# Operations and Algebraic Thinking

Previously, students applied **operations** (addition, subtraction, multiplication, and division) as they worked with whole numbers. In fifth grade, they develop fluency with addition and subtraction of whole numbers and learn about multiplication and division of fractions.

Students in fifth grade develop flexibility in writing and interpreting numerical expressions. They work with expressions that include grouping symbols as they refine their use of the order of operations. Students work with whole numbers in the beginning and then progress to working with decimals and fractions.

**Algebraic thinking** has two components–the use of mathematical thinking tools and the study of fundamental algebraic ideas. Mathematical thinking tools include the analytical habits of mind (e.g., problem solving and reasoning skills). Algebraic ideas consist of the content domain in which mathematical thinking tools are applied. Fifth graders use their algebraic thinking skills in this domain to analyze patterns and relationships. As fourth graders, they generated numerical patterns when given one rule. Now they generate patterns given two rules, and they graph the output and make sense of the relationship it expresses.

**The Grade 5 Common Core State Standards for Operations and Algebraic Thinking specify that students should–**

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

The following hands-on activities will help students explore the concepts of operations and algebraic thinking in meaningful ways. Using concrete models, students will more readily formulate important generalizations about numerical expressions and patterns. And by posing structured questions and encouraging discussion, teachers can help students discover and build their own understanding.

# Operations and Algebraic Thinking

## Contents

LESSON 1

## Objective

Use the order of operations to simplify expressions.

## Common Core State Standards

- **5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Operations and Algebraic Thinking

# Order of Operations

The order of operations makes the language of mathematics more universal. Knowing these rules helps students to communicate more accurately as they gain fluency in manipulating symbolic relationships. The sequence for the order of operations is—

1. Calculate inside parentheses.
2. Multiply and divide in order, from left to right.
3. Add and subtract in order, from left to right.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *Why do the models have different solutions?*
- **Ask:** *Why is it necessary to follow the order of operations when simplifying an expression?*
- Write *5 + 2 × 6 – 8* on the board. **Ask:** *How does the value of this expression differ when using the order of operations versus solving from left to right? Explain.*

## Solve It

Reread the problem with students. Have students draw a picture of the solution to the problem. Then have them write a short paragraph explaining how to use the order of operations to solve the problem.

## More Ideas

For other ways to teach the order of operations—

- Write *20 – 12 ÷ 4* on the board. Have students use Snap Cubes® to model the expression and compute using the order of operations. Repeat with other expressions.
- Use Two-Color Counters to model the problem 5 – 3 + 6 ÷ 2 = 4. Have students use the counters to help them decide where parentheses should be inserted into the equation.

## Formative Assessment

Have students try the following problem.

*Simplify: 20 – 8 ÷ 4 × 2.*

**A.** 1.5 **B.** 6 **C.** 9 **D.** 16

## Try It! 20 minutes | Groups of 4

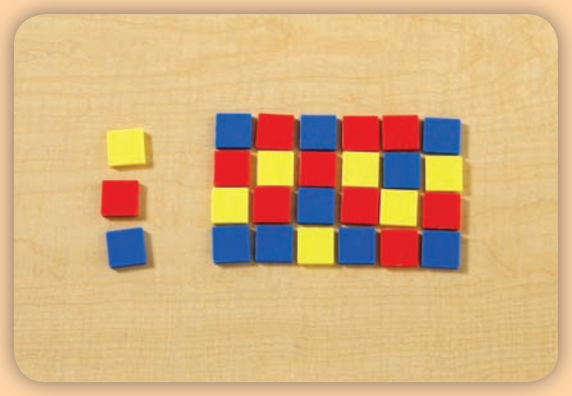
Here is a problem about the order of operations.

*Jay brought some juice boxes to soccer practice to share with his teammates. He had 3 single boxes and 4 multi-packs. There are 6 single boxes in each multi-pack. To determine how many boxes of juice Jay brought to practice, evaluate $3 + 4 \times 6$.*

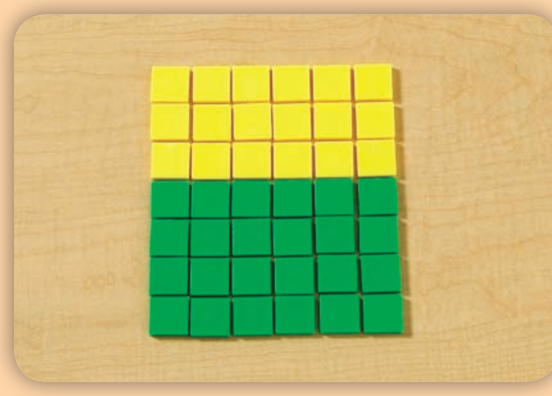
Introduce the problem. Then have students do the activity to solve the problem. Distribute Color Tiles, paper, and pencils to students. Explain that the order of operations provides rules for simplifying expressions. Discuss the rules.

### Materials

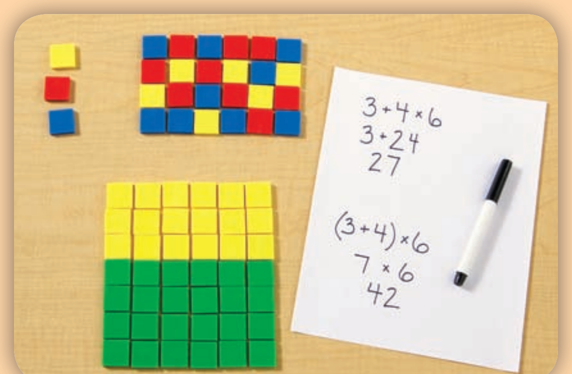
- Color Tiles (100 per group)
- paper (1 sheet per group)
- pencils (1 per group)



**1.** Write *3 + 4* × *6* on the board. Have students start by laying down 3 tiles. Then have students add a 4-by-6 array. **Ask:** *How many tiles are shown in the model?*



**2.** Have students show 3 + 4 using a different color of tile for each addend. Then have them build an array to show this quantity times six. **Ask:** *How many tiles are shown in the model?*




**3. Say:** *You built two models.* **Ask:** *How are they different?* Have students write the expressions to represent the models. **Ask:** *Which model is correct?*

### ⚠ Look Out!

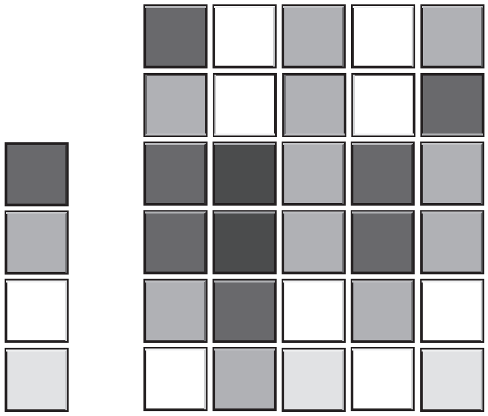
Because we read English from left to right, some students may continue to simplify expressions by performing operations in that order. Suggest that students write the order of operations at the top of their papers and then refer to the steps as they simplify expressions. Some students find a mnemonic, such as *Please My Dear Aunt Sally (Parentheses Multiplication Division Addition Subtraction),* helpful in remembering the order.

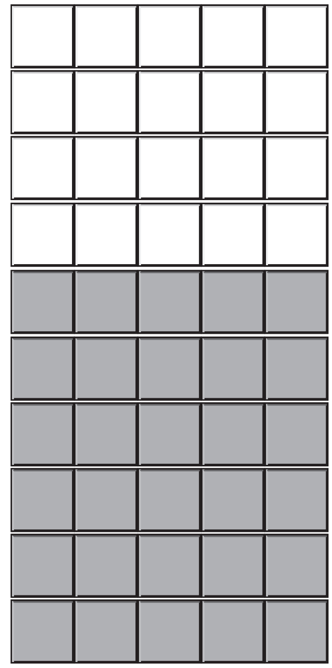
**Answer Key**

**Use Color Tiles to model each expression. Write expressions for the models. Circle the model that shows 4 + (6 × 5) = 34.**

(Check students' work.)

**1.**





4 + 6 × 5 = 34; (4 + 6) × 5 = 50; The model on the left should be circled.

**Using Color Tiles, model the expression using the Order of Operations. Sketch the model and write the answer.**

**2.** 8 + 5 × 3

(Check students' models.)

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**Use the Order of Operations to find each answer.**

**3.** 6 + 4 × 3 18

**4.** 4 × 2 + 10 18

**5.** 9 × 2 ÷ 6 3

**6.** 7 × 5 – 10 25

**7.** 6 ÷ 2 + 4 7

**8.** 3 × 4 + 9 21

## *Answer Key*

**Challenge!** Write the sequence of the Order of Operations. Show an expression where the answer is different when you use the Order of Operations compared to working from left to right. Show an expression where the answer is the same when you use the Order of Operations and when you work left to right. Explain how the second expression works both ways.

Challenge: (Sample) 1. Calculate inside parentheses, 2. Multiply and divide, 3. Add and subtract; The expression $6 + 4 \times 2$ equals 14 when you use the Order of Operations. When you work left to right, it equals 20, which is not the correct answer. The expression $3 \times 5 + 6$ equals 21 using the Order of Operations and when you work left to right. The operations are listed in the correct order for the Order of Operations when you read from left to right.

LESSON 2

## Objective

Evaluate numerical expressions with parentheses and brackets.

## Common Core State Standards

- **5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.



# Using Parentheses and Brackets

In the previous lesson, students investigated the order of operations by working with relatively simple expressions. Some situations call for more extensive use of parentheses and other grouping symbols, such as brackets and braces. In this lesson, students evaluate expressions with these symbols.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *Why do the models have different solutions?*
- **Ask:** *Why do we have to add the numbers inside the brackets, 4 + 6, first? What is the next step? What is the final step?*
- Write *[4(3 + 6)] – 8* on the board. Have students identify which part of the expression to evaluate first, second, and third.

## Solve It

Reread the problem with students. Have students draw a picture of the tables and chairs as a solution to the problem. Have them explain to their partners how their pictures represent the expression.

## More Ideas

For other ways to teach about using parentheses and brackets—

- Write *[3(7 – 5)] + 18* and *[4(2 + 5)] – 18* on the board. Have pairs predict which expression will be greater. Next, have each student use Color Tiles to model one of the expressions and explain their model to their partner. Then have them evaluate which expression is greater and compare their findings to their prediction.
- Write *35 – 4 × 9 – 1 = 3* on the board. Have students use Two-Color Counters to model the expression and write the expression with parentheses and brackets.

## Formative Assessment

Have students try the following problem.

*Simplify: 12 – [2(8 – 5)]*

**A.** 1 **B.** 6 **C.** 18 **D.** 24

## Try It! 20 minutes | Groups of 4

Here is a problem about using parentheses and brackets.

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*There are 4 tables in the library. Each table has 4 red chairs and 6 blue chairs. There are 6 extra chairs in the back of the room. How many chairs are in the library?*

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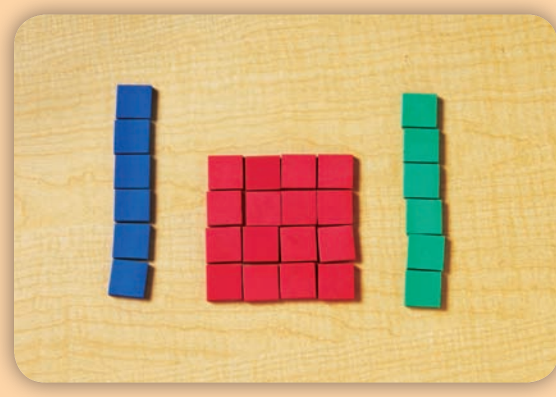
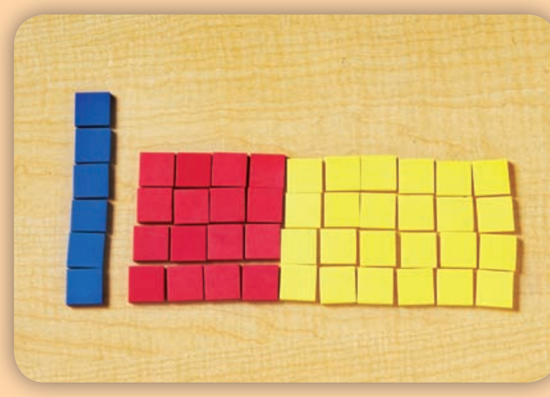
Introduce the problem. Then have students do the activity to solve the problem. Distribute Color Tiles, paper, and pencils to students.

### Materials

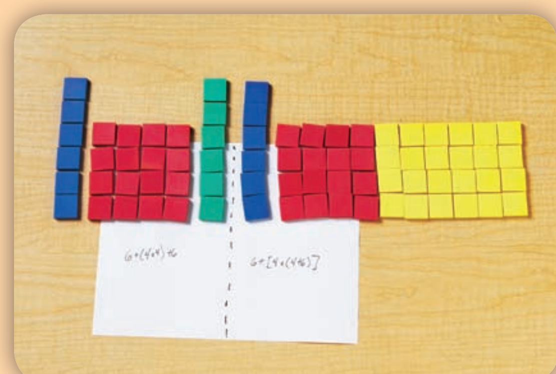
- Color Tiles (100 per group)
- paper
- pencils



**1.** Write *6 + [4 × (4 + 6)]* on the board. Have students start by laying down a column of 6 tiles. Next, have students add a 4-by-4 array. Then have them lay down another column of 6 tiles. **Ask:** *How many tiles are shown in this model?*



**2.** Have students lay down a column of 6 tiles. Next, have students show 4 + 6 in a row using a different color of tile for each addend. Then have them build an array to show 4 times this quantity. **Ask:** *How many tiles are shown in this model?*



**3. Say:** *You built two models.* **Ask:** *How are they different?* Have students write expressions to represent the models. **Ask:** *Which model correctly represents the problem scenario?*

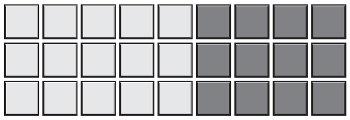
## ⚠ Look Out!

Students might have difficulty simplifying expressions in the proper order. Suggest they write the expression on a piece of paper and use highlighters to mark quantities in brackets one color and quantities in parentheses another color. Tell them to work from inside out.

**Answer Key**

**Use Color Tiles to build the model. Evaluate the expression.** (Check students' work.)

**1.** $3 \times (5 + 4) =$ 27



**Using Color Tiles, model the expression. Sketch the model. Evaluate the expression.**

**2.** $4 \times (6 + 3) =$ 36

**3.** $3 \times [8 - (4 + 2)] =$ 6

(Check students' models.)

**Evaluate each expression.**

**4.** $2 \times (9 + 1) =$ 20

**5.** $8 \times (7 - 2) + 12 =$ 52

**6.** $8 + (3 \times 4) =$ 20

**7.** $3 \times [5 + (8 - 3)] =$ 30

**8.** $5 \times [8 - (9 - 4)] =$ 15

**9.** $4 + [12 - (4 + 2)] =$ 10

## *Answer Key*

**Challenge!** Using one set of parentheses, make the expression $7 + 5 \times 3 + 8$ equal to 44.

Challenge: $(7 + 5) \times 3 + 8$

LESSON 3

## Objective

Identify a numerical pattern and graph it on a coordinate plane.

## Common Core State Standards

- **5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Operations and Algebraic Thinking

# Graphing Number Patterns

Understanding how to generate number patterns and graph them on a coordinate plane is an important foundational skill for later work with linear functions. In this lesson, students will generate number patterns using given rules, form ordered pairs of corresponding terms, graph the pairs, and identify relationships between the terms.

**Try It!** *Perform the Try It! activity on the next page.*

## Talk About It

Discuss the Try It! activity.

- **Ask:** *What pattern does each column of the table show? What is the relationship between columns? Why are the y-values twice the corresponding x-values?*
- **Ask:** *How do you show the ordered pairs on the XY Coordinate Pegboard?*
- **Ask:** *How could we use the grid paper to extend the relationship?*
- **Ask:** *Why is the line on your graph straight? What does that tell you about the number pattern?*

## Solve It

Reread the problem with students. Have students plot the ordered pairs from the function table onto the Centimeter Grid (BLM 6) and draw a line through them. Then have them write two sentences comparing the corresponding terms and explaining why the *y*-terms are twice the *x*-terms.

## More Ideas

For other ways to teach about graphing number patterns—

- Have students use Centimeter Cubes on a grid to plot coordinates from a function table.
- Have students who are ready create different lines on the XY Coordinate Pegboard, and then use the coordinates of the lines to create function tables and determine the number patterns they have created.

## Formative Assessment

Have students try the following problem.

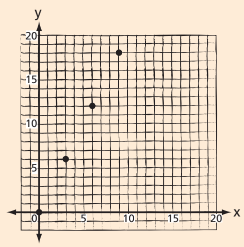
*Determine the relationship between the terms given by the ordered pairs (0, 0), (3, 6), (6, 12), and (9, 18).*

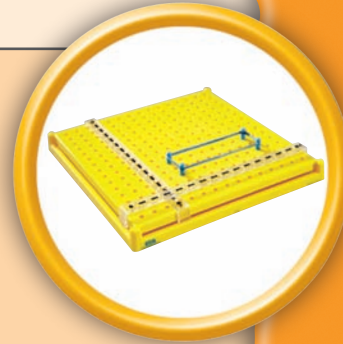
**A.** The *y*-terms are 3 more than the *x*-terms.

**B.** The *x*-terms are 3 more than the *y*-terms.

**C.** The *y*-terms are double the *x*-terms.

**D.** The *y*-terms are four times the *x*-terms.





## Try It!

**15 minutes | Groups of 4**

Here is a problem about graphing number patterns.

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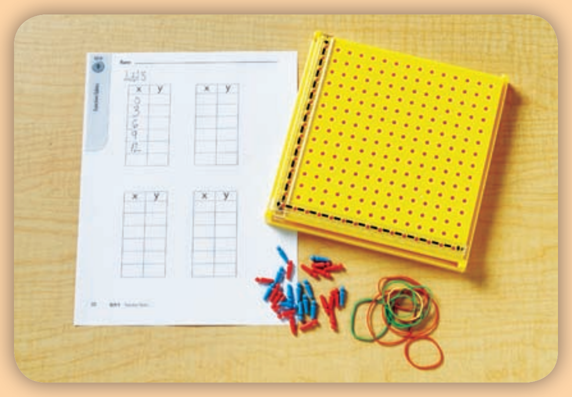
*An older machine can make 3 bike chains each minute. A new machine can make 6 bike chains each minute. How can you graph and describe the relationship between the productivity of the machines?*

---

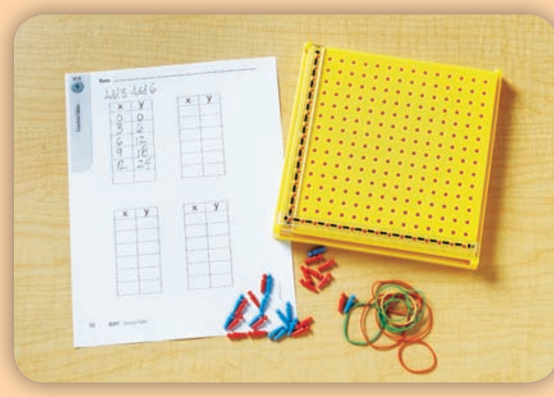
Introduce the problem. Then have students do the activity to solve the problem. Distribute XY Coordinate Pegboards, function tables, and centimeter grids to students.

### Materials

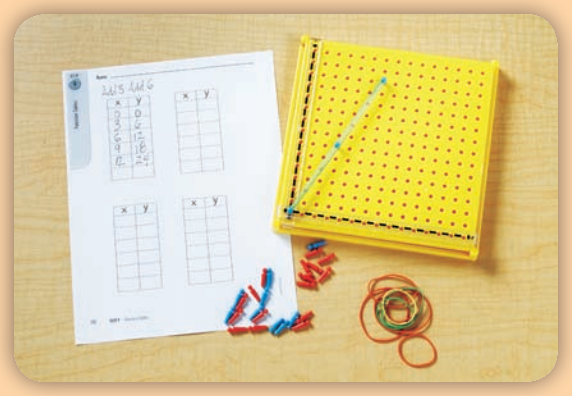
- XY Coordinate Pegboard (1 per group)
- Function Tables (BLM 9)
- Centimeter Grid (BLM 6)



**1. Say:** *Let's make a table. Let* x *represent the output of the older machine and* y *represent the output of the new machine. We can generate each output using a rule.* Elicit that the rule for the older machine is "Add 3" and have students write *Add 3* above the *x*-column. Then have students generate five values for *x*, starting with 0.



**2. Ask:** *What is the rule for the output of the new machine?* Elicit that the rule is "Add 6" and have students write *Add 6* above the *y*-column. Have students generate five values for *y*, starting with 0. **Say:** *The* x *and* y *values form ordered pairs. We can plot the ordered pairs on our XY Coordinate Pegboard and on grid paper.*



**3.** Have students set the axes on the XY Coordinate Pegboard to show the first quadrant of a coordinate plane. Have students plot as many points from the table as they can using blue pegs and add a rubber band to create a line along the points.

### ⚠ Look Out!

Students may be confused by what each axis represents and how to peg values from the function table. Have them write "Older machine" on a sticky note and place it under the *x*-axis. Have them write "Newer machine" on a sticky note and place it to the left of the *y*-axis.

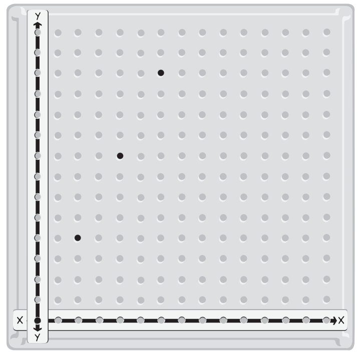
**Answer Key**

**Complete the number pattern for *y*. Use an XY Coordinate Pegboard to plot the ordered pairs.**

(Check students' work.)

**1.**

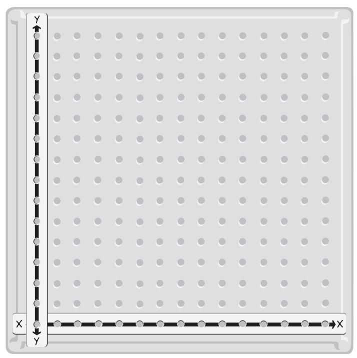
| *x* "Add 2" | *y* "Add 4" |
|---|---|
| 0 | 0 |
| 2 | 4 |
| 4 | 8 |
| 6 | 12 |



**Complete the number pattern for *y*. Use an XY Coordinate Pegboard to plot the ordered pairs. Sketch the graph.**

**2.**

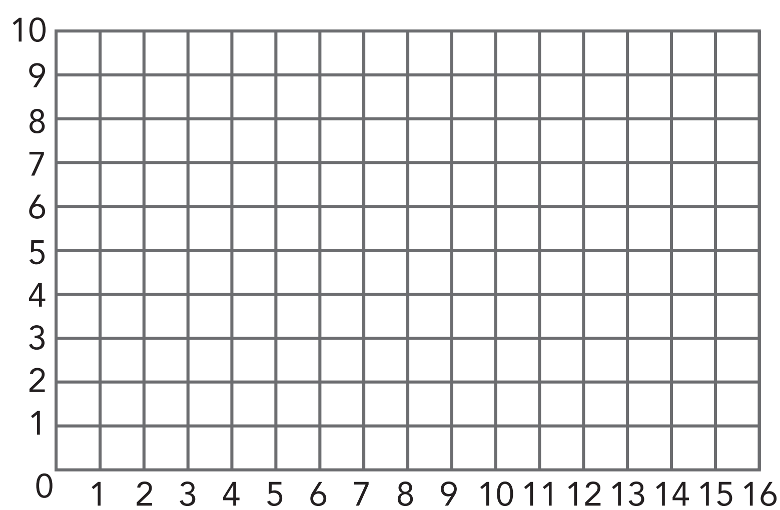
| *x* "Add 1" | *y* "Add 2" |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |



**Complete the number patterns for *x* and *y*. Graph the ordered pairs.**

**3.**

| *x* "Double" | *y* "Double – 1" |
|---|---|
| 2 | 2 |
| 4 | 3 |
| 8 | 5 |
| 16 | 9 |



## *Answer Key*

**Challenge!** Starting with (1, 1), make an *x*-*y* function table using the patterns "double" for the *x*-coordinate and "double + 1" for the *y*-coordinate. Graph the ordered pairs in the work space below or on grid paper and describe the relationship between corresponding terms.

Challenge: $x$ = 1, 2, 4, 8, 16; $y$ = 1, 3, 7, 15, 31; the *y*-terms are one less than the *x*-terms doubled. The *y*-terms grow faster than the *x*-terms because "double + 1" is a larger growth rate than "double."