

PROBLEMS

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that

$$f(x+y) = f(x) + f(y) \quad \text{for all } x, y \in \mathbb{R}.$$

$$f(x) = x \quad \text{for all } x \in \mathbb{Q}.$$

Prove that $f(x) = x$ for all $x \in \mathbb{R}$.

2. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that

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Prove that $f(x) = x$ for all $x \in \mathbb{R}$.