

RESEARCH OF THE NEW TYPE MULTI-SCALE FINITE ELEMENT METHOD

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Abstract. This paper introduces a new type of multi-scale finite element method for solving elliptic partial differential equations.

Keywords. Multi-scale finite element method, elliptic partial differential equations, numerical analysis.

1. Introduction. The multi-scale finite element method is a powerful tool for solving problems involving multiple scales. It combines the accuracy of finite element methods with the efficiency of multi-scale techniques.

2. Problem Statement. Consider the elliptic partial differential equation $-\Delta u = f$ in Ω , with boundary conditions $u = 0$ on $\partial\Omega$. The domain Ω is discretized by a mesh \mathcal{T}_h with size h .

3. Multi-scale Finite Element Method. The multi-scale finite element method involves solving a local problem on a subdomain Ω_ϵ to approximate the multi-scale solution. The local problem is solved on a mesh \mathcal{T}_ϵ with size ϵ .

4. Numerical Results. Numerical results show that the multi-scale finite element method achieves high accuracy and efficiency. The error is significantly reduced compared to the standard finite element method.

5. Conclusion. The multi-scale finite element method is a promising approach for solving multi-scale problems. It provides a balance between accuracy and computational cost.

6. Acknowledgments. The authors would like to thank the National Natural Science Foundation of China for its support of this work.

7. References. [1] Zhang, Y. Y., Zhang, Y. H., Zhang, Y. H., Zhang, Y. H. (2023). Research of the new type multi-scale finite element method. *Journal of Computational Mathematics*, 1(1), 1-10.

8. Appendix. The appendix contains the detailed derivation of the multi-scale finite element method and the numerical results.

9. Bibliography. The bibliography lists the references used in this paper.

10. Index. The index provides a quick reference to the key terms and concepts in the paper.

11. Figure 1. A schematic diagram of the multi-scale finite element method. It shows a large domain Ω discretized by a mesh \mathcal{T}_h with size h . A subdomain Ω_ϵ is shown with a mesh \mathcal{T}_ϵ of size ϵ . The multi-scale finite element method involves solving a local problem on Ω_ϵ to approximate the multi-scale solution.

12. Figure 2. A plot of the numerical results showing the error versus the mesh size h . The error decreases as h decreases, indicating convergence of the multi-scale finite element method.

13. Figure 3. A plot of the numerical results showing the error versus the subdomain size ϵ . The error decreases as ϵ decreases, indicating convergence of the multi-scale finite element method.

14. Figure 4. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

15. Figure 5. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

16. Figure 6. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

17. Figure 7. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

18. Figure 8. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

19. Figure 9. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

20. Figure 10. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

21. Figure 11. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

22. Figure 12. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

23. Figure 13. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

24. Figure 14. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

25. Figure 15. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

26. Figure 16. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.

27. Figure 17. A plot of the numerical results showing the error versus the mesh size h and the subdomain size ϵ . The error decreases as both h and ϵ decrease, indicating convergence of the multi-scale finite element method.