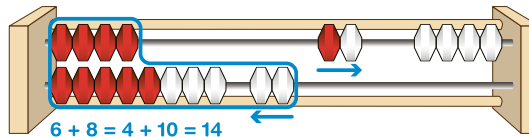
**Teacher:** How many beads? How did you see them? **Goal:** 12; I slid 3 beads from the top and put it on the bottom to make 10. Now there are 2 on top and 10 on the bottom, which is 12.



$$6 + 8$$

Show 6 on top, 8 on the bottom for 3 seconds

**Teacher:** How many beads? How did you see them? **Goal:** 14; I slid 2 beads from the 6 on top to make 10 on the bottom, so 4 beads on top and 10 on the bottom is 14.



## Strategies Taught

- Subitizing
- Counting On and Counting Back
- Use Five/Use Ten
- Use Doubles
- Get to Ten
- Use Ten and Adjust
- Use Know Facts

## Manipulatives included

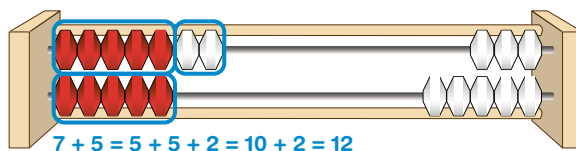
- Demonstration Rekenrek
- Number Path Pocket Chart
- Picture/Dot Subitizing Cards
- Five/Ten/Double Ten-Frame Cards
- Demonstration Open Number Line Poster

# Math Talks

## 7 + 5

### Use the Five/Ten Structure

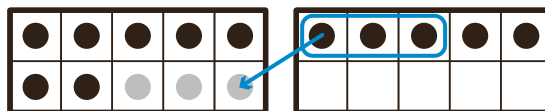
12; I think about the 7 as 5 and 2. I know 5 and 5 is 10, and 2 more is 12.



$$7 + 5 = 5 + 5 + 2 = 10 + 2 = 12$$

### Get to Ten

12; I take 3 from the 5 and give it to the 7 to make 10. That leaves 2 from the 5, so  $10 + 2 = 12$ .



$$7 + 5 = 10 + 2 = 12$$

### Use Known Facts

12; I make it into a double. I take 1 from the 7 and give it to the 5, which makes  $6 + 6$ .

$$\begin{array}{r} 7 + 5 \\ -1 + 1 \\ \hline 6 + 6 = 12 \end{array}$$

$$7 + 5 = 6 + 6 = 12$$

### Counting On

12; I hold 7 in my head and count on: 8, 9, 10, 11, 12.

### Teacher Notes

Start the math talk by writing  $7 + 5$ . Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represents the strategy.

**Facilitating Questions:** **1.** Can you find two strategies that are similar? How are they the same? **2.** Are there any strategies that are more efficient than the others? Why? **3.** After observing other strategies, did you revise your thinking? How?



## Get to a Friendly Number

55

<b>99 + 1</b> Write problem	<b>Teacher:</b> What is $99 + 1$ ? <b>Goal:</b> 100; I know 1 more than 99 is 100.	 $99 + 1 = 100$
<b>99 + 11</b> Write problem	<b>Teacher:</b> What is $99 + 11$ ? <b>Goal:</b> 110; I know $99 + 1$ is 100 and 10 more is 110.	 $99 + 11 = 110$
<b>99 + 17</b> Write problem	<b>Teacher:</b> What is $99 + 17$ ? <b>Goal:</b> 116; I know $99 + 1$ is 100 and I have 16 more to go, which is 116.	 $99 + 17 = 116$
<b>68 + 32</b> Write problem	<b>Teacher:</b> What is $68 + 32$ ? <b>Goal:</b> 100; I start at 68 and jump 2 to get to 70. Then I see I have 30 left, and I know 70 and 30 is 100.	 $68 + 32 = 100$
<b>68 + 37</b> Write problem	<b>Teacher:</b> What is $68 + 37$ ? <b>Goal:</b> 105; I know $68 + 2$ is 70. Then I add 30 to get to 100, but I have a jump of 5 still to go, which gets me to 105.	 $68 + 37 = 105$



## Strategies Taught

- Subitizing
- Counting On and Counting Back
- Use Five/Use Ten
- Use Doubles
- Get to Ten
- Use Ten and Adjust
- Use Know Facts
- Splitting

## Manipulatives included

- Demonstration Rekenrek
- Five/Ten Double Ten-Frame Cards
- Magnetic Demonstration 120-Bead Rekenrek Line
- Demonstration Open Number Line Poster



## 68 + 29

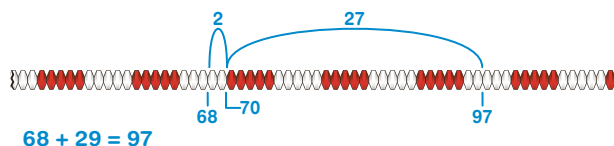
### Give and Take

97; I notice 29 is really close to 30. I take 1 from 68 and give it to 29. Then I have a new problem  $67 + 30$ . I can do this problem in my head. It is 97.

$$\begin{array}{r} 68 + 29 \\ -1 \quad +1 \\ \hline 67 + 30 = 97 \end{array}$$

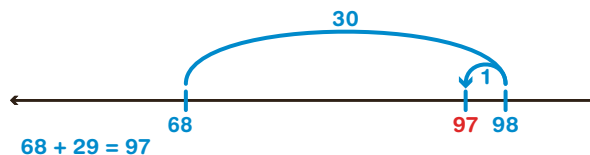
### Get to a Friendly Number

97; I start at 68 and jump 2 to get to 70. I still have 27 to jump. I know 70 and 27 is 97.



### Over and Adjust

97; I notice 29 is close to 30. I start with 68 and jump 30 to get to 98. I have to jump back 1, because 30 is 1 more than 29. I get to 97.



### Add a Friendly Number

97; I start at 68 and jump 20 to get to 88. I jump 2 to get to 90. Then I jump 7 more to get to 97.






### Teacher Notes

Start the Math Talk by writing  $68 + 29$ . Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represent the strategy.

**Facilitating Questions:** **1.** Can you find two strategies that are similar? How are they the same? **2.** Are there any strategies that are more efficient than another? Why? **3.** After observing other strategies, did you revise your thinking? How?

# Number Strings

**Use Ten Times**

<b><math>10 \times 4</math></b> Show card P-K	<i>Teacher: How many buttons? How do you know? Goal: 40; I see 10 rows of 4, and I know <math>10 \times 4</math> is 40, because I can think of it as 4 tens.</i>	 $10 \times 4 = 40$
<b><math>9 \times 4</math></b> Show card P-L	<i>Teacher: How many buttons with yellow thread? Goal: 36; I see 10 rows of 4, but one row has black thread. I know <math>10 \times 4</math> is 40, so <math>9 \times 4</math> is 36, because it is 4 less.</i>	 $9 \times 4 = (10 \times 4) - (1 \times 4) = 40 - 4 = 36$
<b><math>10 \times 6</math></b> Show card P-M	<i>Teacher: How many apples? How do you know? Goal: 60; I see 5 baskets with 6 apples in each on top, and I know that is 30. The same is on the bottom, so I double it to get 60.</i>	 $10 \times 6 = 60$
<b><math>9 \times 6</math></b> Show card P-N	<i>Teacher: How many apples? How do you know? Goal: 54; I still see 10 baskets, but 1 basket is missing apples. I know <math>10 \times 6</math> is 60, and one group less of 6 is 54.</i>	 $9 \times 6 = (10 \times 6) - (1 \times 6) = 60 - 6 = 54$
<b><math>9 \times 8</math></b> Show card P-O	<i>Teacher: How many muffins? How do you know? Goal: 72; I see 10 pans with 8 muffins in each, but one pan is missing muffins. <math>10 \times 8</math> is 80, and one group less of 8 is 72.</i>	 $9 \times 8 = (10 \times 8) - (1 \times 8) = 80 - 8 = 72$

## Strategies Taught

- Splitting
- Use a Friendly Number
- Get to a Friendly Number
- Give and Take
- Over and Adjust
- Find the Distance
- Keep the Same Distance
- Double

## Manipulatives included

- Flexitable Grid Arrays
- Picture/Dot Multiplication Cards
- Magnetic Demonstration 120-Bead Rekenrek Line
- Demonstration Open Number Line Poster
- Ratio Table Poster



# Math Talks

## 4 x 9

### Double

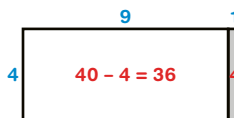
36; I know when I multiply by 4 I can use double, double. I double 9 to get 18 and double 18 to get 36.



$$4 \times 9 = (2 \times 2) \times 9 = 2 \times (2 \times 9) = 2 \times 18 = 36$$

### Use Ten Times

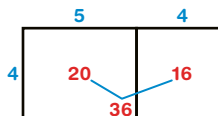
36; I think about 9 as 10. I know  $4 \times 10$  is 40. Then I have to subtract a group of 4 from 40 to get 36.



$$4 \times 9 = (4 \times 10) - (4 \times 1) = 40 - 4 = 36$$

### Use Five Times

36; I think about 9 being composed of 5 and 4. Then I multiply  $4 \times 5$  to get 20, and  $4 \times 4$  to get 16. So, 20 and 16 is 36.



$$4 \times 9 = 4 \times (5 + 4) = (4 \times 5) + (4 \times 4) = 20 + 16 = 36$$

### Double and Halve

36; I double 9 to get 18 and halve 4 to get 2. Then I solve the problem  $18 \times 2$ . I know when I multiply by 2, I just have to double the other factor. Double 18 is 36.

### Teacher Notes

Start the Math Talk by writing  $4 \times 9$ . Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represent the strategy.

**Facilitating Questions:** **1.** Can you find two strategies that are similar? How are they the same? **2.** Are there any strategies that are more efficient than another? Why? **3.** After observing other strategies, did you revise your thinking? How?

<b>7 x 40</b> Write problem	<b>Teacher:</b> What is 7 x 40? <b>Goal:</b> 280; I think about 40 as 4 x 10. I multiply 7 x 4 to get 28. Then I multiply by 10 to get 280.	$7 \times 40 = 280$
<b>7 x 39</b> Write problem	<b>Teacher:</b> What is 7 x 39? <b>Goal:</b> 273; Since 7 x 40 is the same as 40 sevens, 39 sevens is 1 seven less than 40 sevens, so 280 - 7 is 273.	$7 \times 39 = 273$
<b>7 x 400</b> Write problem	<b>Teacher:</b> What is 7 x 400? <b>Goal:</b> 2,800; I know that 7 x 4 is 28. Then I have to multiply it by 100 to get 2,800.	$7 \times 400 = 2,800$
<b>7 x 399</b> Write problem	<b>Teacher:</b> What is 7 x 399? <b>Goal:</b> 2,793; Since 7 x 400 is the same as 400 sevens, 399 sevens is 1 seven less than 400 sevens, so 2,800 - 7 is 2,793.	$7 \times 399 = 2,793$
<b>6 x 299</b> Write problem	<b>Teacher:</b> What is 6 x 299? <b>Goal:</b> 1,794; I know 6 x 300 is 1,800. I only need to multiply 6 x 299, so I subtract 6 from 1,800 to get 1,794.	$6 \times 299 = 1,794$

## Strategies Taught

- 3-Digit Addition and Subtraction
- 4-Digit Addition and Subtraction
- Decimal Addition and Subtraction
- Basic Multiplication-Doubles x2, Doubles plus Another Group, x3, Double Double x4, Double Double Double x8, x5, x6, x7, x9, x10
- Multiplication 2-Digit by 2-Digit
- Basic Division Facts
- Division with Remainders

## Manipulatives included

- Picture/Dot Multiplication Cards
- Flexitable Grid Arrays
- Demonstration Open Number Line Poster
- Ratio Table Poster

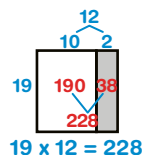




## 19 x 12

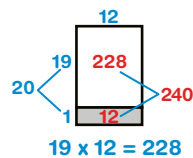
### Use Partial Products

228; I think about 12 as 10 and 2. I multiply  $19 \times 10$  to get 190 and  $19 \times 2$  to get 38. So,  $190 + 38 = 228$ .



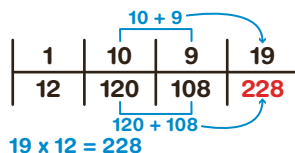
### Use Partial Products

228; I think about 19 as 20. I multiply  $20 \times 12$  to get 240. Then I subtract a group of 12 to get to 228.



### Use Partial Products

228; I multiply  $10 \times 12$  to get 120 and  $9 \times 12$  to get 108. So, 120 and 108 is 228.



### Double and Halve

228; I double 19 to get 38 and halve 12 to get 6. Then I double 38 to get 76 and halve 6 to get 3. Then I solve the problem  $76 \times 3$ , which is 228.

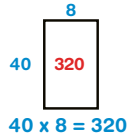
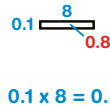
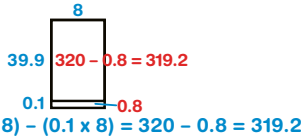
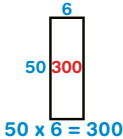
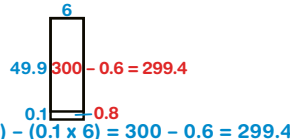
### Teacher Notes

Start the Math Talk by writing  $19 \times 12$ . Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represents the strategy.

**Facilitating Questions:** **1.** Can you find two strategies that are similar? How are they the same? **2.** Are there any strategies that are more efficient than the others? Why? **3.** After observing other strategies, did you revise your thinking? How?

## Use Partial Products

77

<p><b>40 x 8</b></p> <p>Write problem</p>	<p><b>Teacher:</b> What is 40 x 8? <b>Goal:</b> 320; I know 4 x 8 is 32. Then I multiply 32 x 10 to get 320.</p>	 <p><math>40 \times 8 = 320</math></p>
<p><b>0.1 x 8</b></p> <p>Write problem</p>	<p><b>Teacher:</b> What is 0.1 x 8? <b>Goal:</b> 0.8; I think about 0.1 x 8 as 1/10 of 8. It is 0.8.</p>	 <p><math>0.1 \times 8 = 0.8</math></p>
<p><b>39.9 x 8</b></p> <p>Write problem</p>	<p><b>Teacher:</b> What is 39.9 x 8? <b>Goal:</b> 319.2; I think about 39.9 as 40. I know 40 x 8 is 320, but this is one tenth too many. I have to subtract 0.8 from 320 to get 319.2.</p>	 <p><math>39.9 \times 8 = (40 \times 8) - (0.1 \times 8) = 320 - 0.8 = 319.2</math></p>
<p><b>50 x 6</b></p> <p>Write problem</p>	<p><b>Teacher:</b> What is 50 x 6? <b>Goal:</b> 300; I know 5 x 6 is 30. Then I multiply 30 x 10 to get 300.</p>	 <p><math>50 \times 6 = 300</math></p>
<p><b>49.9 x 6</b></p> <p>Write problem</p>	<p><b>Teacher:</b> What is 49.9 x 6? <b>Goal:</b> 299.4; I think about 49.9 as 50. I know 50 x 6 is 300, but this is one tenth too many. I subtract 0.6 from 300 to get 299.4.</p>	 <p><math>49.9 \times 6 = (50 \times 6) - (0.1 \times 6) = 300 - 0.6 = 299.4</math></p>

## Strategies Taught

- Splitting
- Use a Friendly Number
- Get to a Friendly Number
- Give and Take
- Over and Adjust
- Find the Distance
- Keep the Same Distance
- Double

## Manipulatives included

- Magnetic Demonstration Fraction Circles
- Magnetic Demonstration Two Color Counters
- Magnetic Demonstration Cuisenaire® Rods
- Demonstration Open Number Line Poster
- Ratio Table Poster



## 6 x 79.9

### Double and Halve

479.4; I halve 6 to get 3 and double 79.9 to get 159.8. Then I can solve an easier problem. I double 159.8 to get 319.6 and add another group of 159.8 to get 479.4.

$$\div 2 \left( \begin{array}{l} 6 \times 79.9 \\ 3 \times 159.8 \end{array} \right) \times 2 = 479.4$$

### Use Partial Products

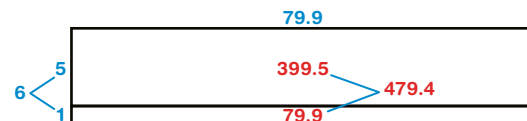
479.4; I think about 79.9 as 80. I multiply  $6 \times 80$  to get 480. Then I have to subtract six-tenths from 480 to get 479.4.



$$6 \times 79.9 = 6 \times (80 - 0.1) = (6 \times 80) - (6 \times 0.1) = 480 - 0.6 = 479.4$$

### Use Partial Products

479.4; I think about 6 as 5 and 1. I multiply  $5 \times 79.9$  to get 399.5. Then I multiply  $1 \times 79.9$ . So,  $399.5 + 79.9 = 479.4$ .



$$6 \times 79.9 = (5 + 1) \times 79.9 = (5 \times 79.9) + (1 \times 79.9) = 399.5 + 79.9 = 479.4$$

### Use Partial Products

479.4; I figure out that  $70 \times 6$  is 420. Next, I multiply  $10 \times 6$  to get 60 and use this to help me figure out  $9 \times 6$ , which is 54. Then I figure out that  $0.9 \times 6$  is 5.4, because it is one-tenth of 54. Last, I add up  $420 + 54 + 5.4$  to get 479.4.

### Teacher Notes

Start the Math Talk by writing  $6 \times 79.9$ . Give students time to mentally solve the problem. Write all answers on the board and then have the students explain their thinking. Model student thinking using a manipulative that will help make the strategy clear for all students to access. Write any equations that represent the strategy.

**Facilitating Questions:** **1.** Can you find two strategies that are similar? How are they the same? **2.** Are there any strategies that are more efficient than another? Why? **3.** After observing other strategies, did you revise your thinking? How?



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