PENTACUBES II

GEOMETRY • LOGIC

- Spatial visualization
- Congruence
- Transformational geometry

Getting Ready

What You'll Need

Snap Cubes, 100 per group

Isometric dot paper, page 90 (optional)

Overview

Children search for different ways to arrange five Snap Cubes so that each arrangement, no matter how it is placed on a table, has at least one cube that does not touch the table. In this activity, children have the opportunity to:

- develop their spatial sense
- see how reflection affects solid geometric figures

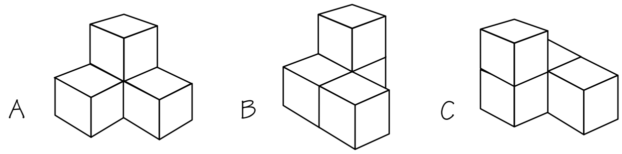
The Activity

This lesson is an extension of Pentacubes I *(page 50). However, the outcome of this lesson does not depend on the outcomes of* Pentacubes I, *and could easily be done independently of that lesson.*

Use Snap Cubes of the same color so children concentrate on the figure, not the color, when deciding whether two figures are the same or not.

Introducing

- Ask children to use their Snap Cubes to determine if it is possible to make a figure in which all four Snap Cubes do not touch the tabletop.
- Have volunteers hold up their different arrangements and talk about them. Three figures are possible.



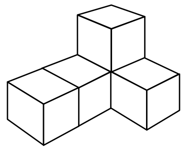
- Some children may initially think that figures B and C are the same. Show them that no matter how you try to rotate or flip these two figures, you cannot get C to look exactly like B.

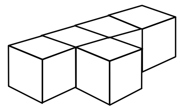
On Their Own

How many different figures can you make with 5 Snap Cubes so that at least 1 of the 5 cubes does not touch the tabletop?

- Work with a group. Each of you makes a figure with 5 cubes. Make the figures so that no matter how you place them on a table at least 1 cube does *not* touch the table.

Okay





Not okay

- Compare your figures.
    - If they are all different, keep them all.
    - If some of them are the same, keep only 1 of each different figure.
- Continue to build and compare figures, keeping only those figures that are different from what you have already built.
- Continue building figures until you can't make any more that are different from what you already have.

The Bigger Picture

Thinking and Sharing

Have volunteers, one at a time, display a figure they found. Call on children to bring up any figures that match the ones on display and group like figures together. Continue until all the different figures found are on display.

Use prompts such as these to promote class discussion:

- What was easy about making the figures? What was hard?
- How are these two figures different from each other?
- What do you notice about these two figures? (Hold up two figures that are mirror images.)
- Did you find any other pairs that were mirror images of each other?
- Did you have a strategy for finding the different figures? If so, what was it?
- How many figures did you find? Do you think you found them all? Why do you think so?

Writing

Have children choose two of the pentacube figures and describe how the two shapes are alike and how they are different.

Extending the Activity

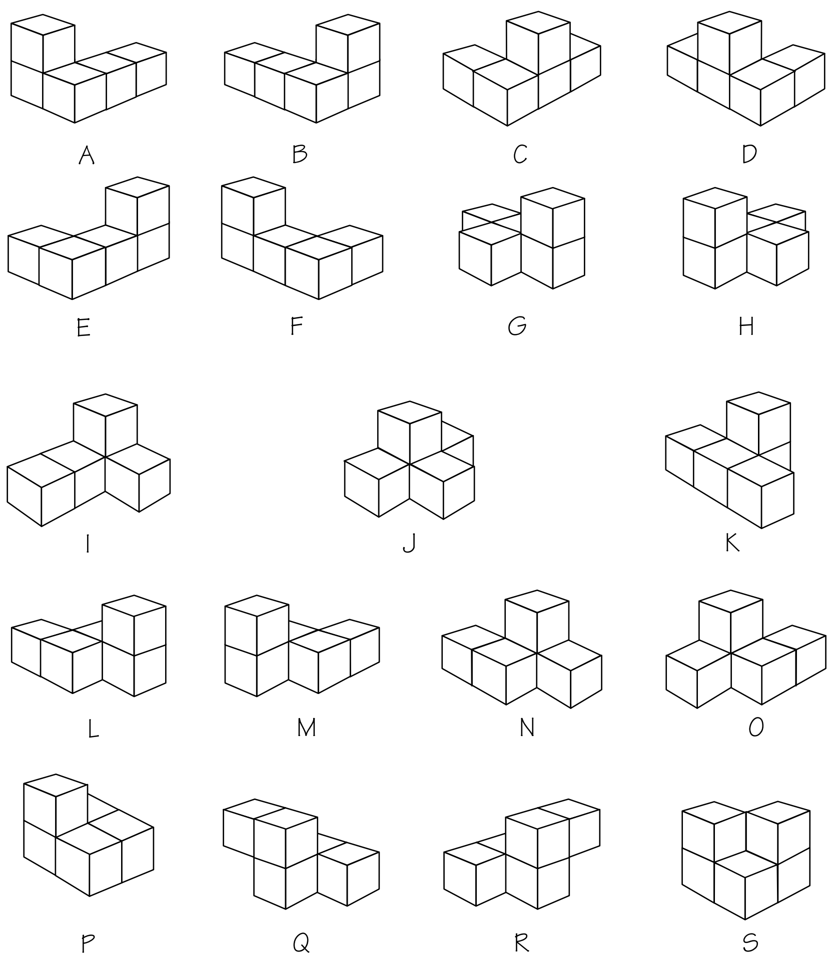
1. If children are familiar with isometric dot paper, have them record their figures on it.

Teacher Talk

Where's the Mathematics?

Children have an easy time building a variety of pentacube figures. The challenge arises in determining whether the shape is unique or not. Many children will need to handle the models physically, turning, flipping, and rotating them before they are convinced of their uniqueness or of their congruency to a shape already made. Studying the models this closely will provide valuable background for later, more formal studies in solid geometry.

The nineteen possible shapes are shown here. Do not expect that all groups will discover all of them.



2. Ask children to select one of the pentacube figures and draw it on isometric dot paper. Have them return the pentacube figure to a pile of figures made by several other children. Then have children exchange isometric drawings to see if they can match the drawings with the actual figures.

Of the nineteen shapes, there are seven pairs of mirror images: A and B, C and D, E and F, G and H, L and M, N and O, and Q and R. No matter how these pairs are turned, flipped, or rotated, it will be impossible to make the two shapes look identical. They will look like the reflection one would see in a mirror.

Initially, many children will approach the task of finding all possible figures in a very random manner. When they begin to wonder whether they have exhausted all the possibilities, they may try to develop a “system”. For example, they may start with four cubes arranged as an “L” on the tabletop and then move the fifth cube to find all of its possible locations that will create a new figure that satisfies the condition that all five cubes may not touch the tabletop. Then they may explore all the figures possible with only three cubes in the row and finally with only two in a row. Other children may make one of the figures and then move only cube to find the next shape and then move only one cube in that shape to find the next one and so forth. For children who have discovered the mirror property that many of the figures possess, they may make a figure and then see if it has a unique mirror partner to go with it.

ISOMETRIC DOT PAPER–1-CM